



EECS 483: Compiler Construction

Lecture 17:
Register Allocation Part 2: Graph Coloring and Code Generation

March 19
Winter Semester 2025

Announcements

- Assignment 4 to be released tonight. Due April 4th
- Exam grades to be released next week.

Register Allocation

3 Steps

1. **Liveness analysis:** identify when each variable's value is needed in the program
2. **Conflict analysis:** identify which variables interfere with each other
3. **Graph Coloring:** assign variables to registers so that interfering registers are assigned different registers.
 1. Spilling: if necessary, assign some variables to stack slots

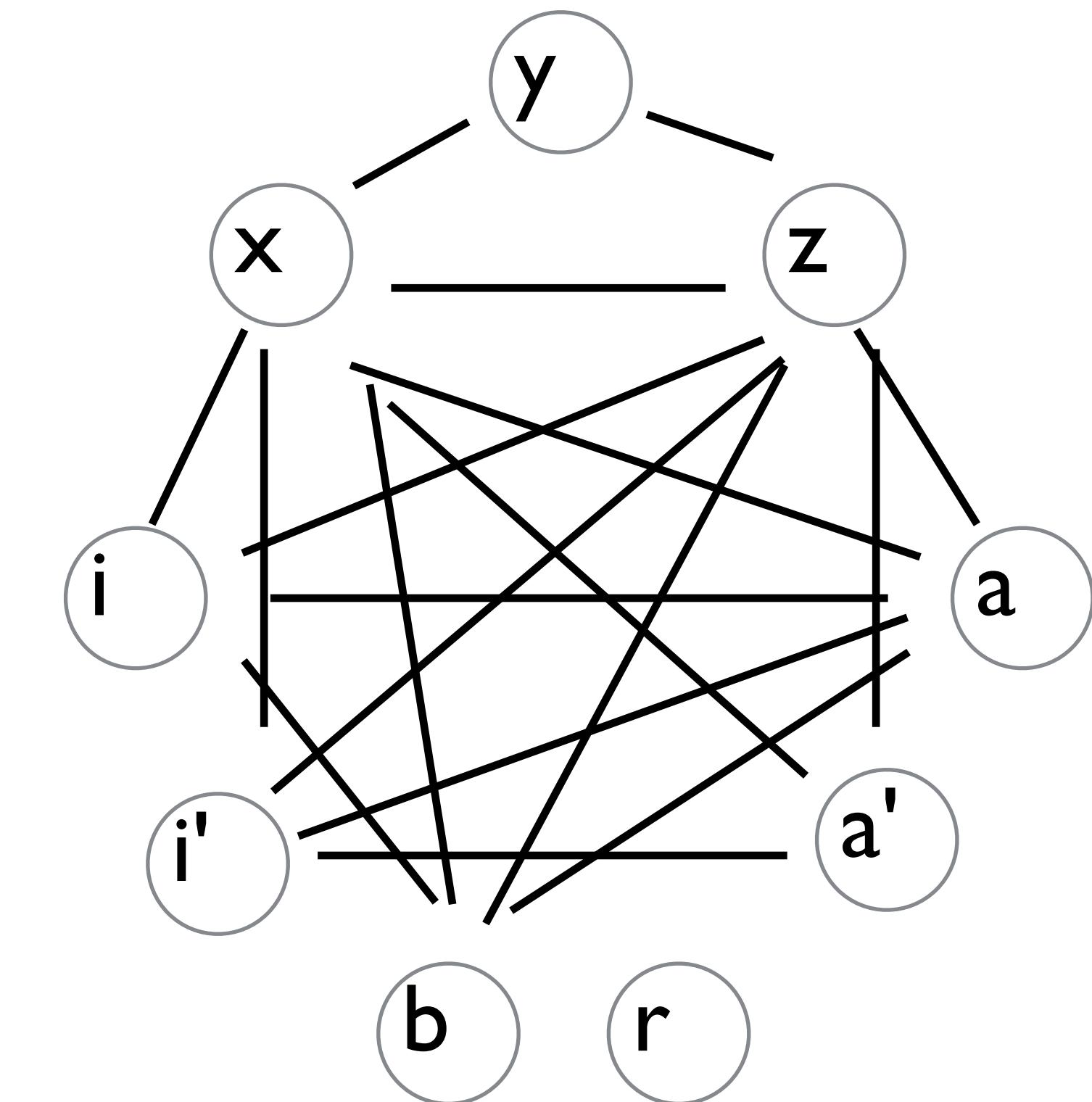
SSA Program

```
f(x,y,z):  
loop(i,a):  
    thn():  
        r = a * z  
        ret r  
    els():  
        i' = i - 1  
        a' = a + x  
        br loop(i', a')  
    b = i == 0  
    cbr b thn() els()  
br loop(y, 0)
```

Liveness Info

1:	{x, z}
2:	{x, y, z}
3:	{a, i, x, z}
4:	{a, b, i, x, z}
5/6:	{a, z}
7:	{r}
8/9:	{a, i, x, z}
10:	{a, i', x, z}
11:	{a', i', x, z}

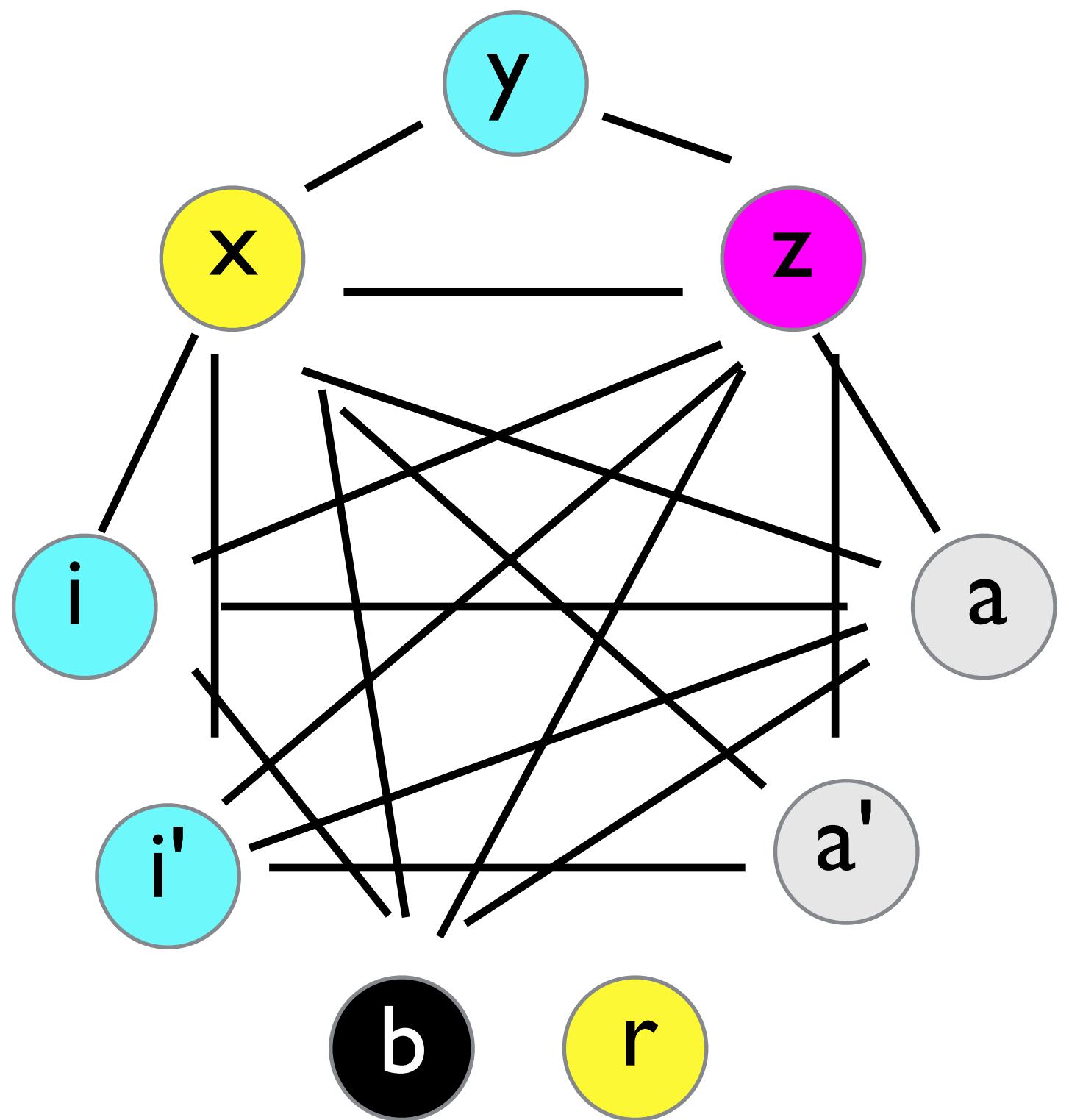
Interference Graph



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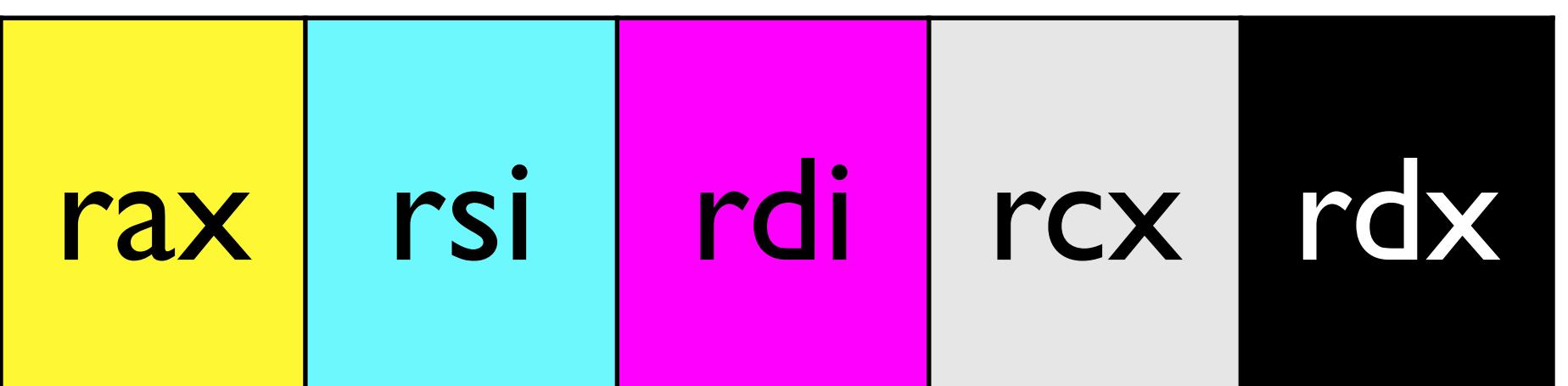
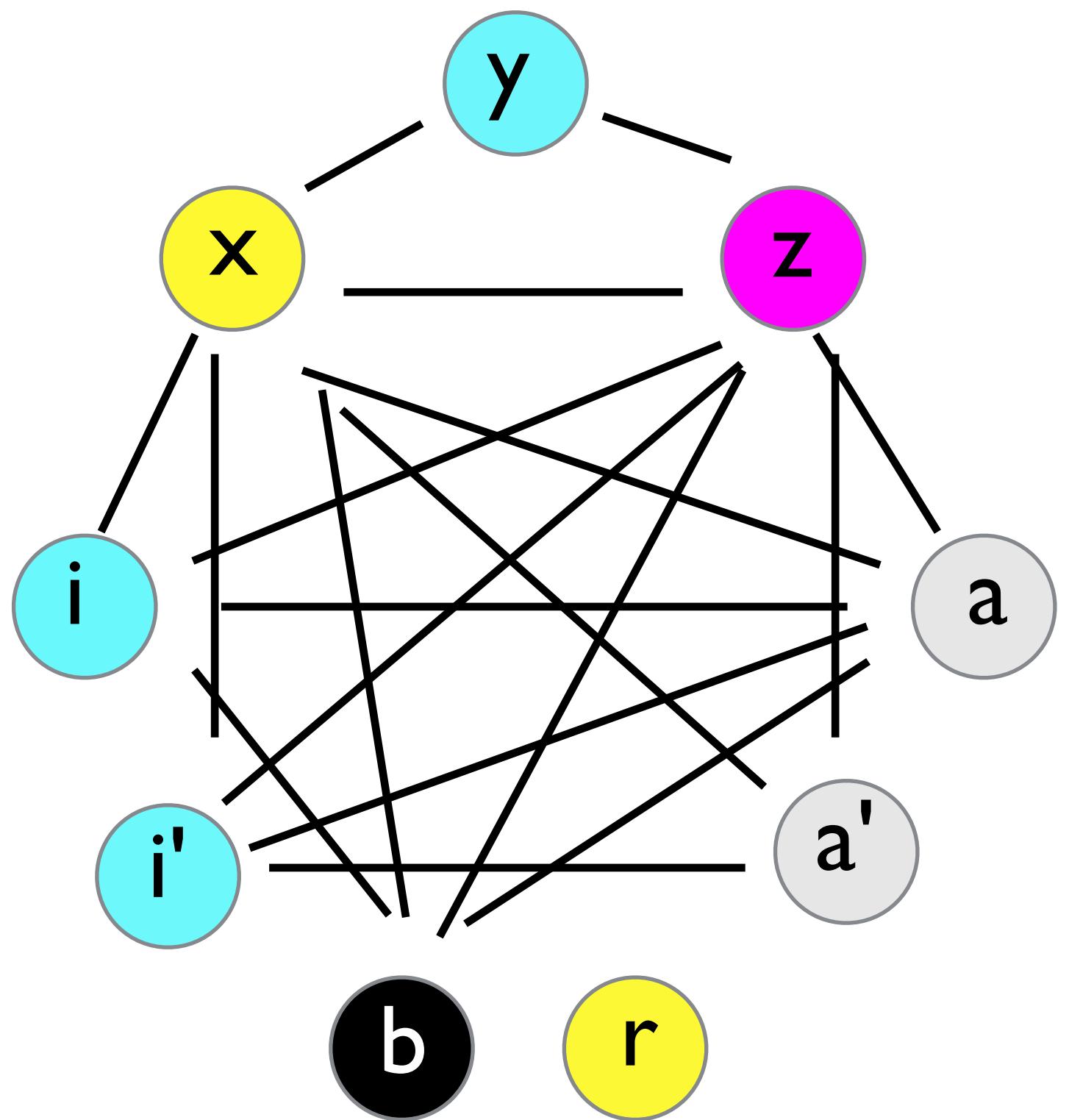
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Interference Graph

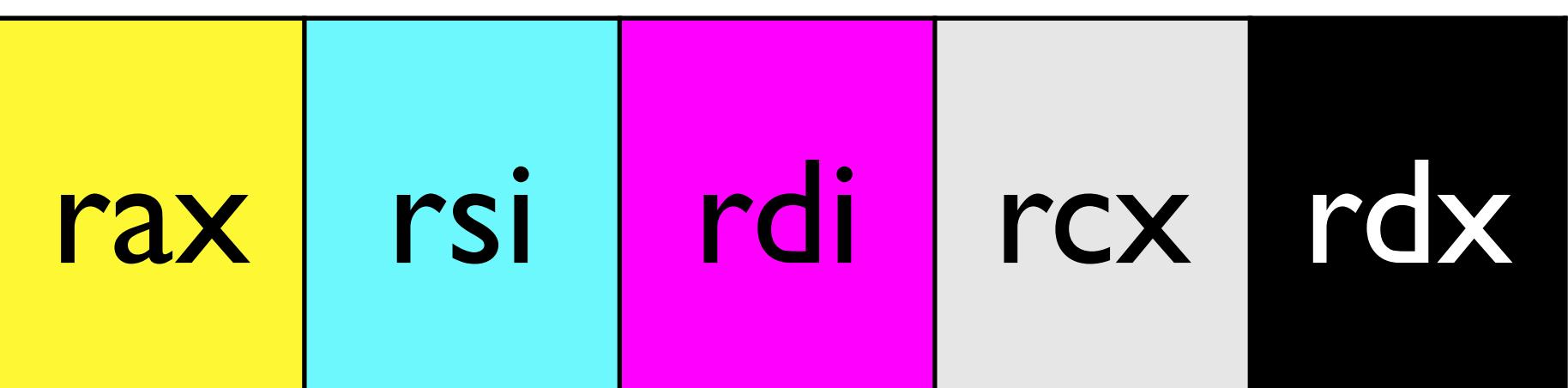
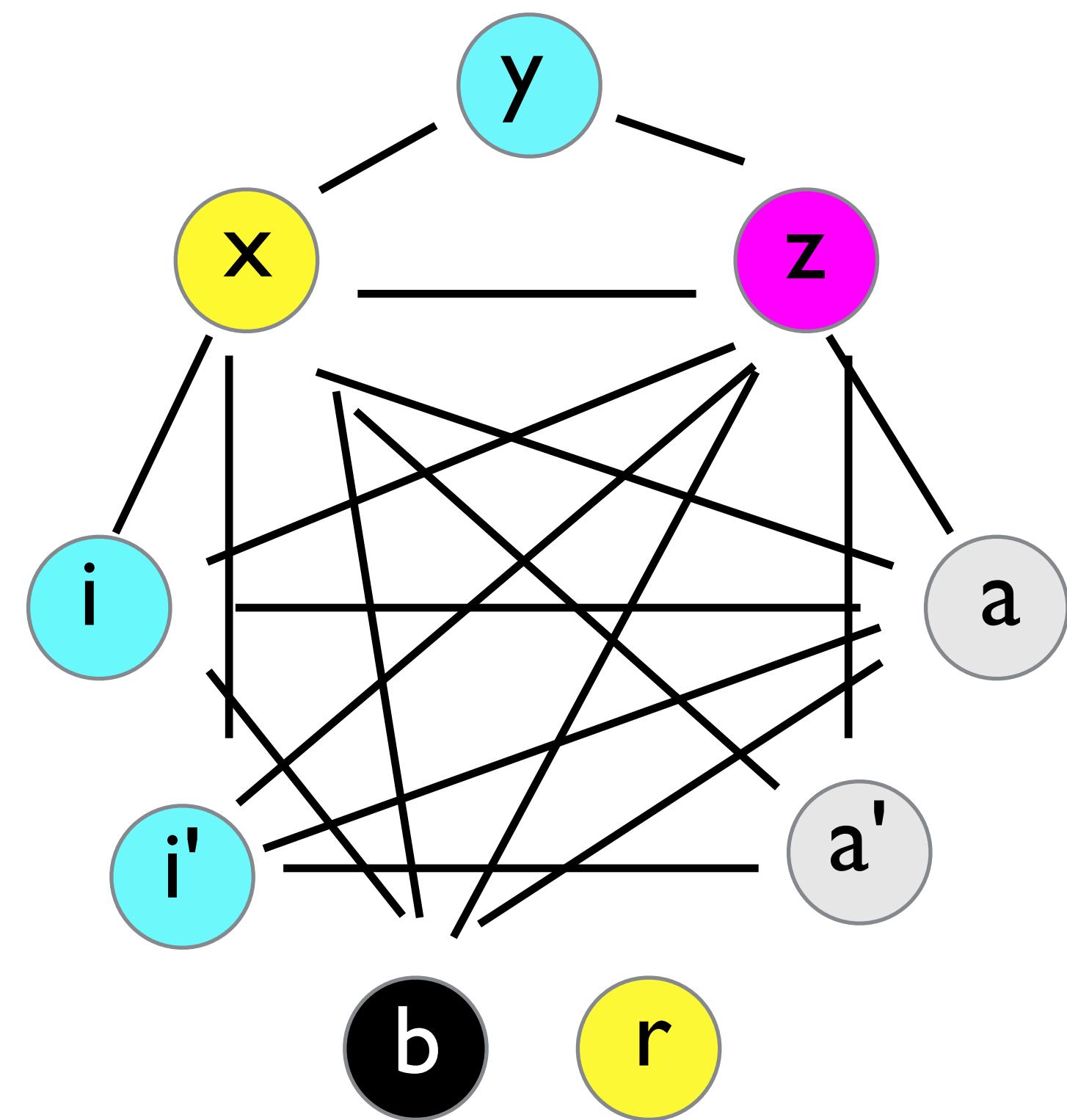


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```

```
f:  
    mov rcx, 0  
    jmp loop  
  
loop:  
    cmp rsi, 0  
    mov rdx, 0  
    sete rdx  
    cmp rdx 0  
    jne thn  
    jmp els  
  
thn:  
    mov rax, rcx  
    imul rax, rdi  
    ret  
  
els:  
    sub rsi, 1  
    add rcx, rax  
    jmp loop
```

Interference Graph



Graph Coloring Register Allocation

Given our register interference graph, want to assign a register to each variable so that no interfering variables are assigned the same register.

Equivalent to graph coloring of the interference graph

- think of each register as a “color” and we want to paint each node so that no adjacent nodes are the same color.

Efficient algorithm for graph coloring -> efficient algorithm for graph coloring!

Graph Coloring is Hard

Determining whether a graph is k -colorable is NP-complete for $k > 2$.

- So no polytime algorithm is known

Does that mean register allocation is NP-hard?

Is Register Allocation Hard?

Chaitin et al, "Register allocation via coloring", *Computer Languages* 1981

- Showed that the register allocation problem for a language with assignments and arbitrary control flow (goto) is **equivalent** to graph coloring
- every graph arises as the interference graph of some program

So register allocation of an imperative language with goto is NP complete.

- But our programs are more restrictive: SSA form...

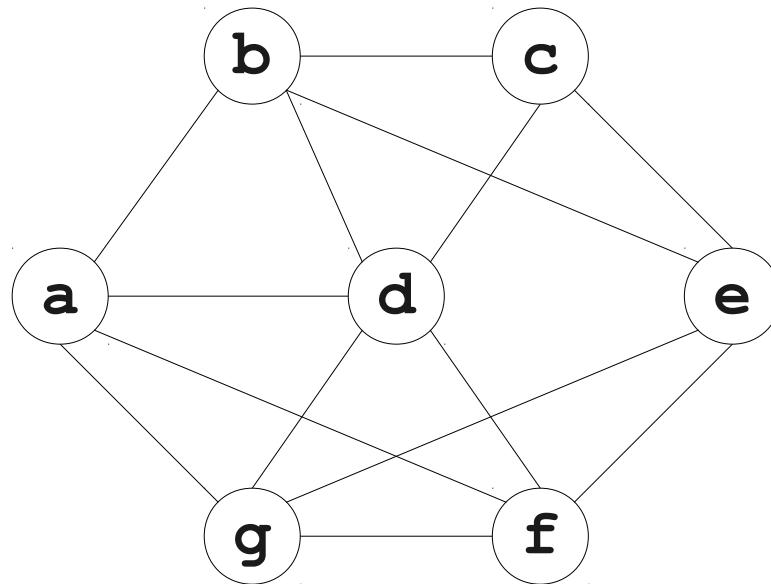
Chaitin's Algorithm

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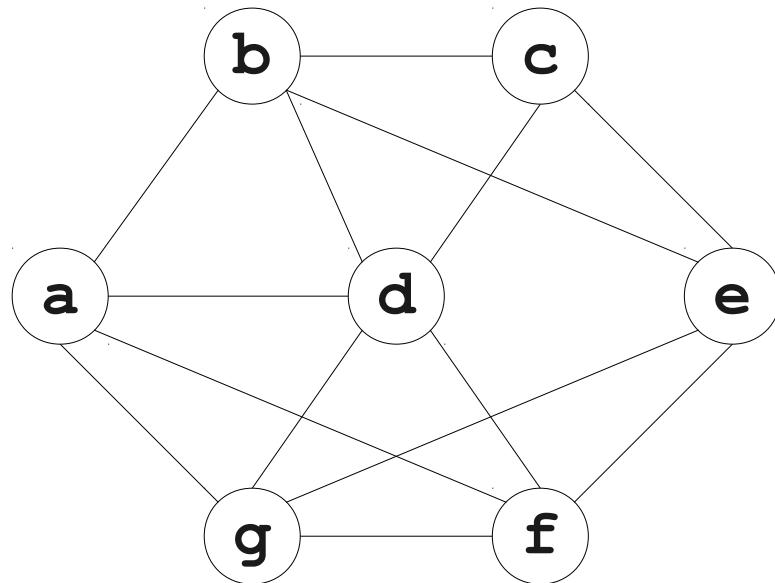
- Intuition:
 - Suppose we are trying to k -color a graph and find a node with fewer than k edges.
 - If we delete this node from the graph and color what remains, we can find a color for this node if we add it back in.
 - Reason: With fewer than k neighbors, some color must be left over.
- Algorithm:
 - Find a node with fewer than k outgoing edges.
 - Remove it from the graph.
 - Recursively color the rest of the graph.
 - Add the node back in.
 - Assign it a valid color.

Chaitin's Algorithm

Chaitin's Algorithm



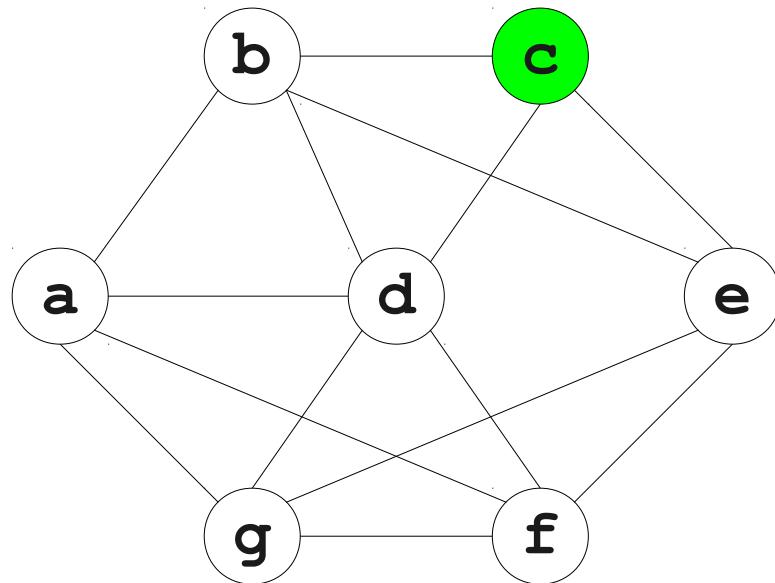
Chaitin's Algorithm



Registers



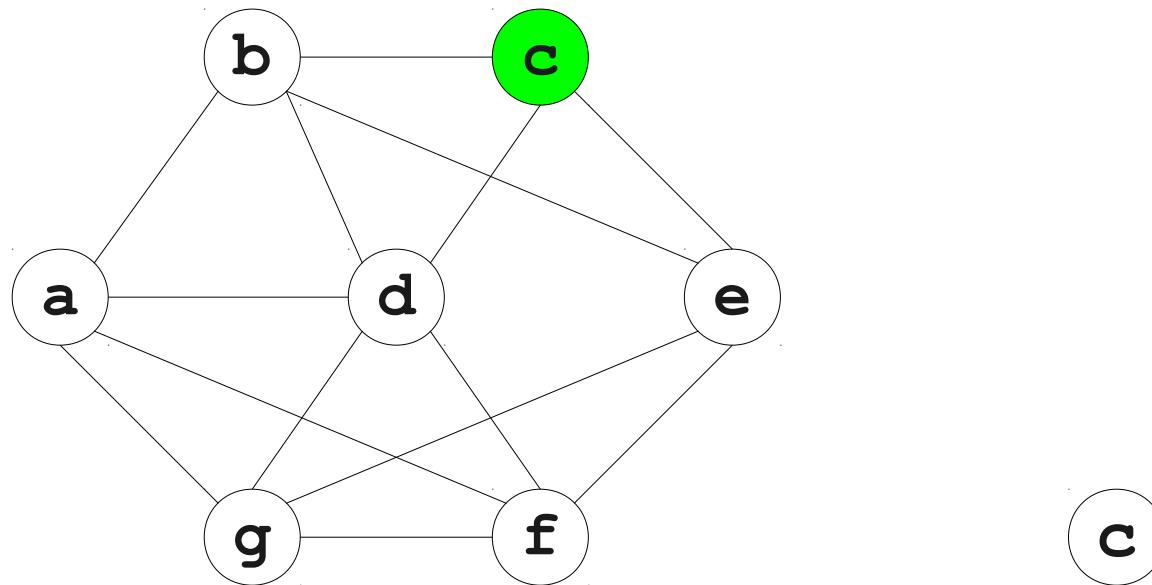
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Registers



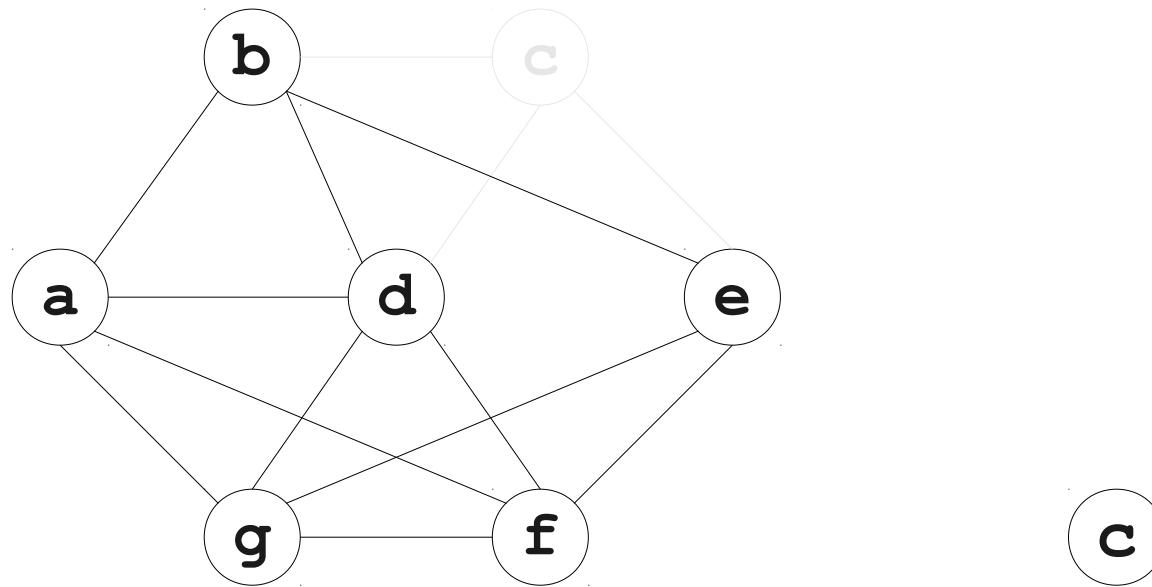
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Registers

R₀	R₁	R₂	R₃
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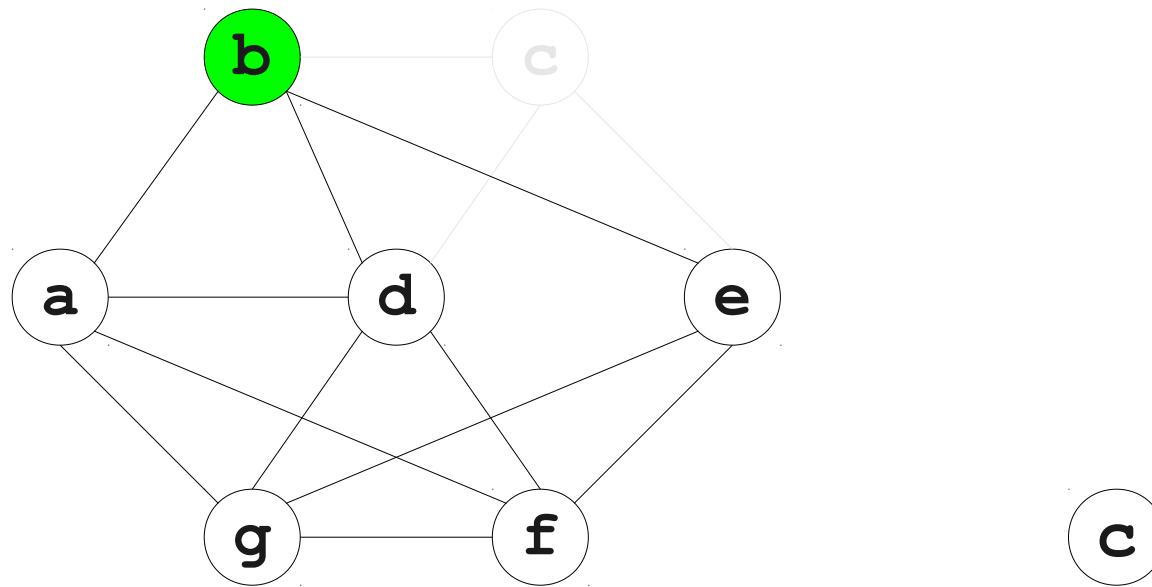
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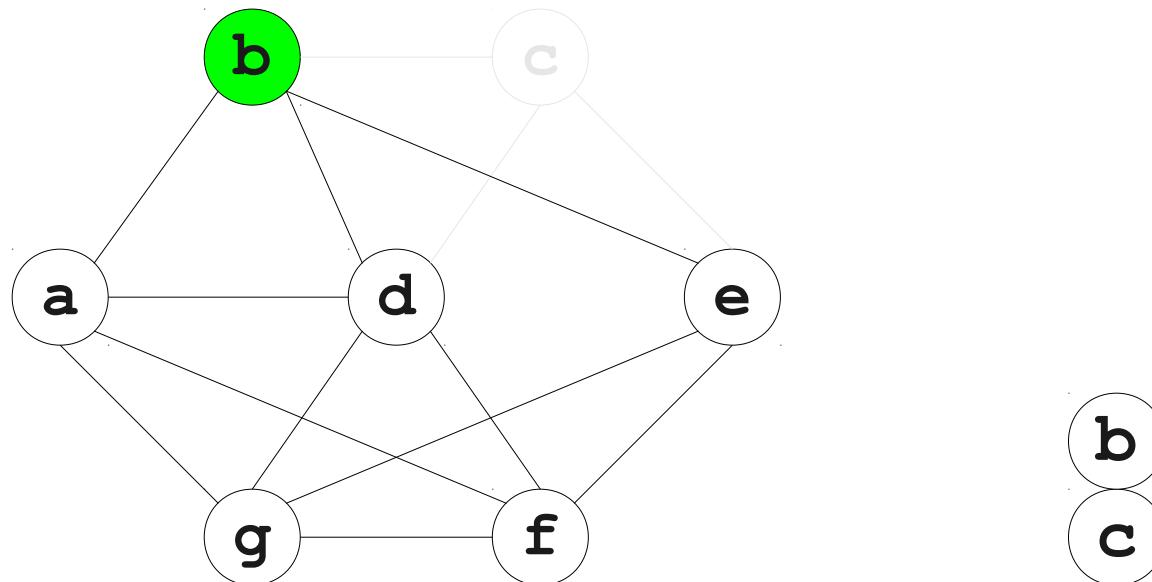
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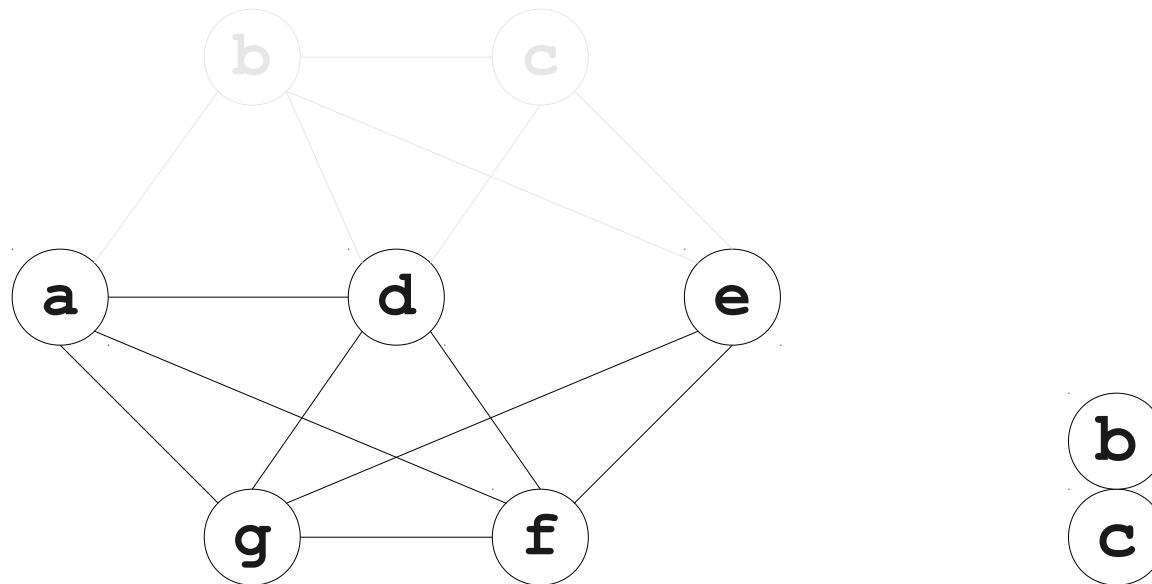
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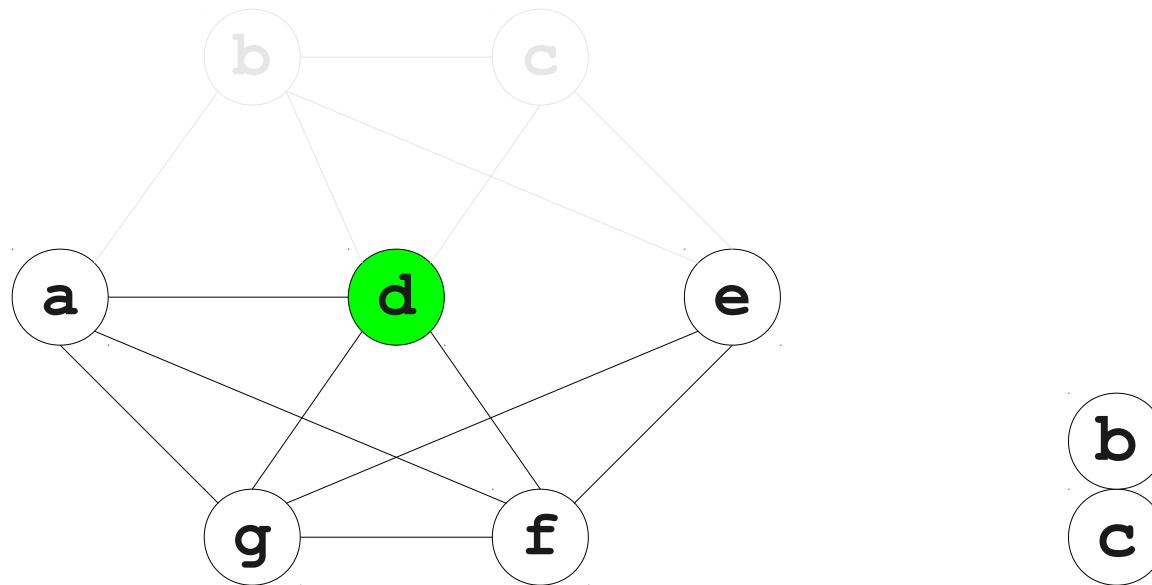
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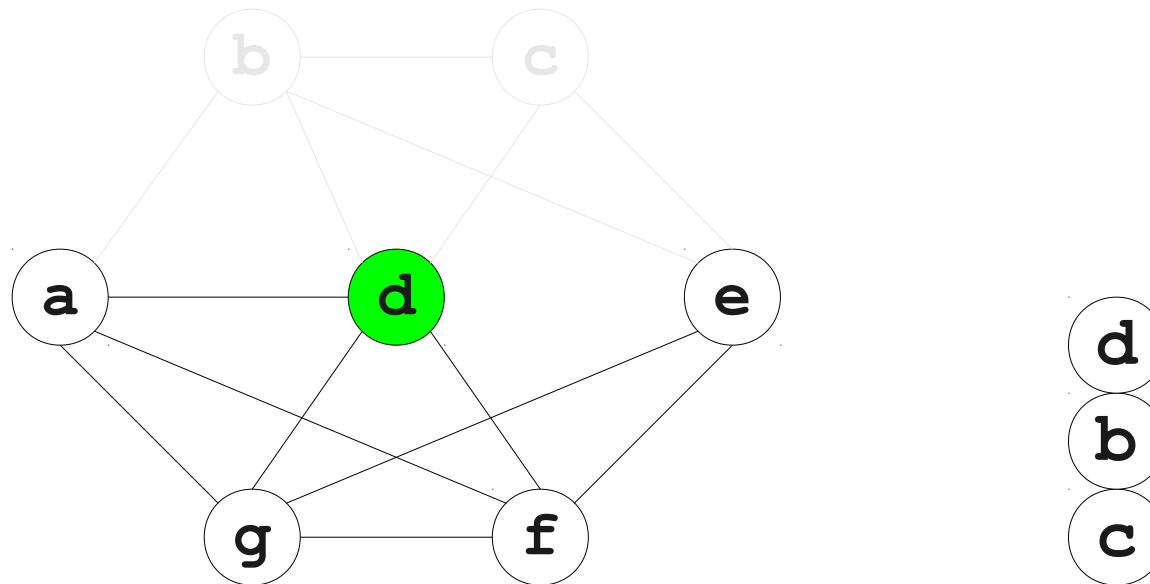
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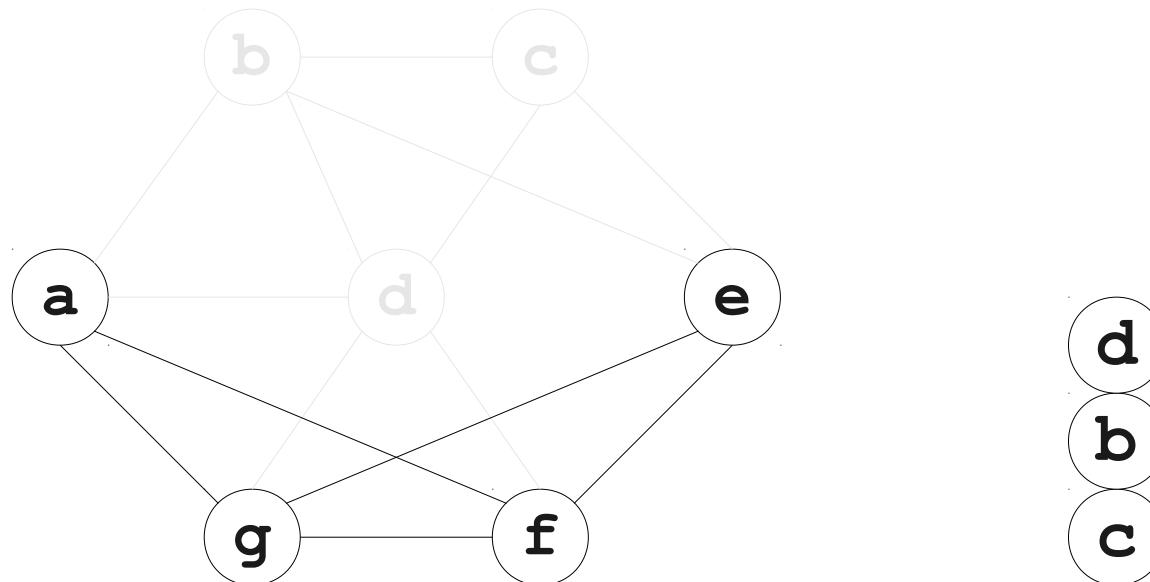
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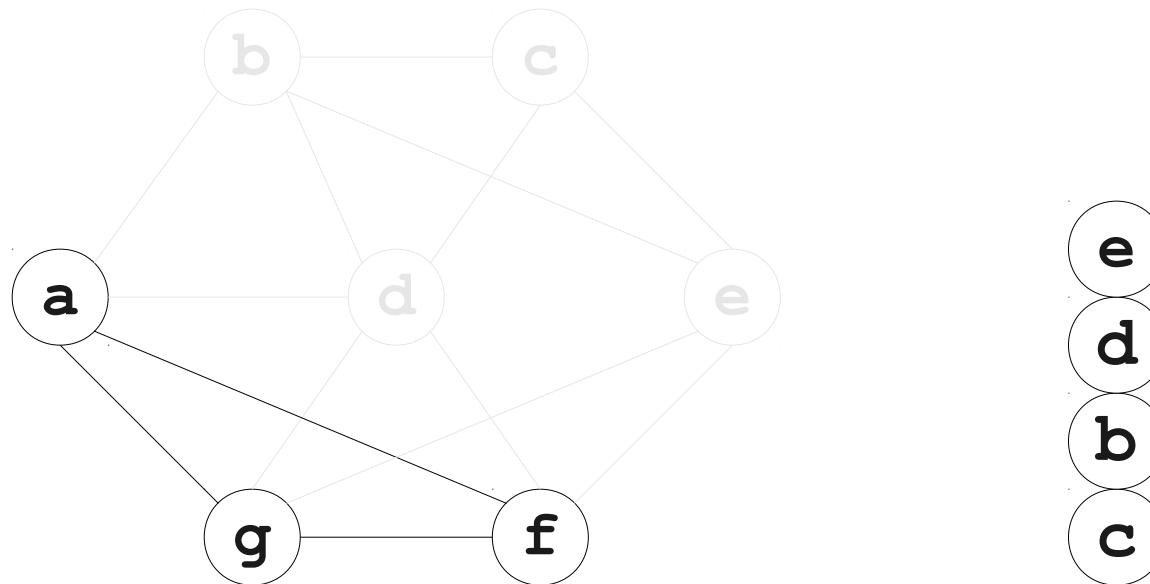
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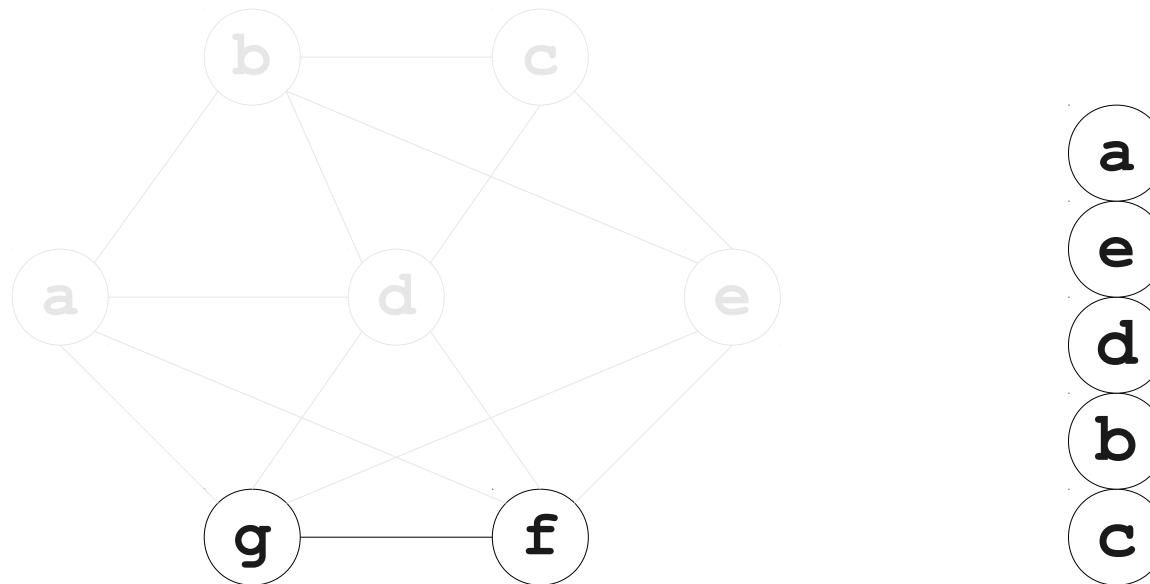
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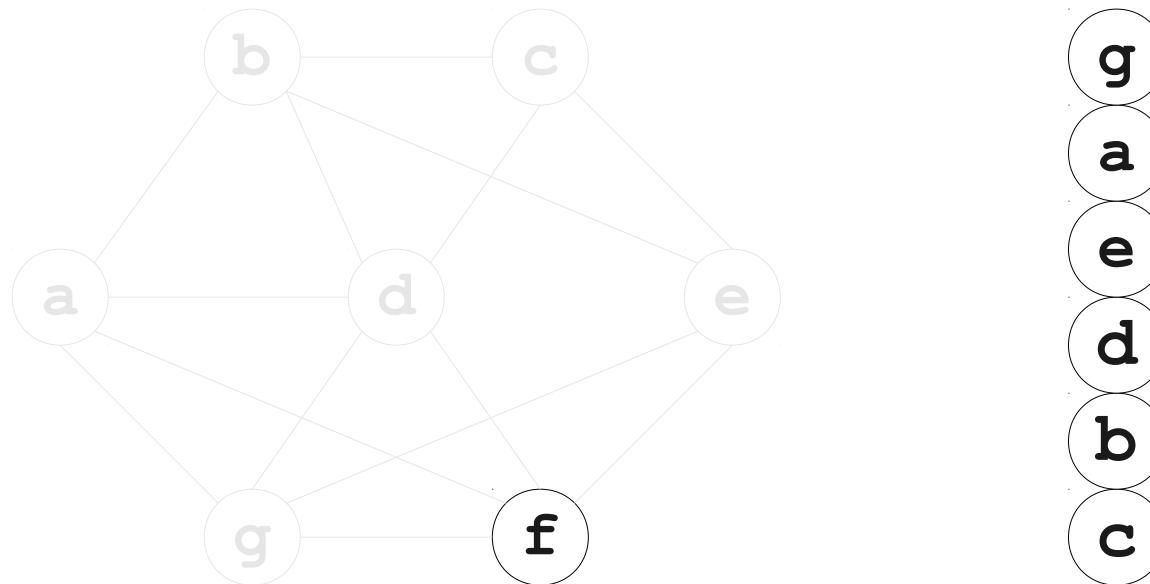
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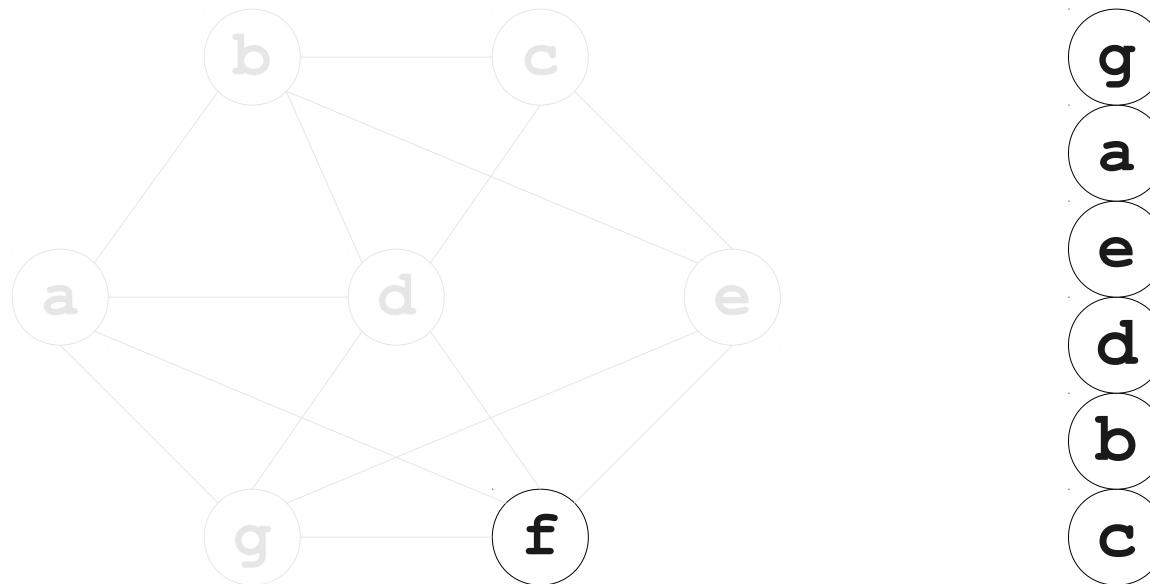
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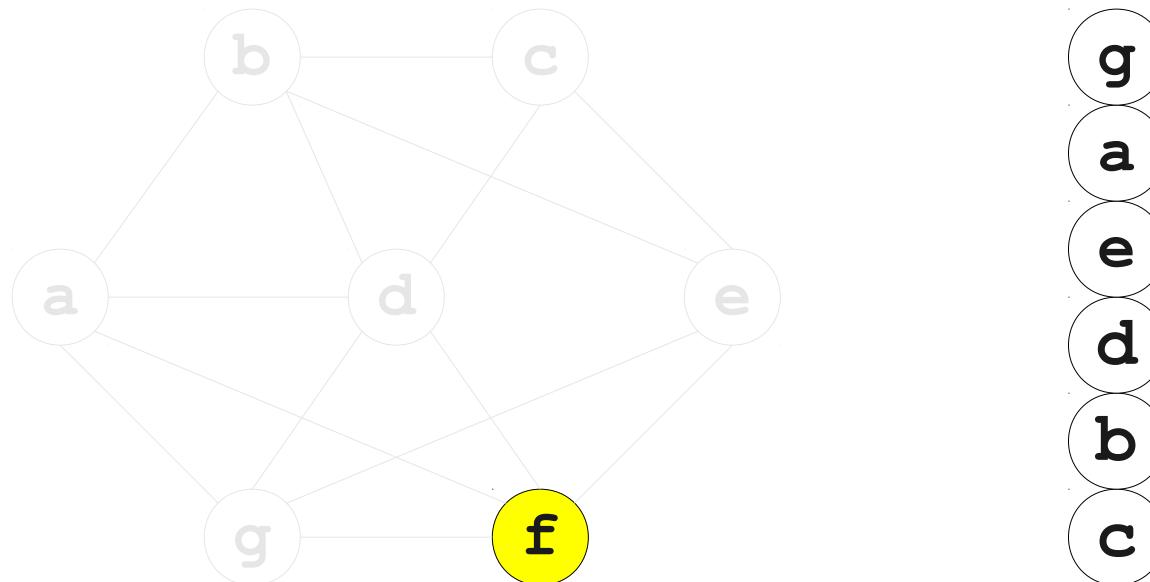
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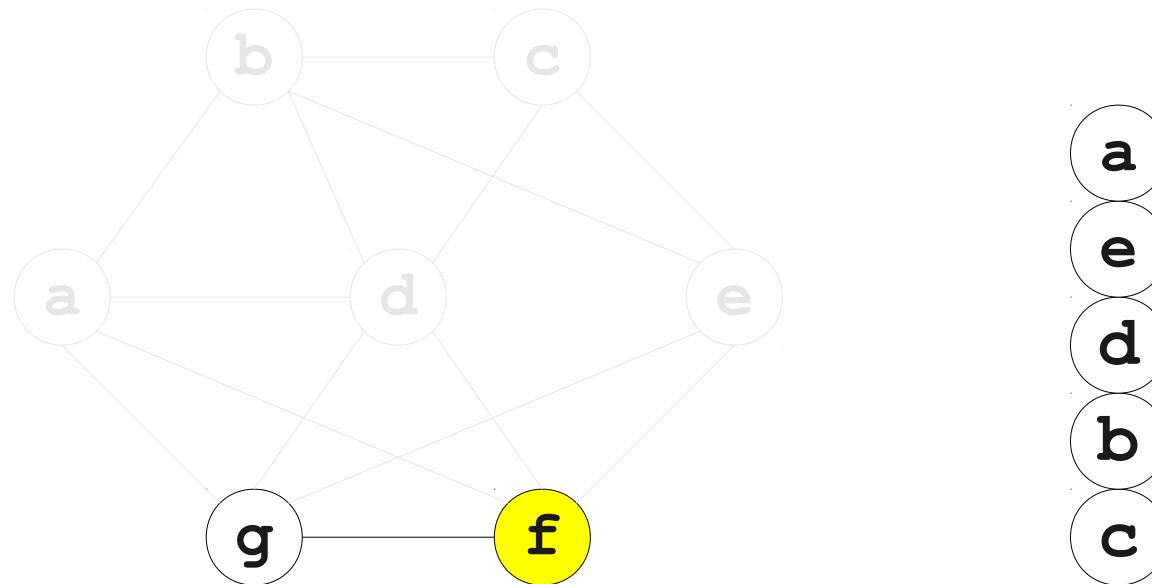
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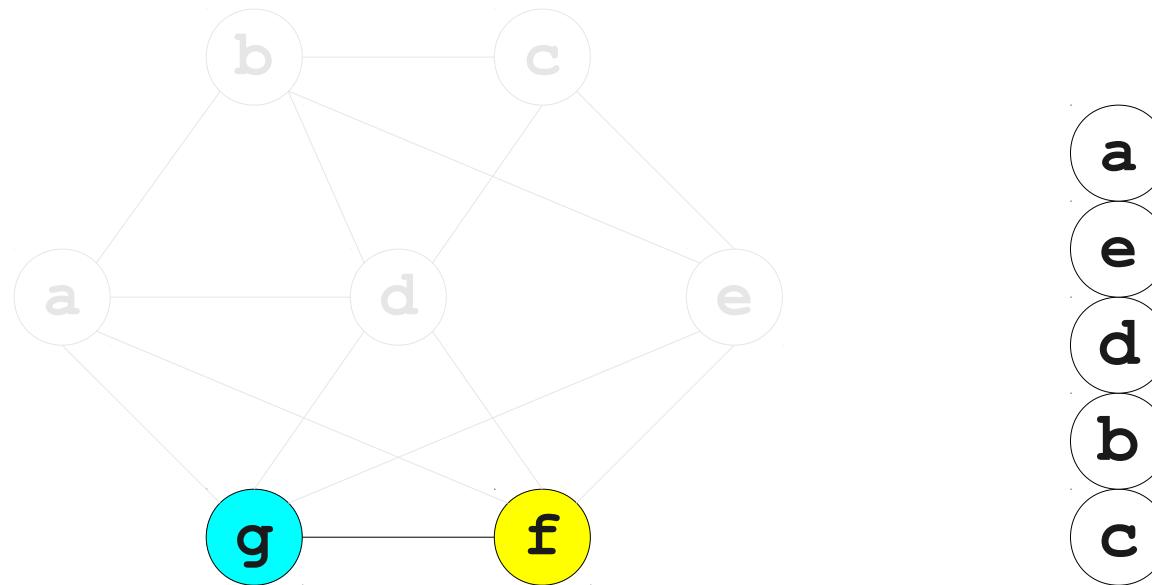
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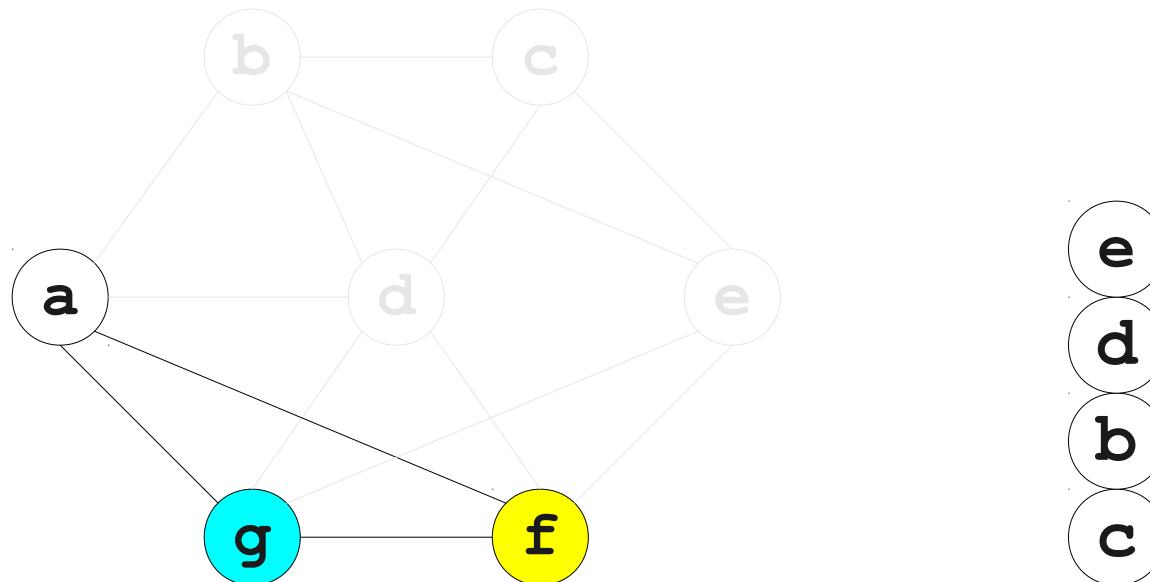
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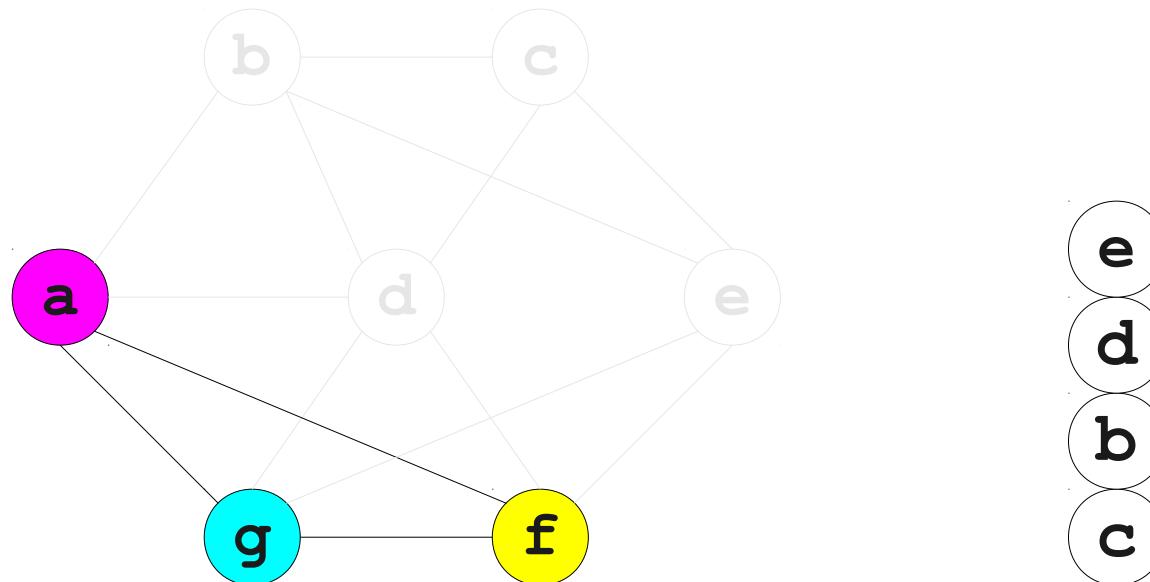
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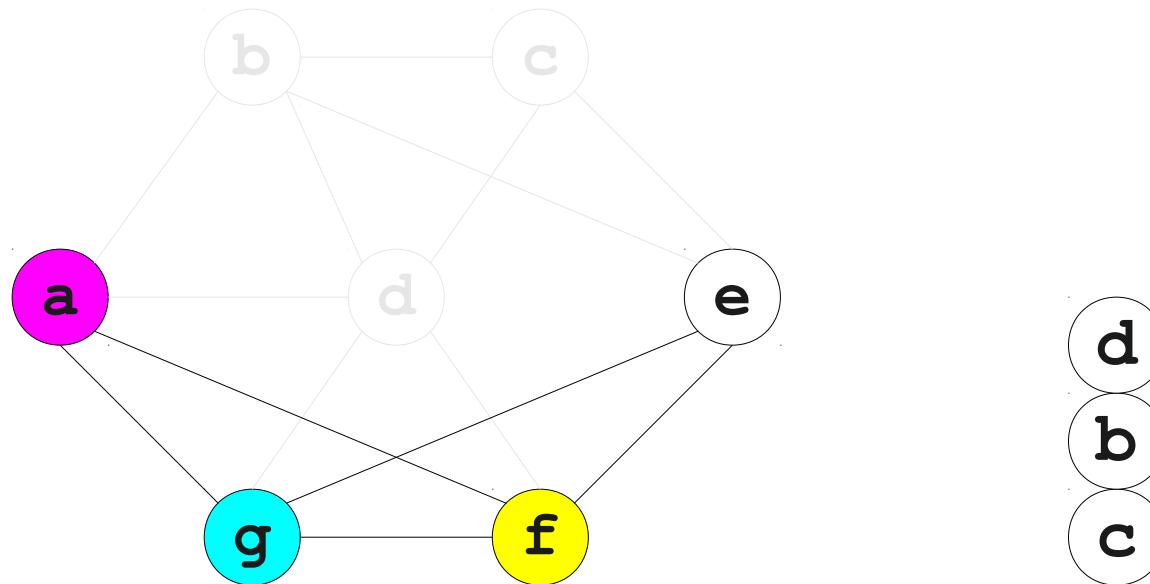
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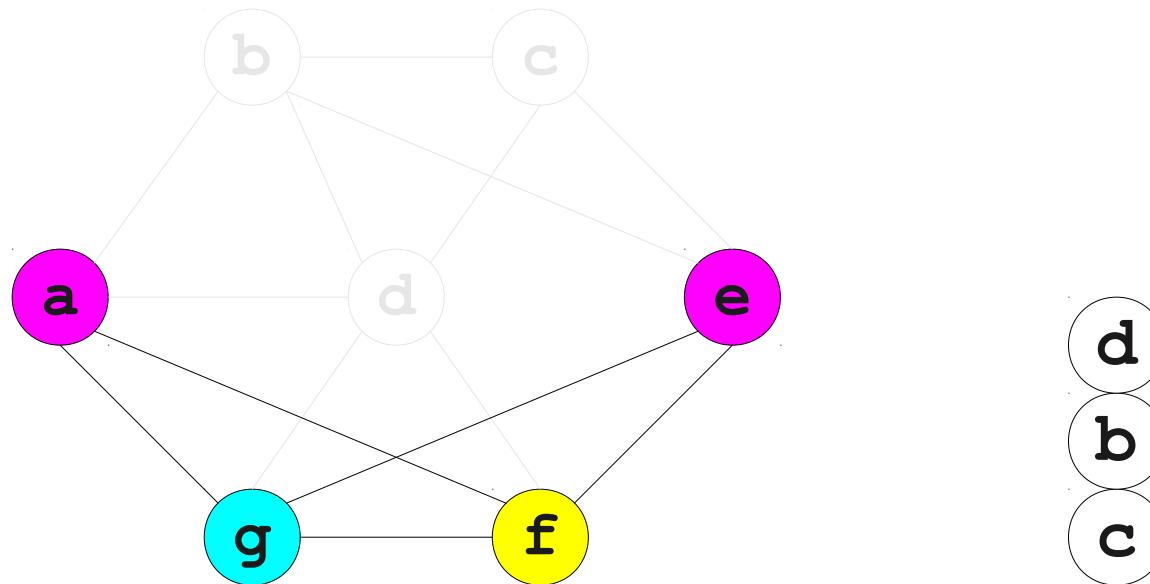
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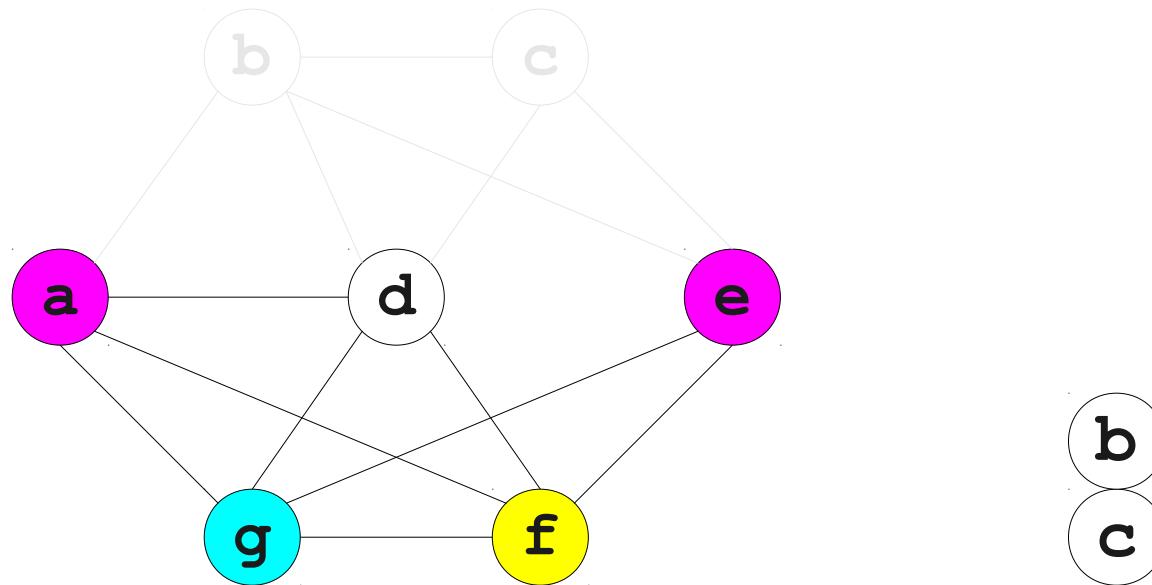
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Chaitin's Algorithm



d
b
c

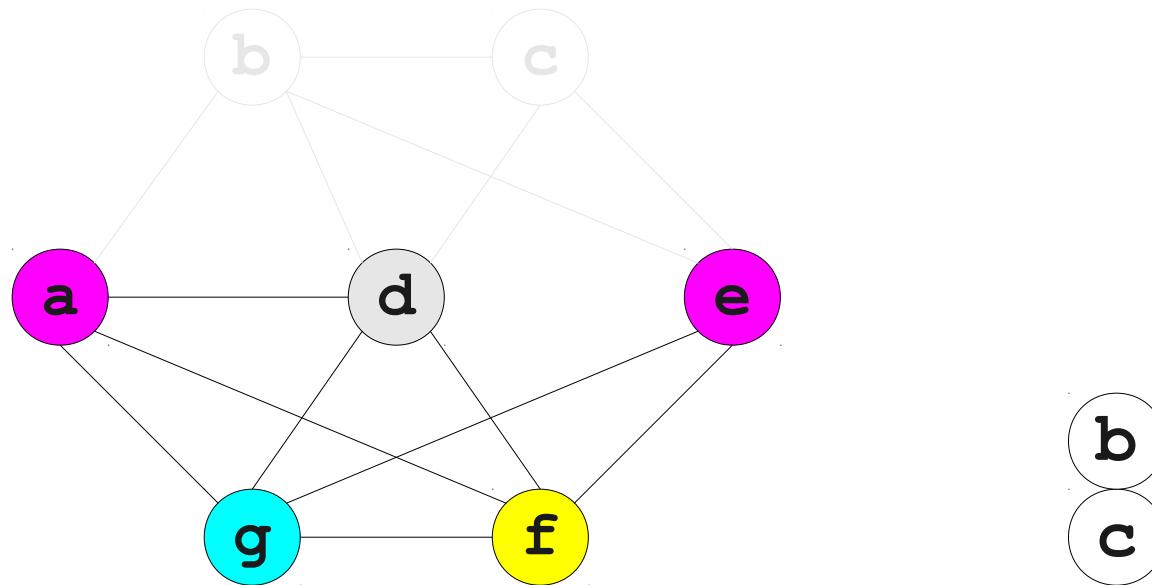
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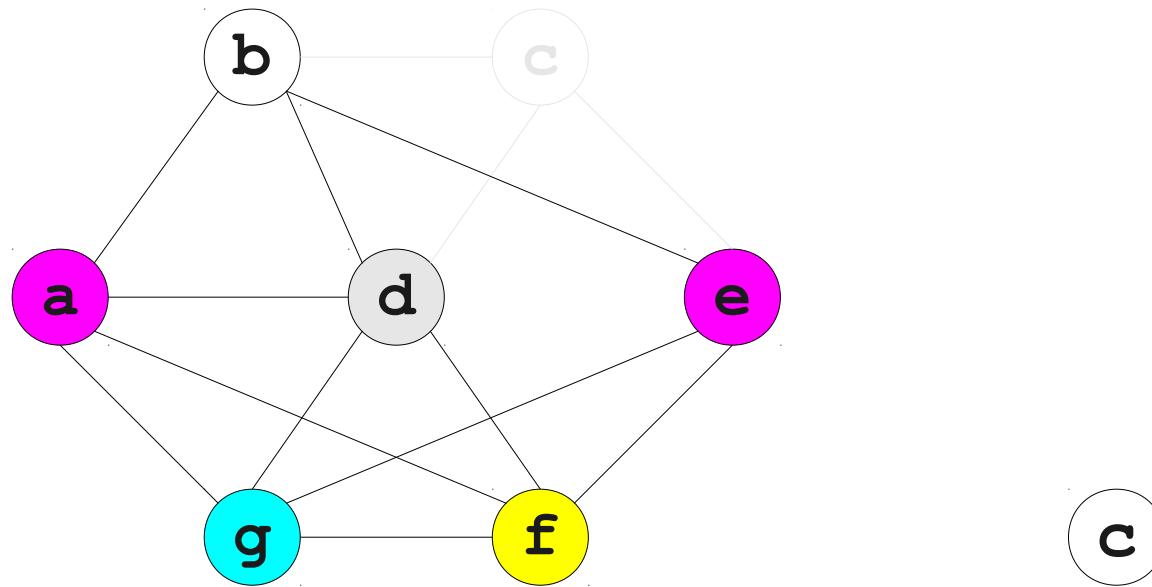
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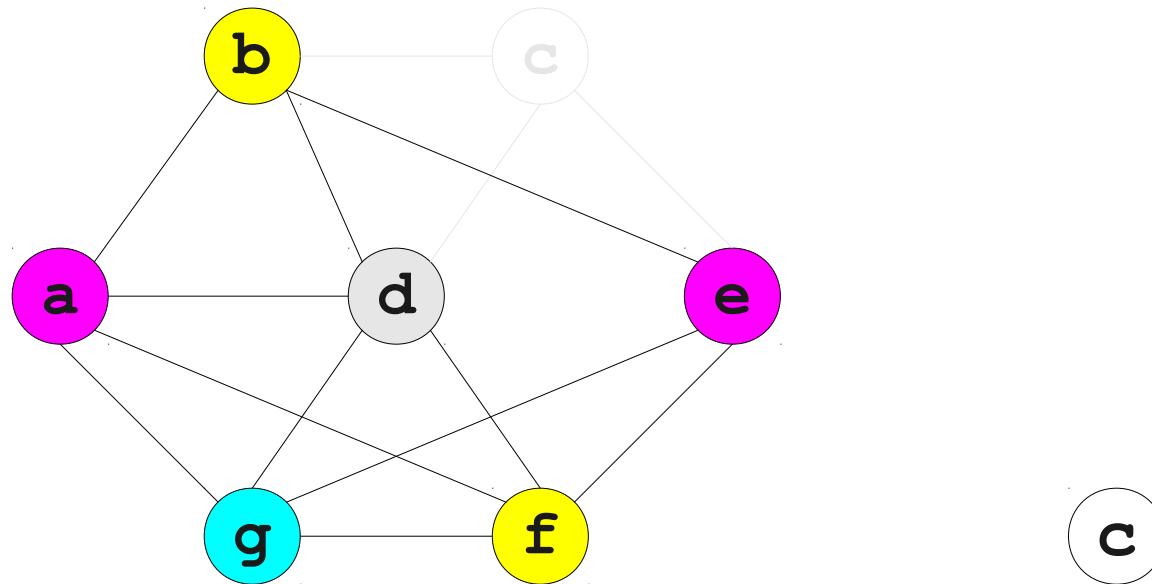
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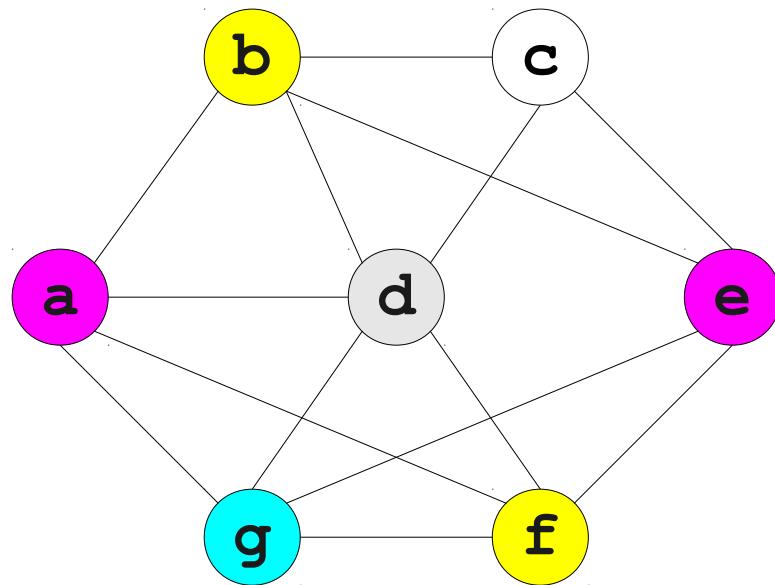
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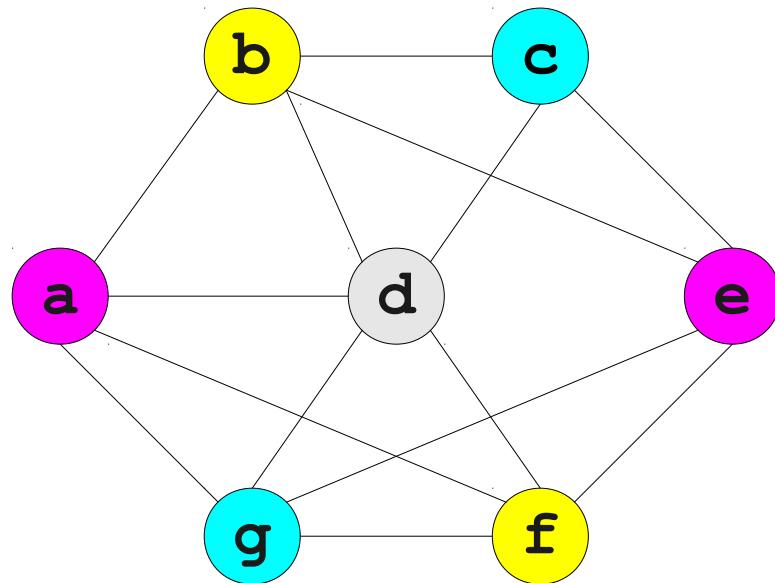
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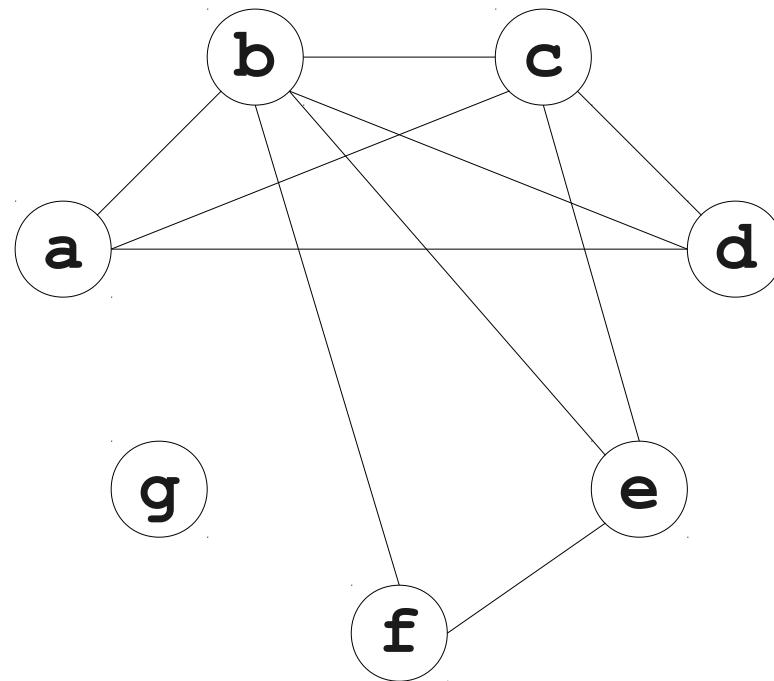
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One Problem

- What if we can't find a node with fewer than k neighbors?
- Choose and remove an arbitrary node, marking it "troublesome."
 - Use heuristics to choose which one.
- When adding node back in, it may be possible to find a valid color.
- Otherwise, we have to spill that node.

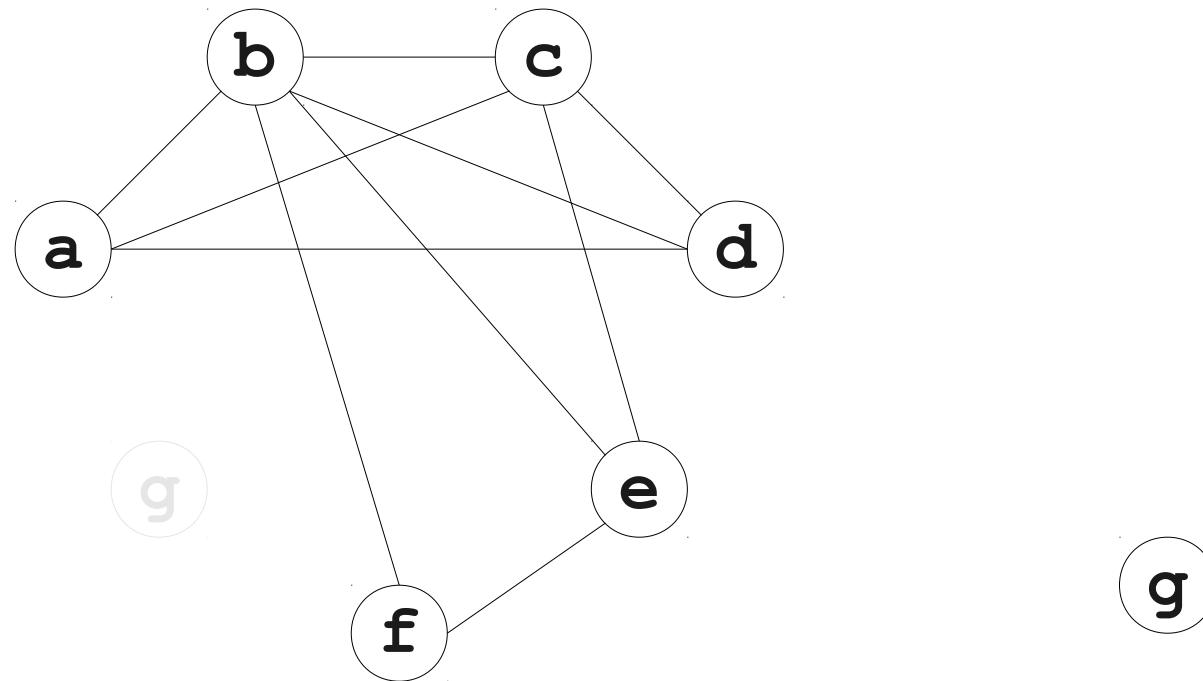
Chaitin's Algorithm Reloaded



Registers

R_0	R_1	R_2
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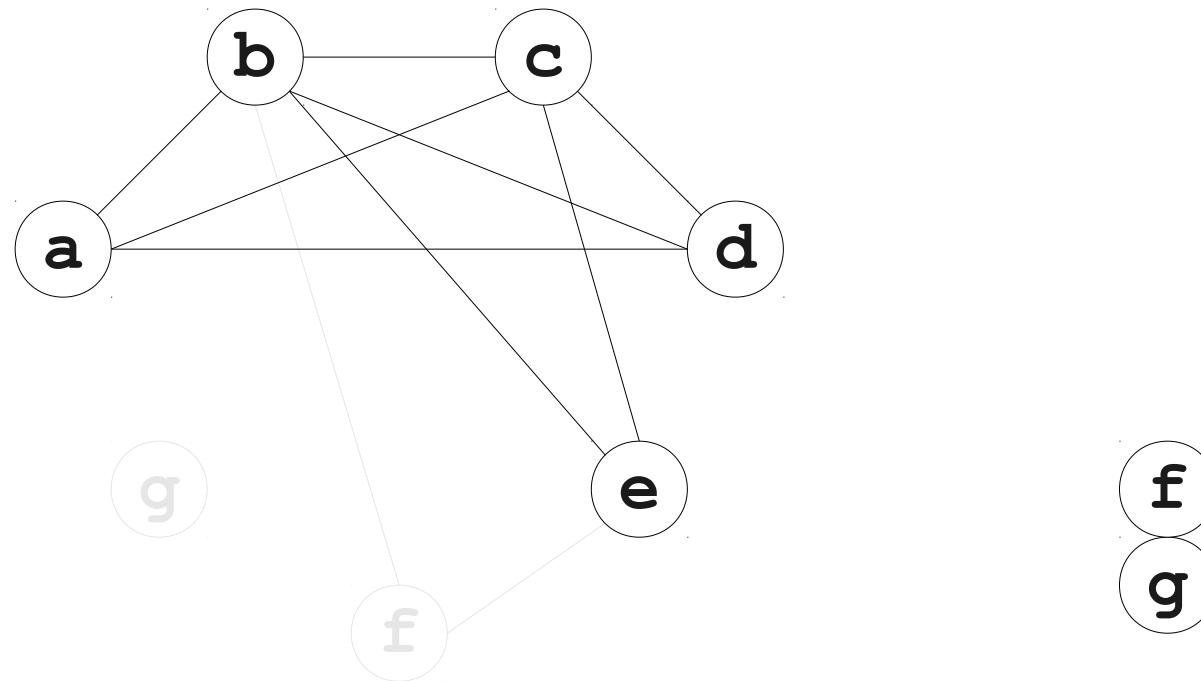
Chaitin's Algorithm Reloaded



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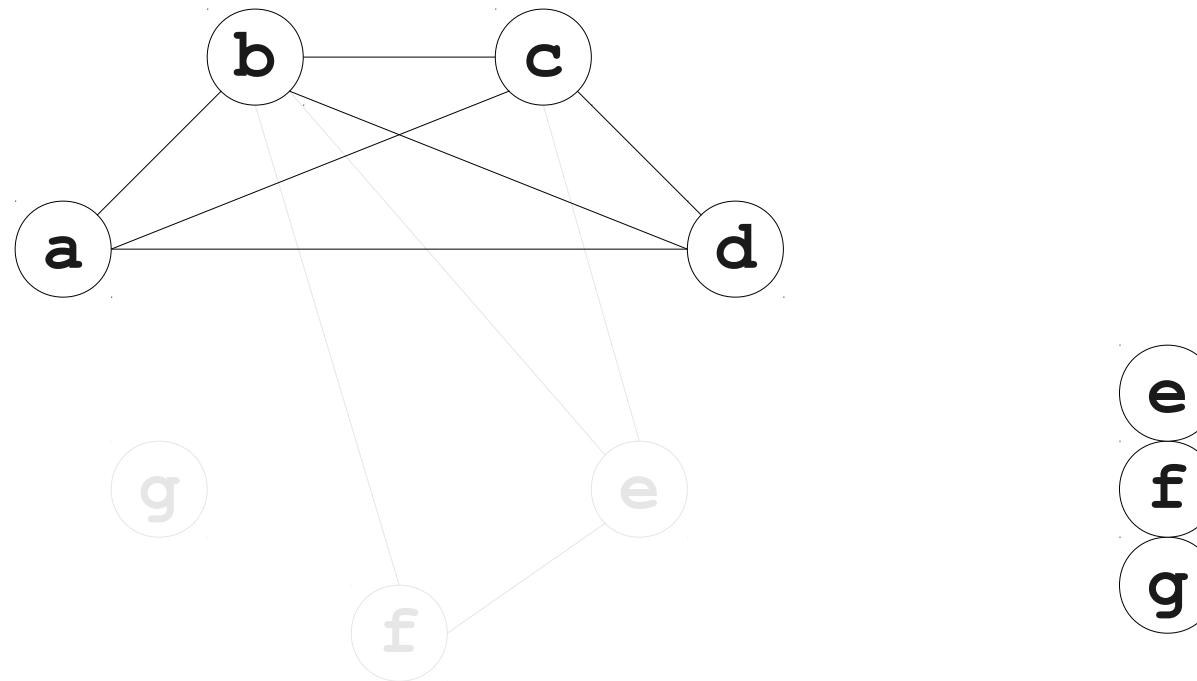
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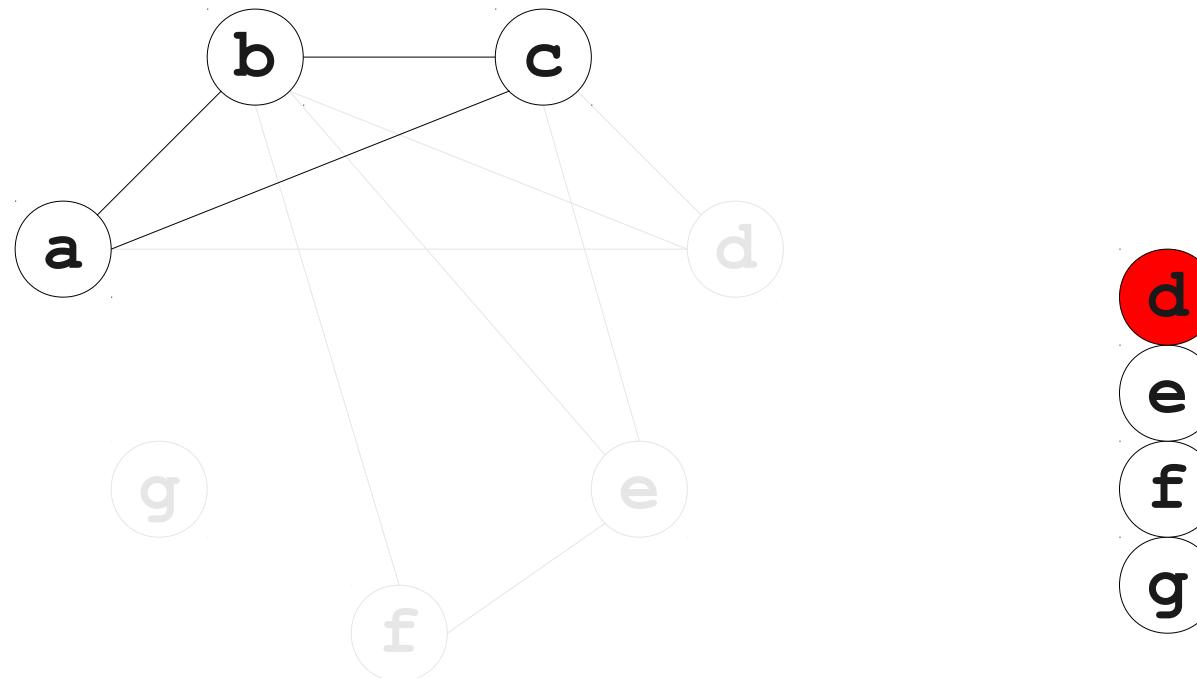
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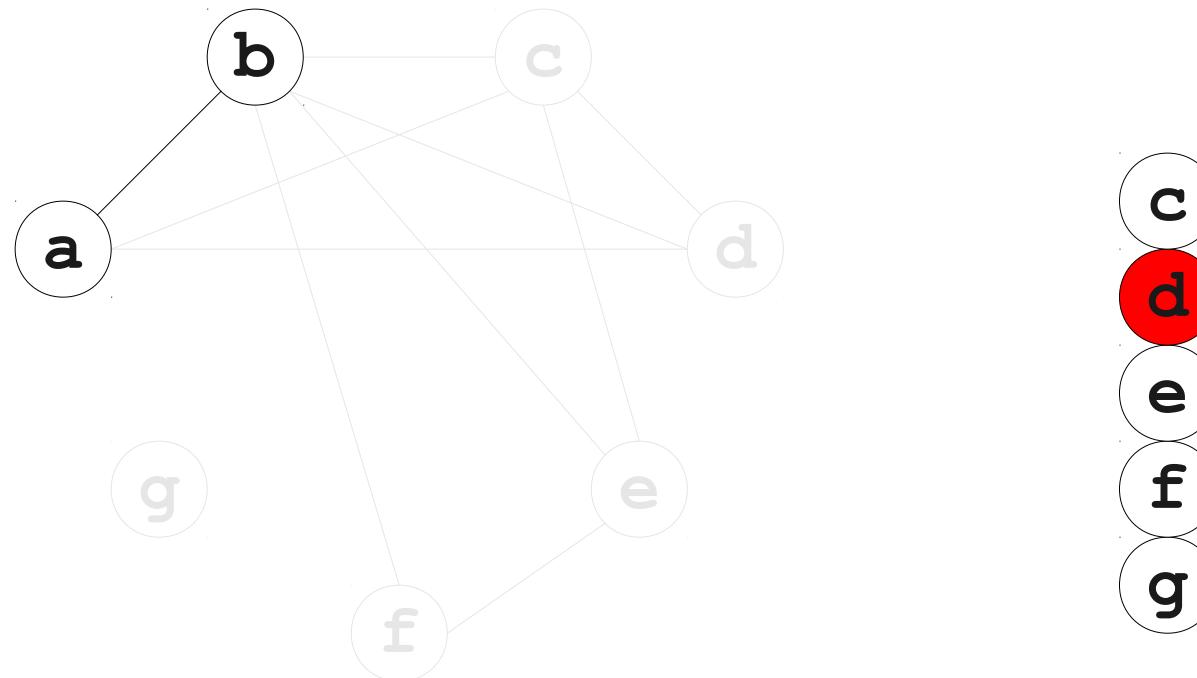
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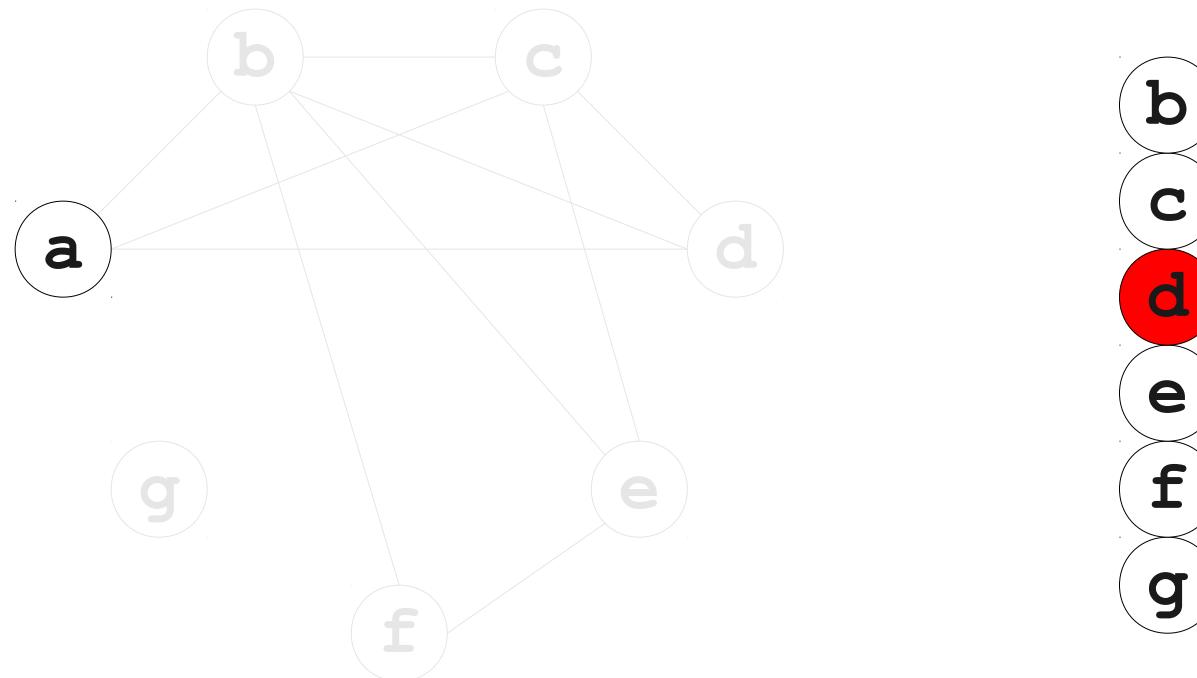
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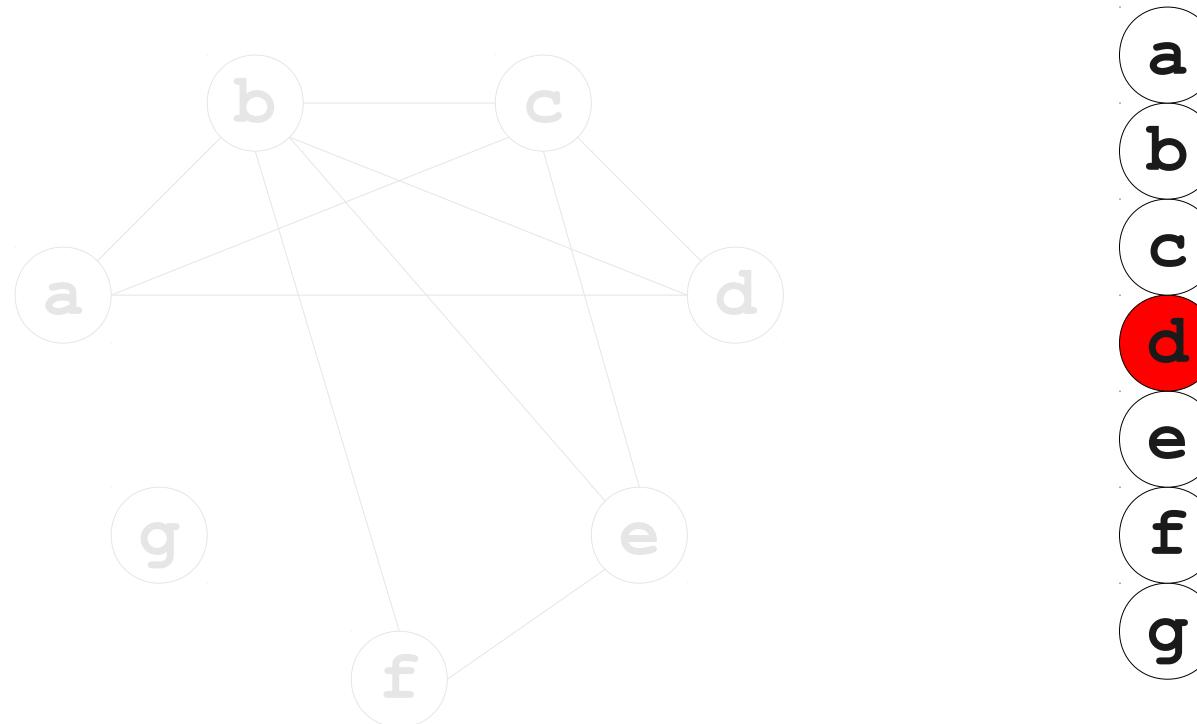
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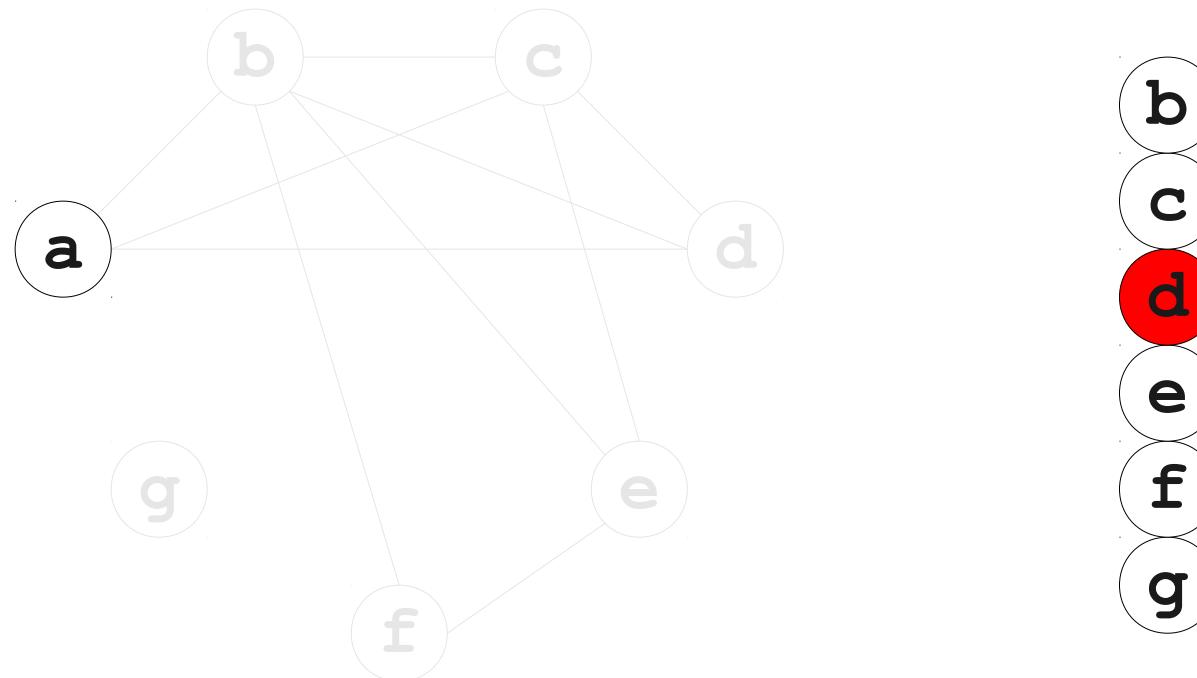
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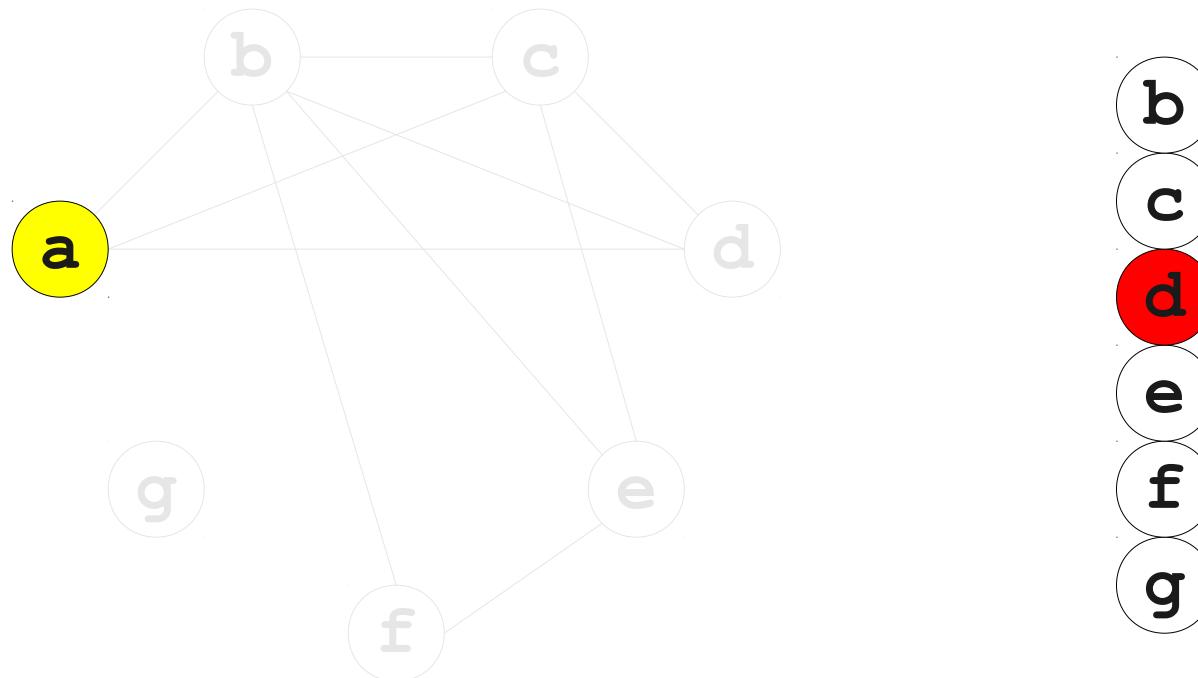
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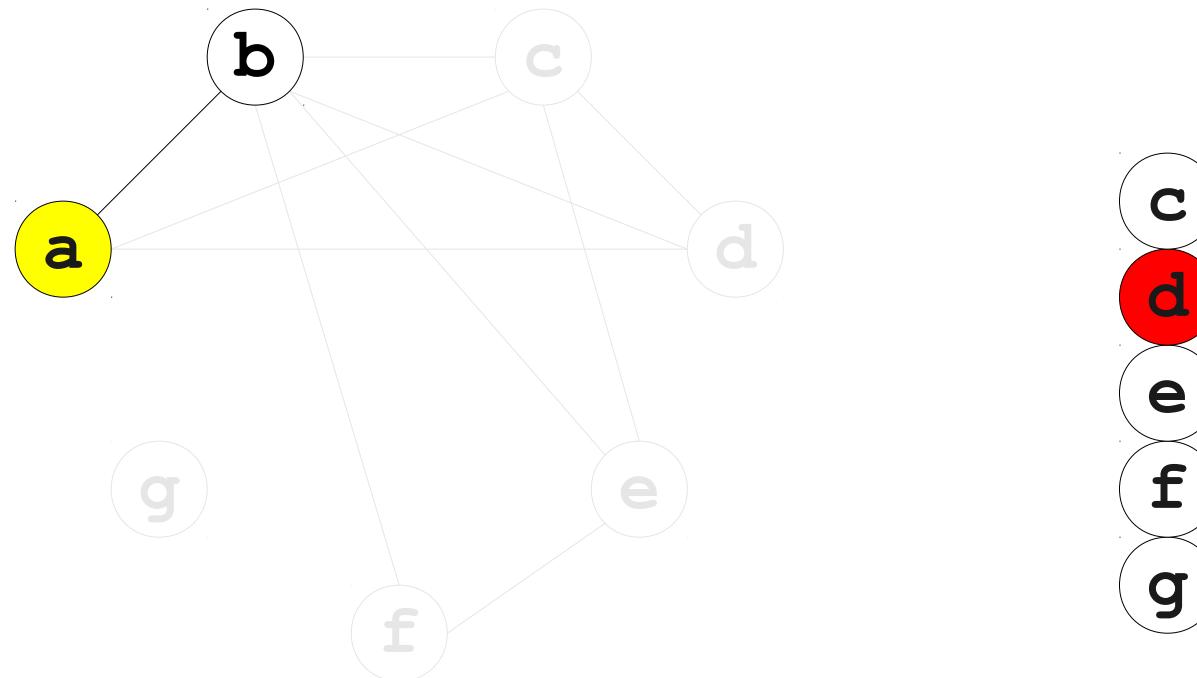
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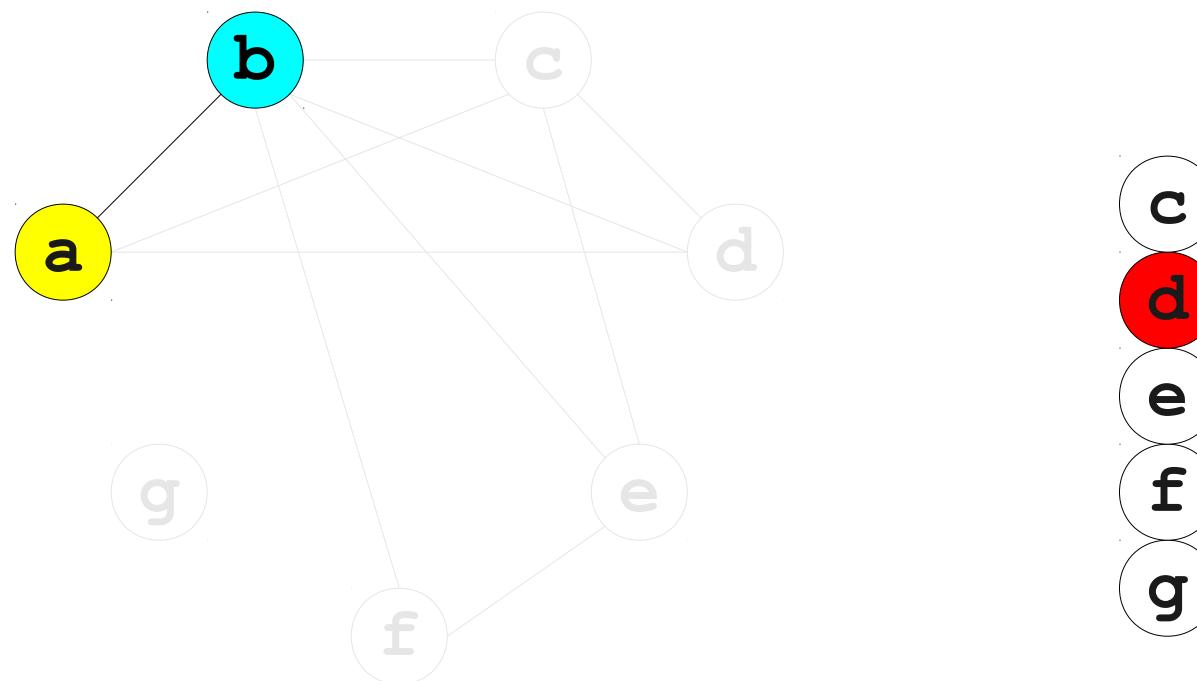
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Registers

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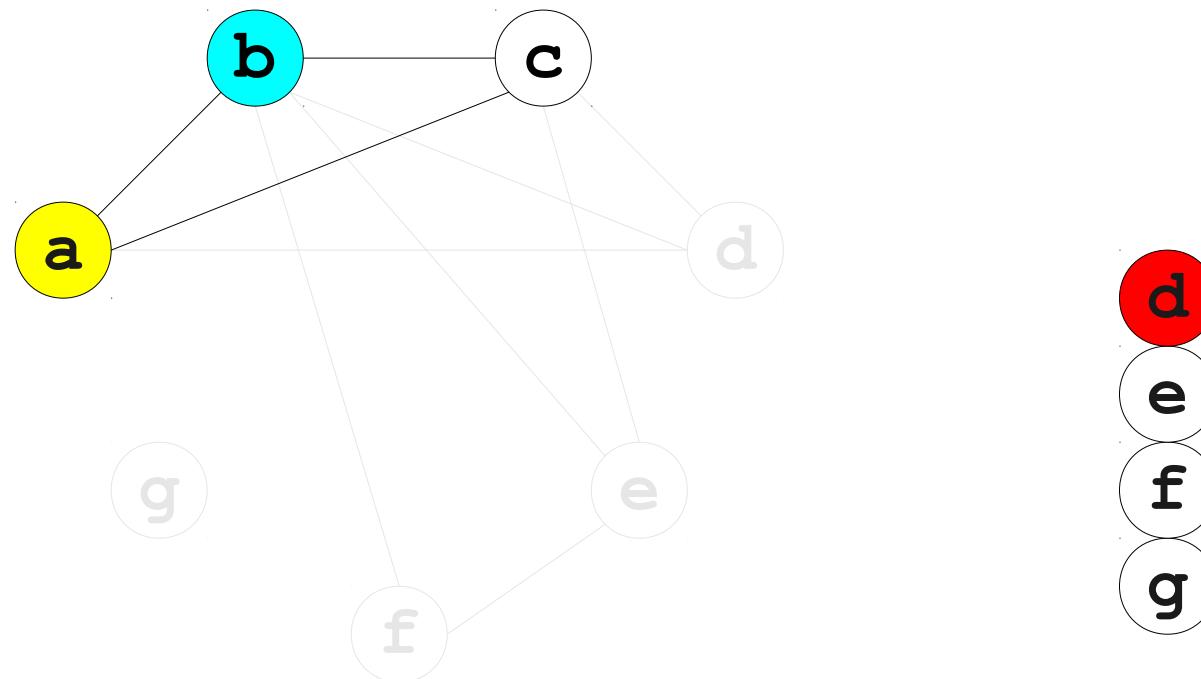
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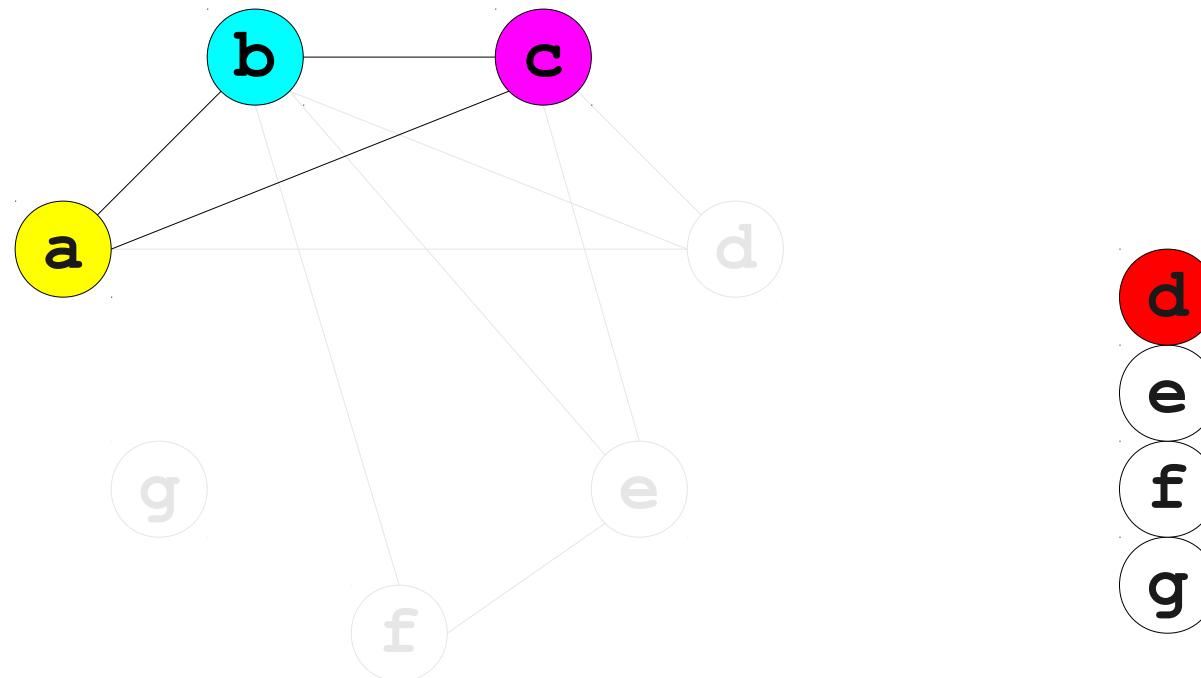
Chaitin's Algorithm Reloaded



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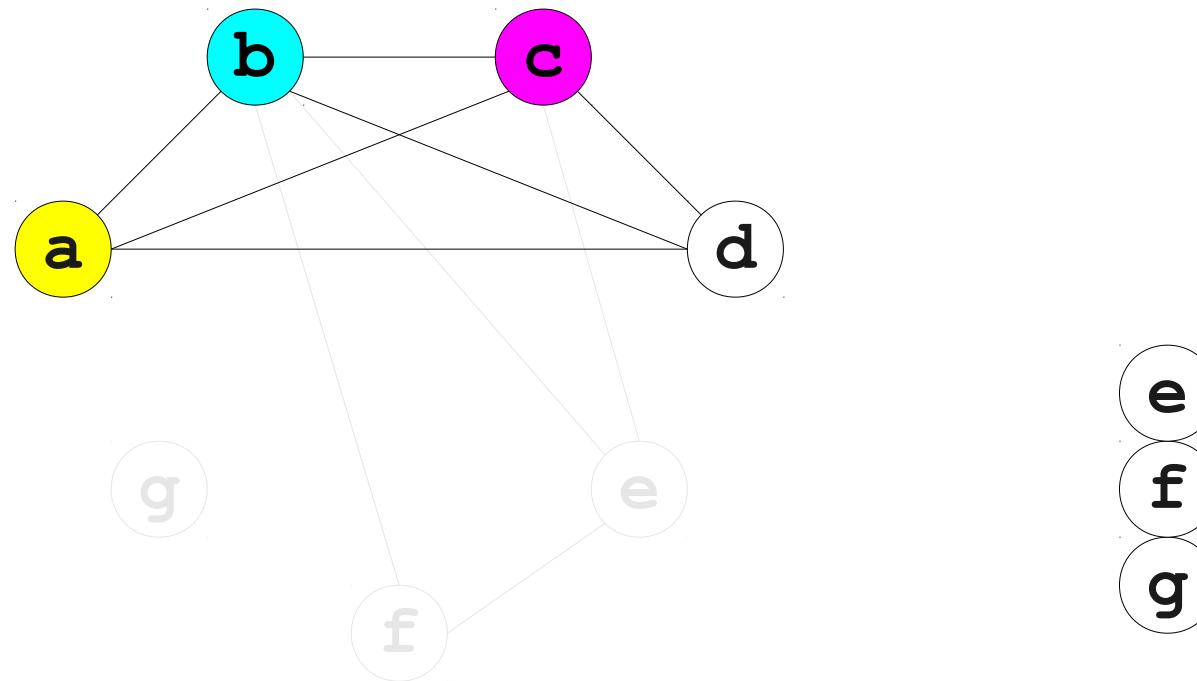
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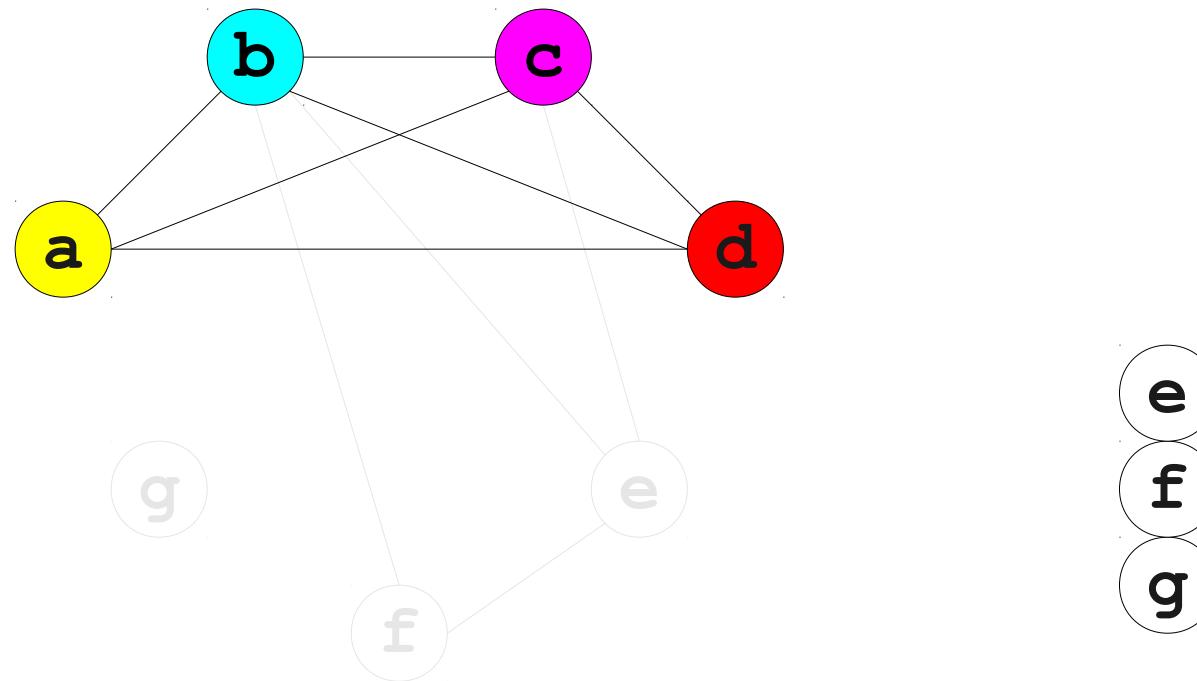
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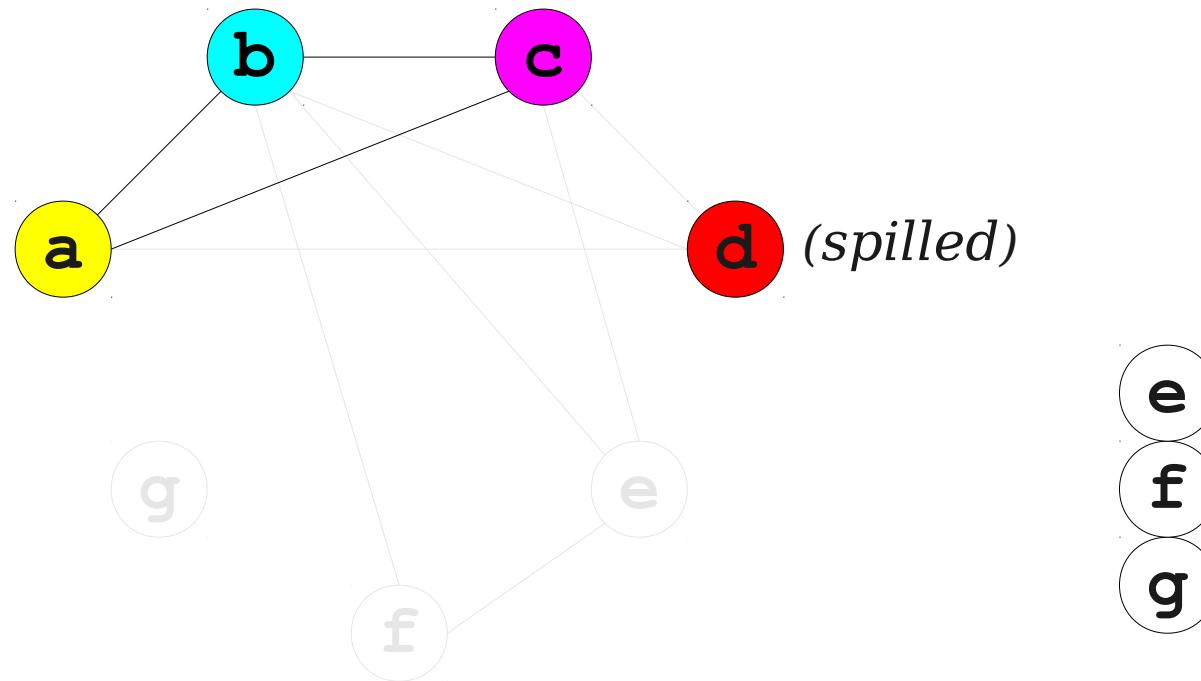
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Registers

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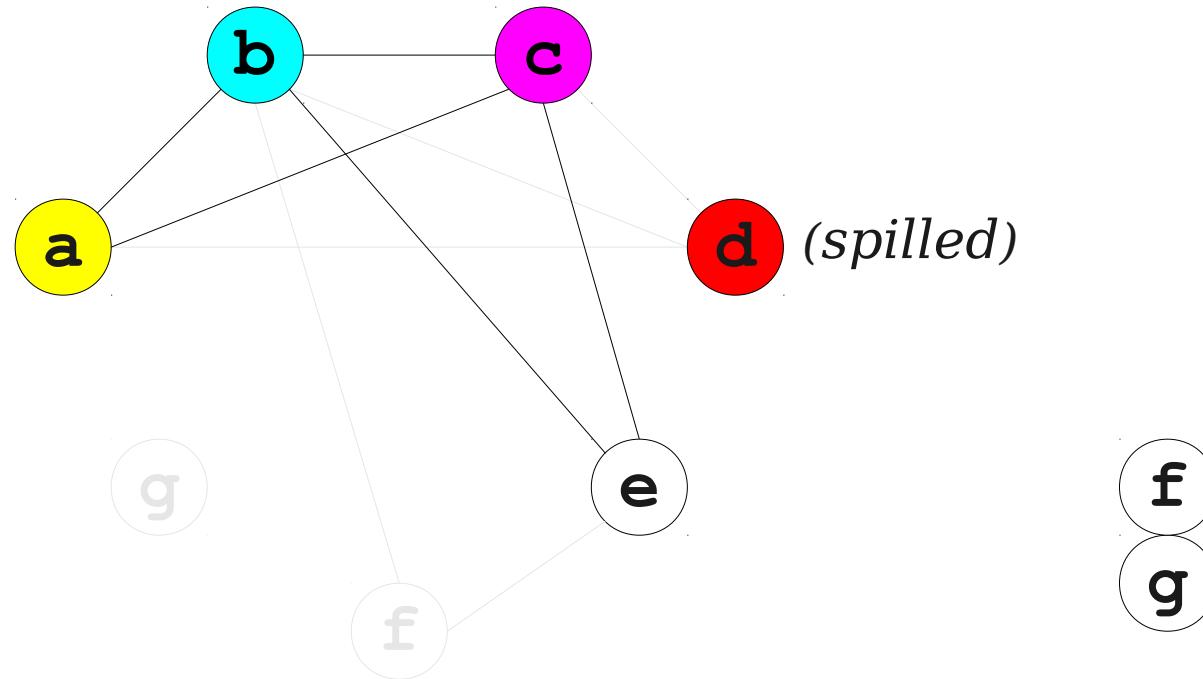
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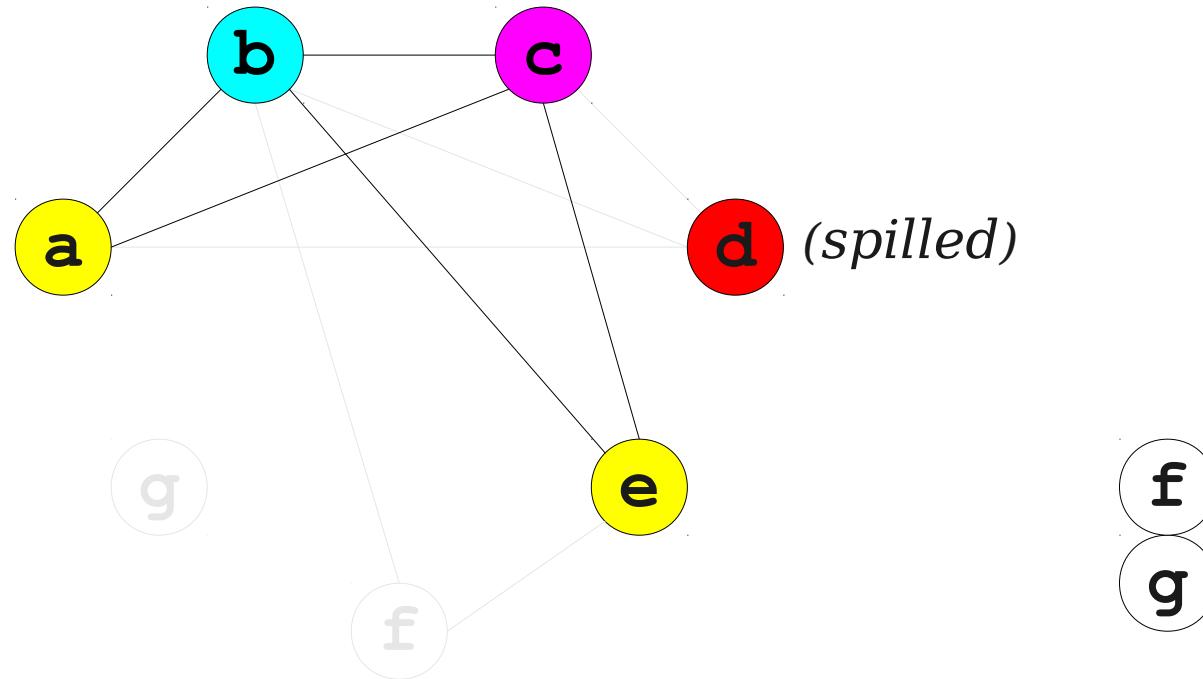
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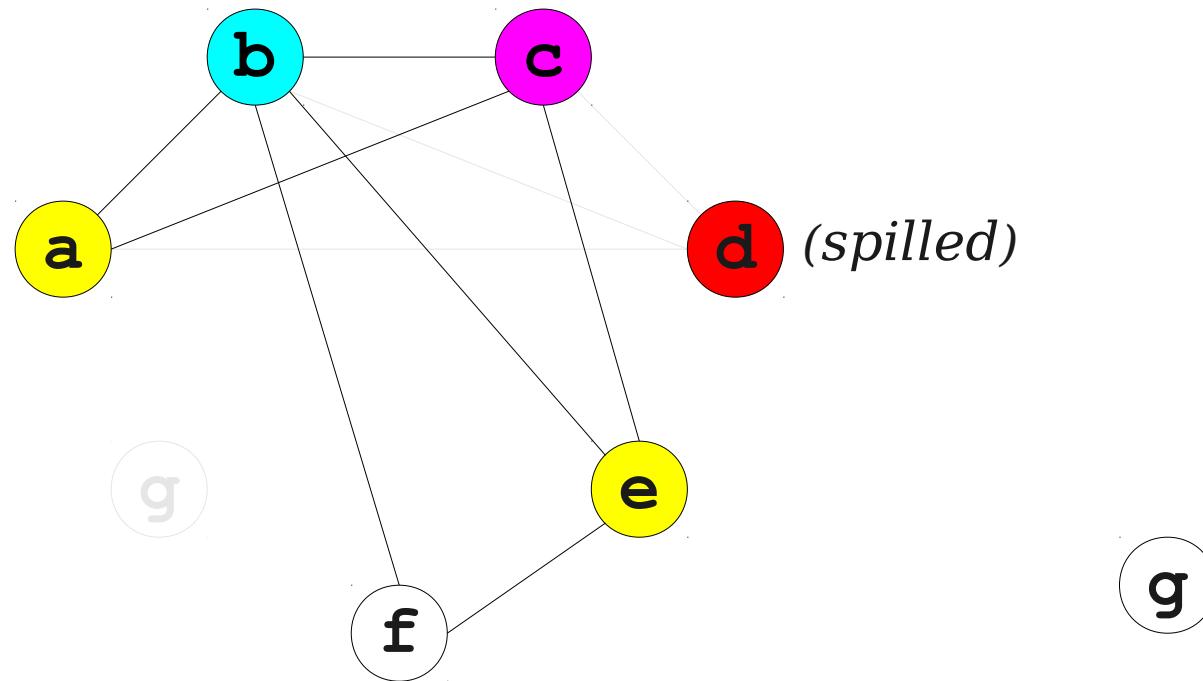
Chaitin's Algorithm Reloaded



Registers

R₀ R₁ R₂

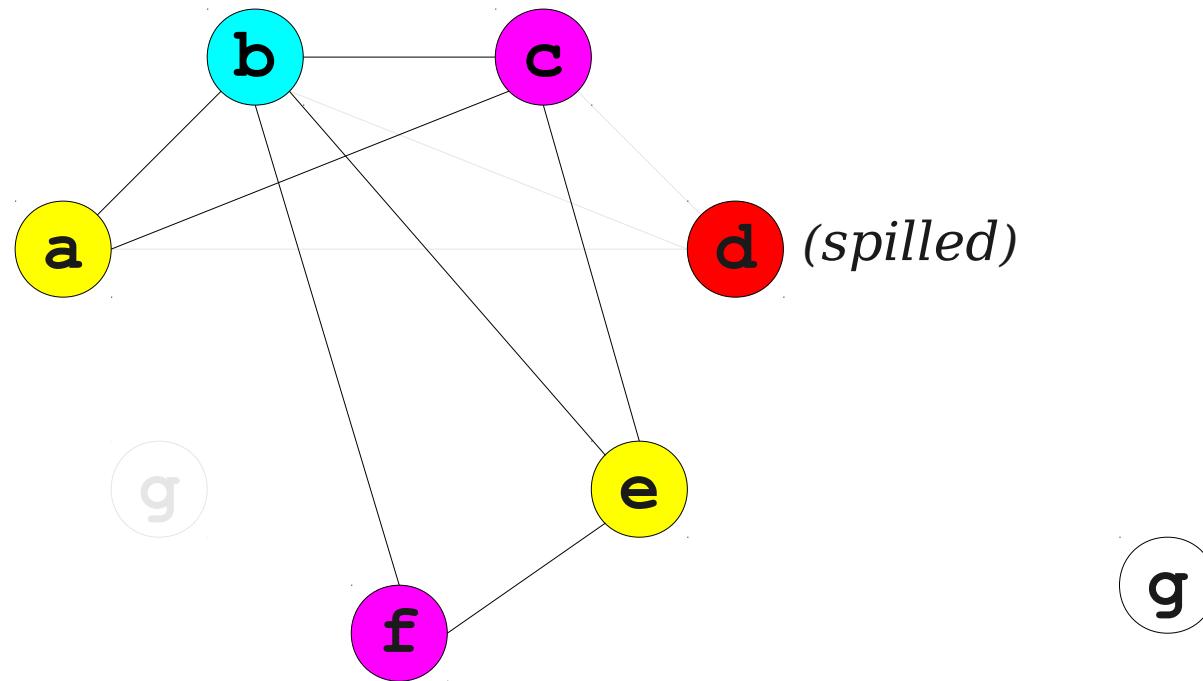
Chaitin's Algorithm Reloaded



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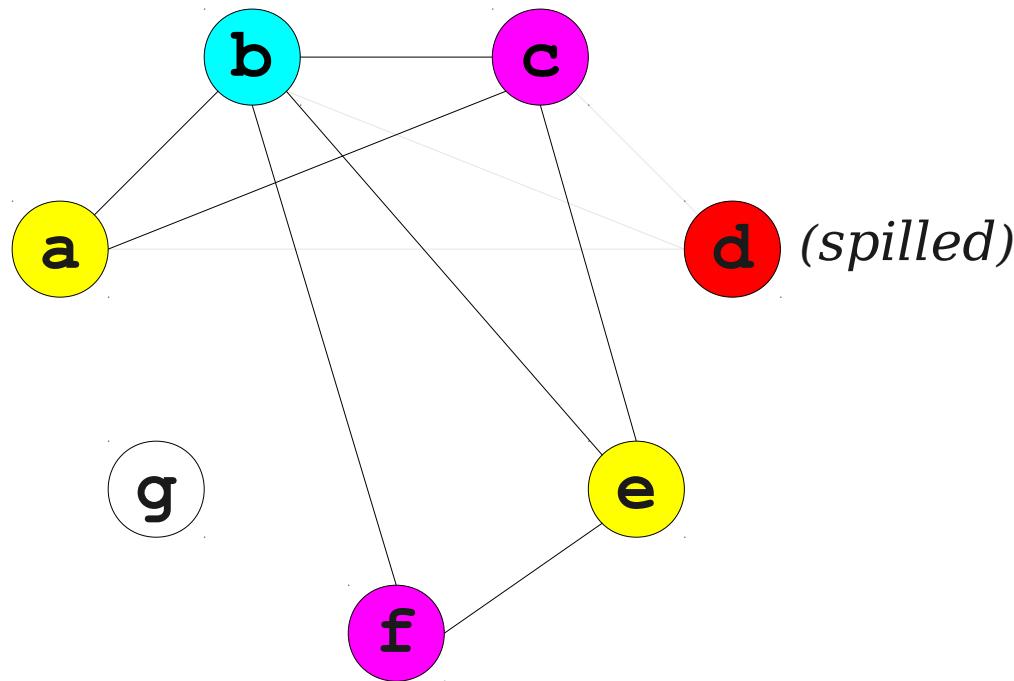
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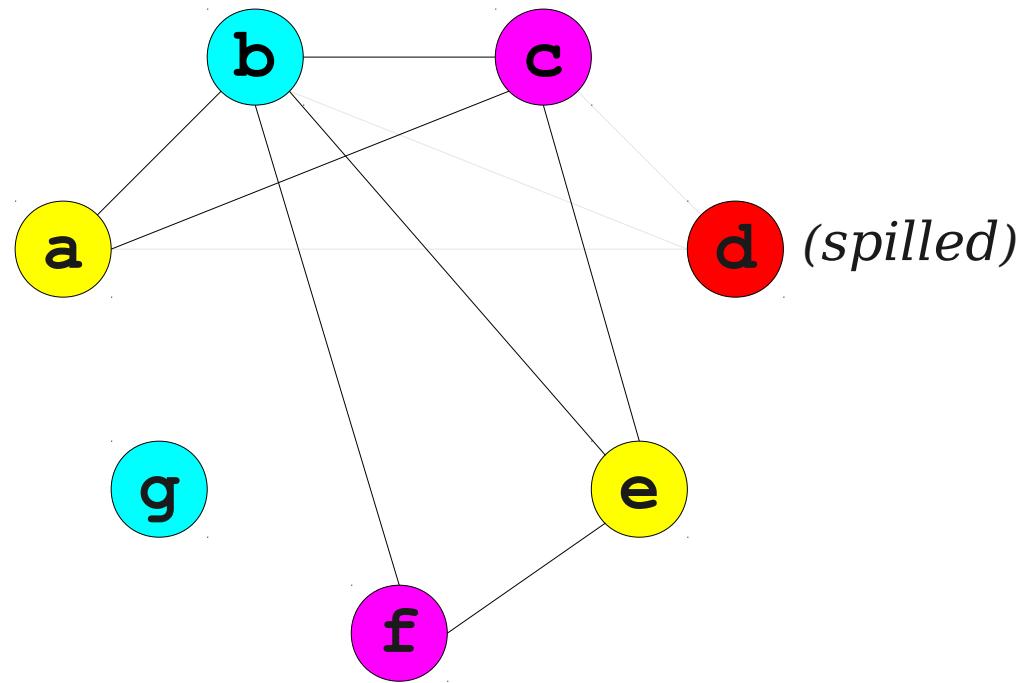
Chaitin's Algorithm Reloaded



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Chaitin's Algorithm Reloaded

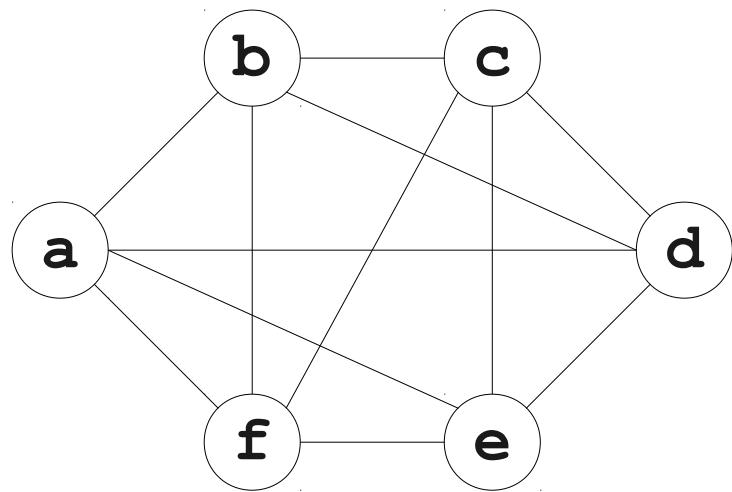


Registers

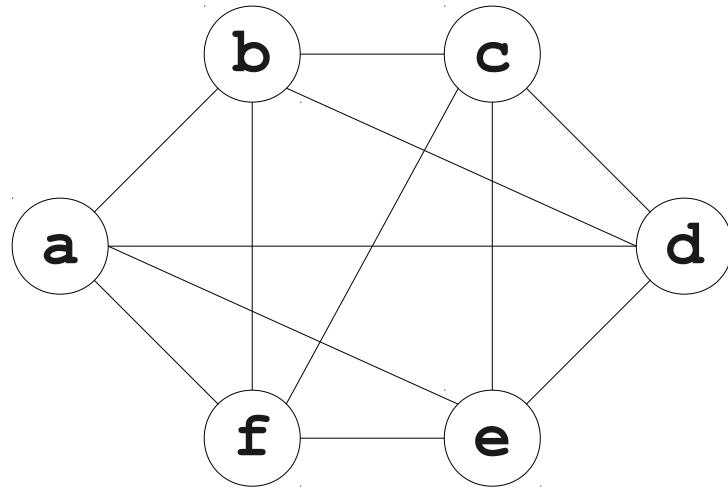
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Another Example

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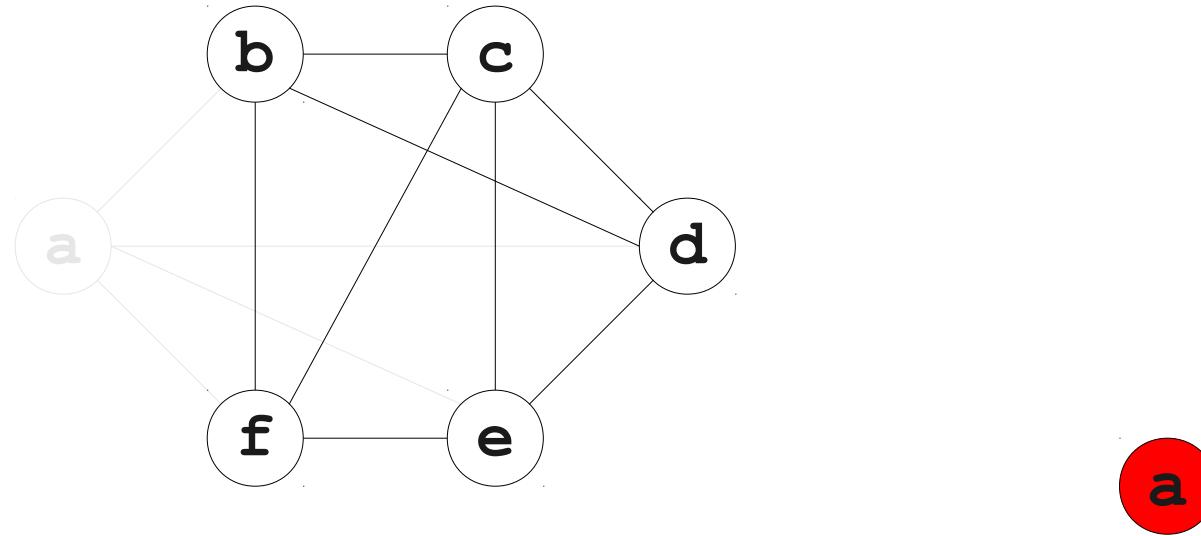
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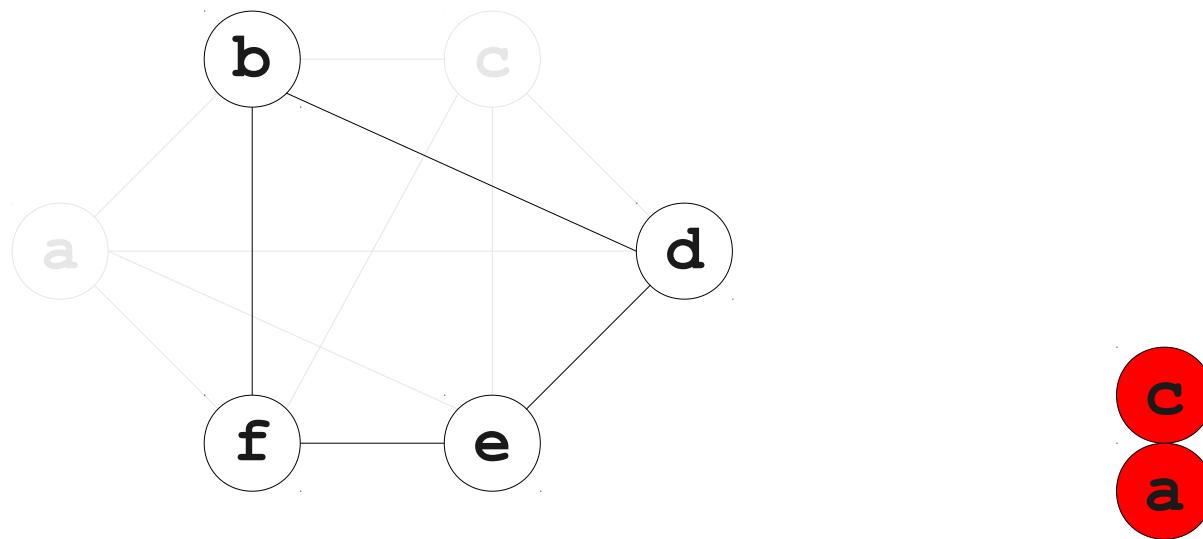


a

Registers

R_0	R_1	R_2
-------	-------	-------

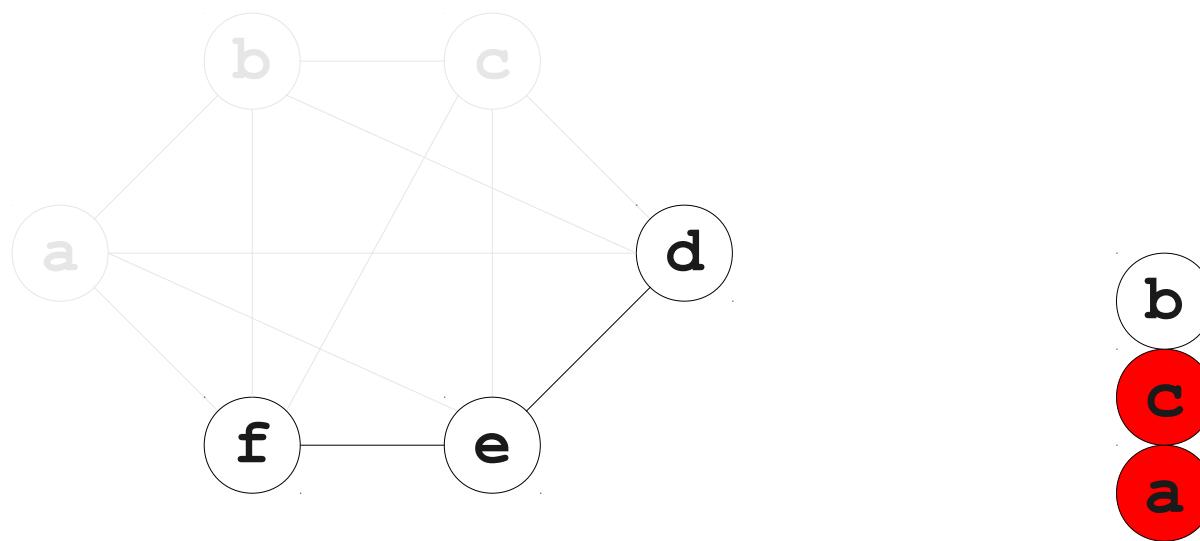
Another Example



Registers

R₀	R₁	R₂
----------------------	----------------------	----------------------

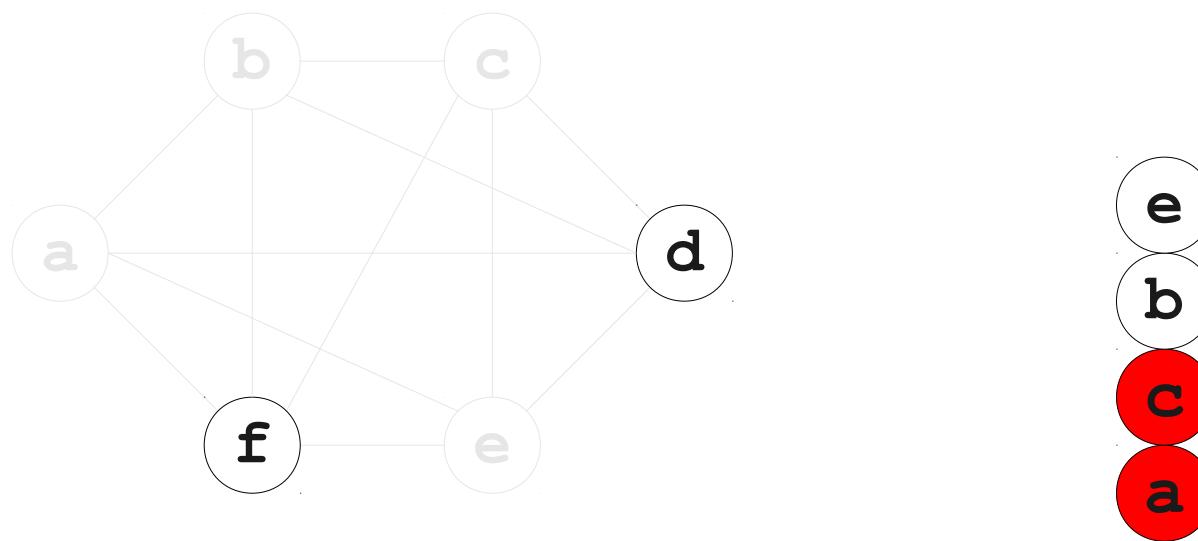
Another Example



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R_0	R_1	R_2
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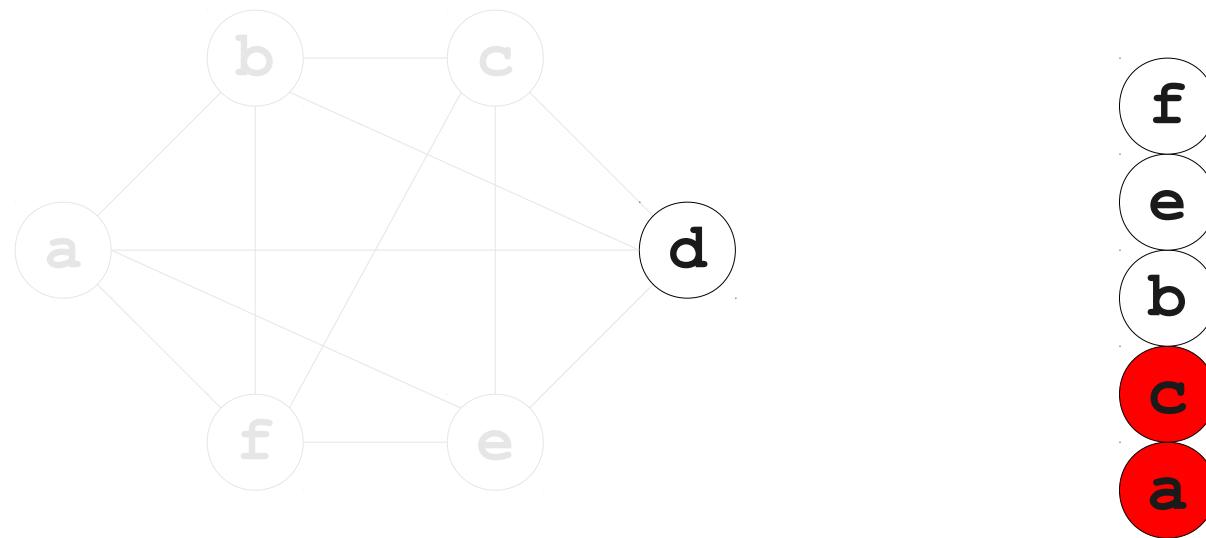
Another Example



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R₀	R₁	R₂
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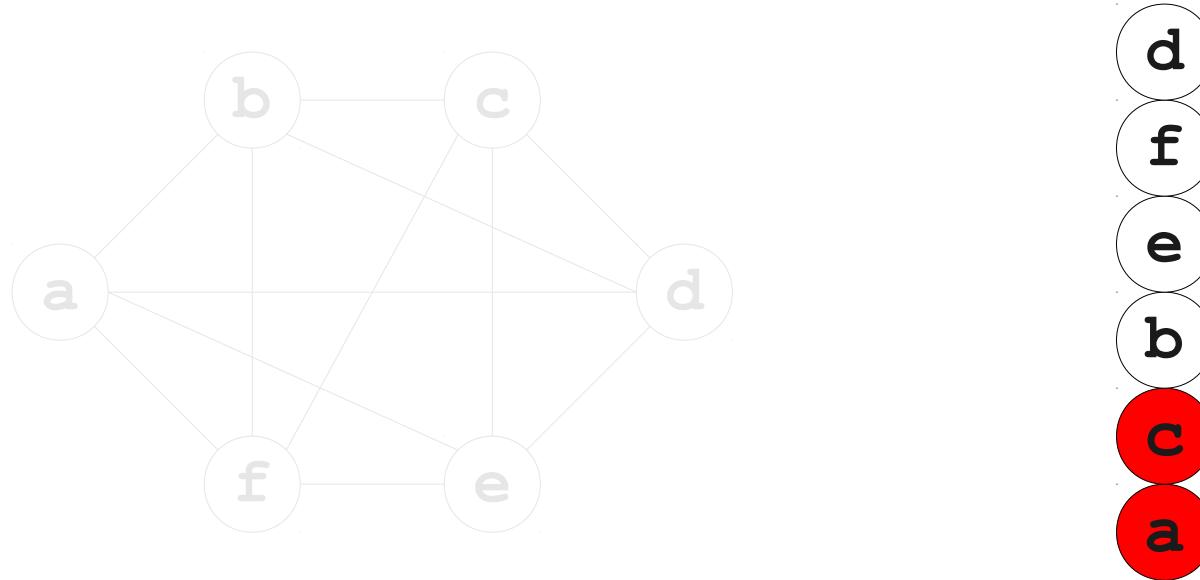
Another Example



Registers

R₀ **R₁** **R₂**

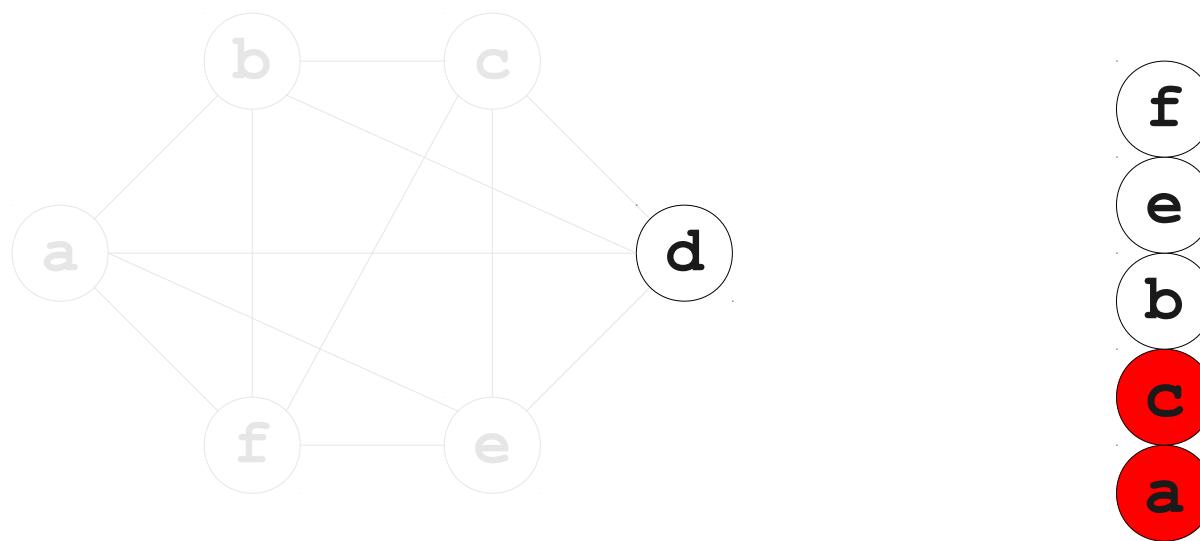
Another Example



Registers

R_0 **R_1** **R_2**

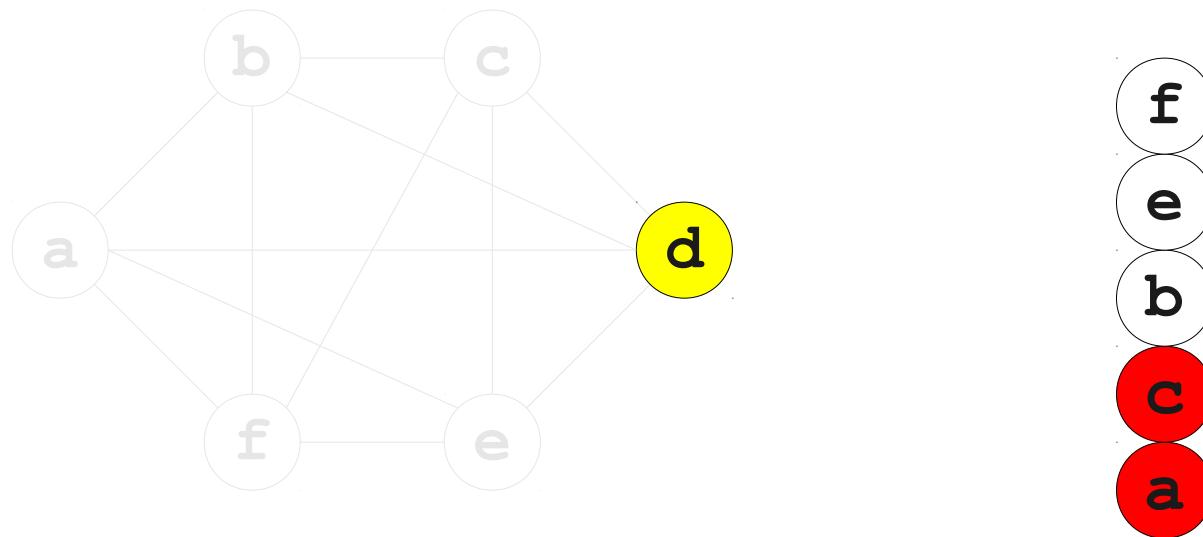
Another Example



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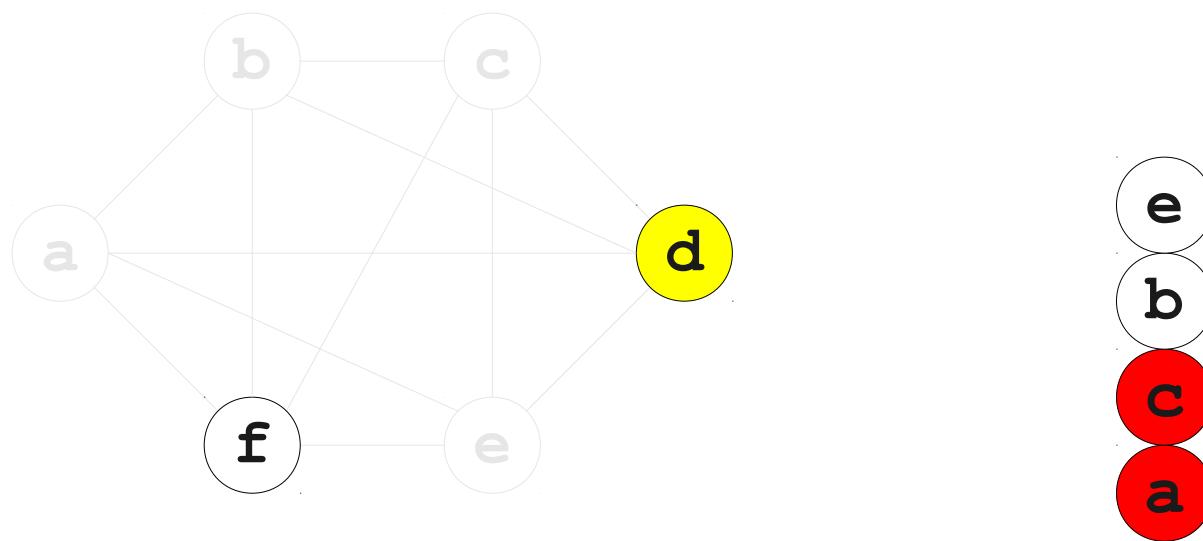
Another Example



Registers

R_0	R_1	R_2
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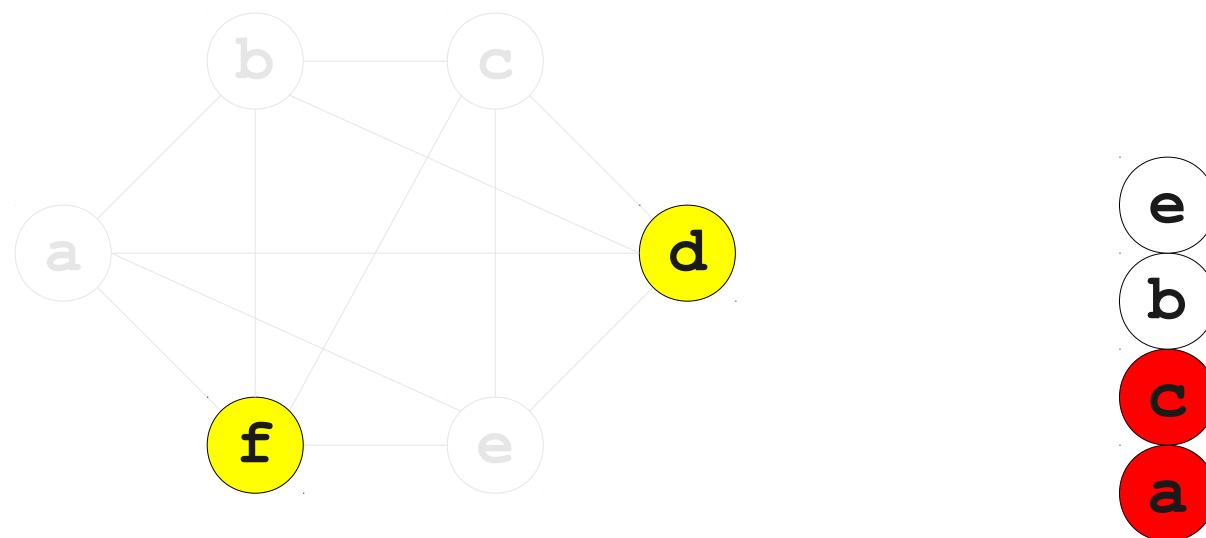
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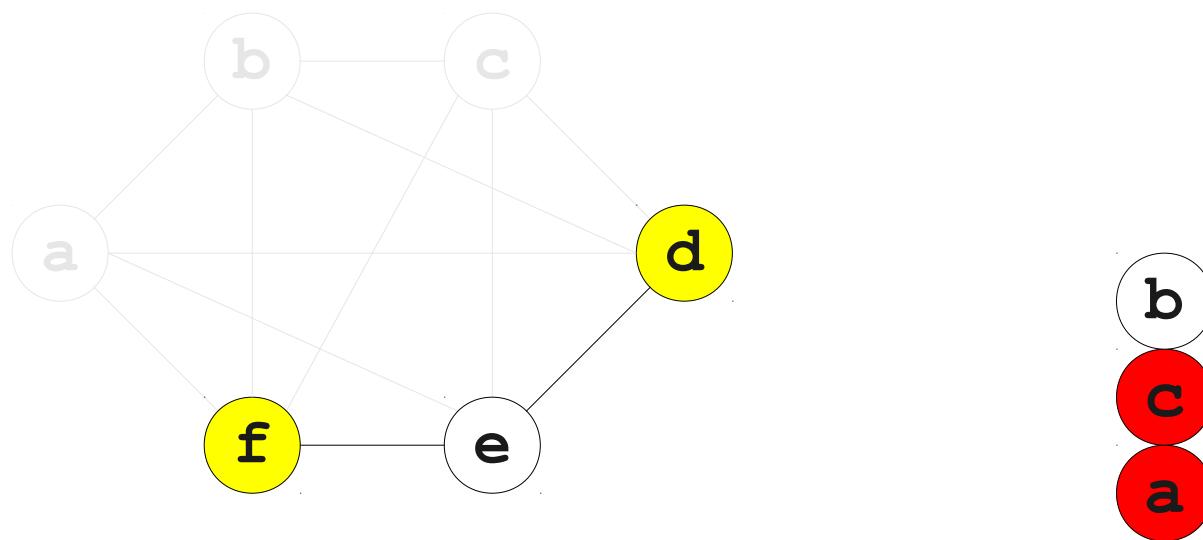
Another Example



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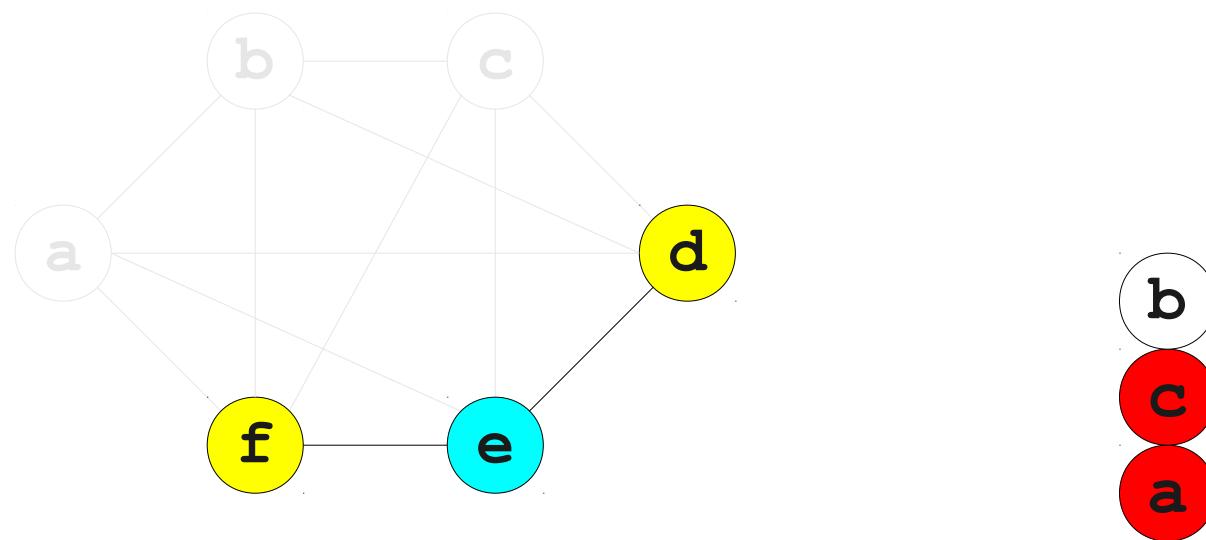
Another Example



Registers

R_0	R_1	R_2
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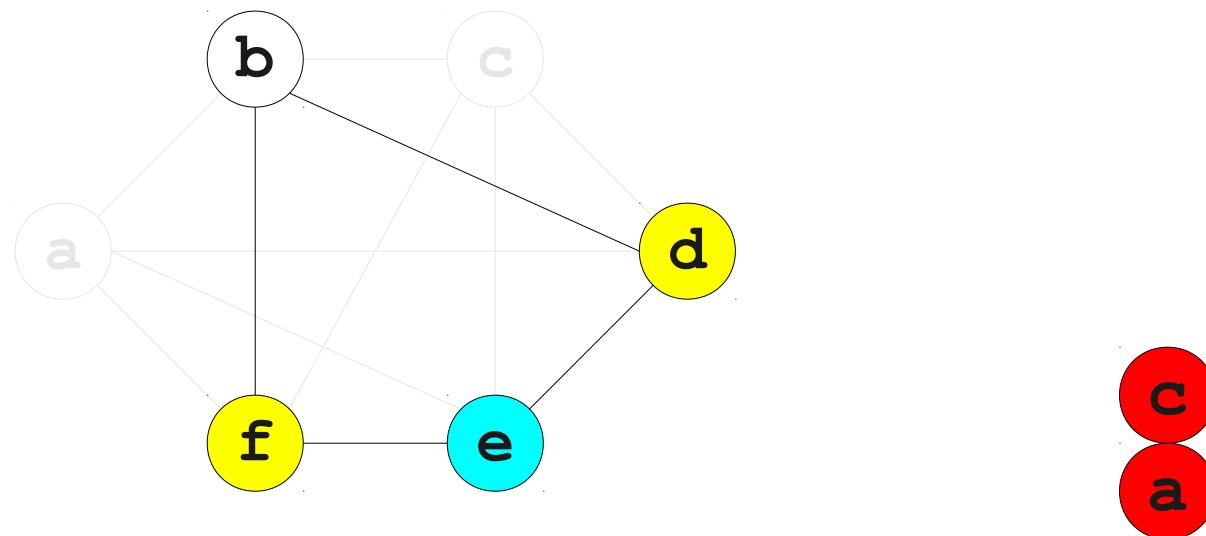
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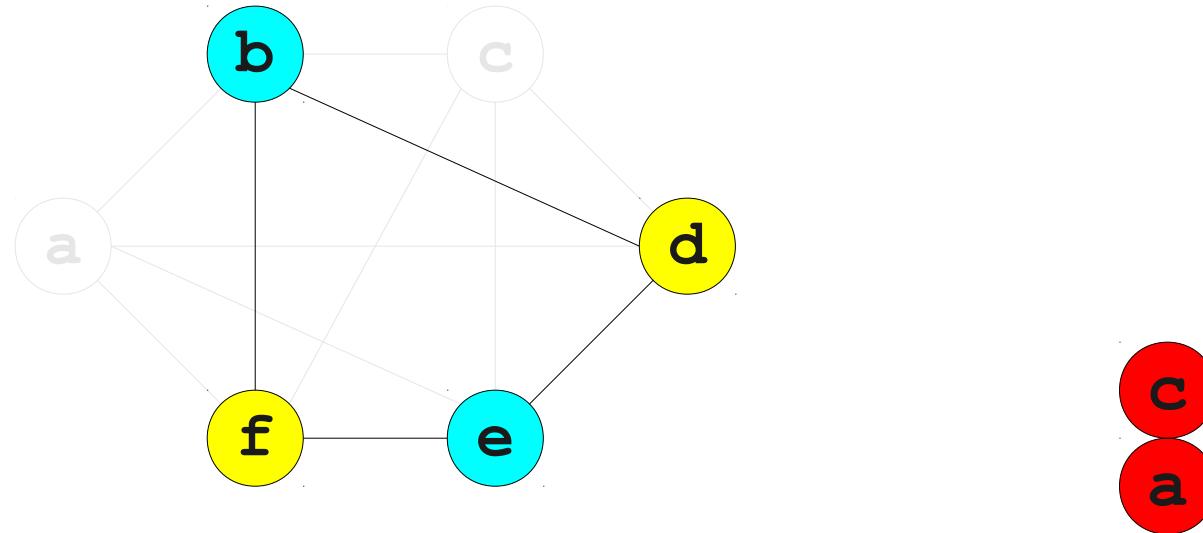
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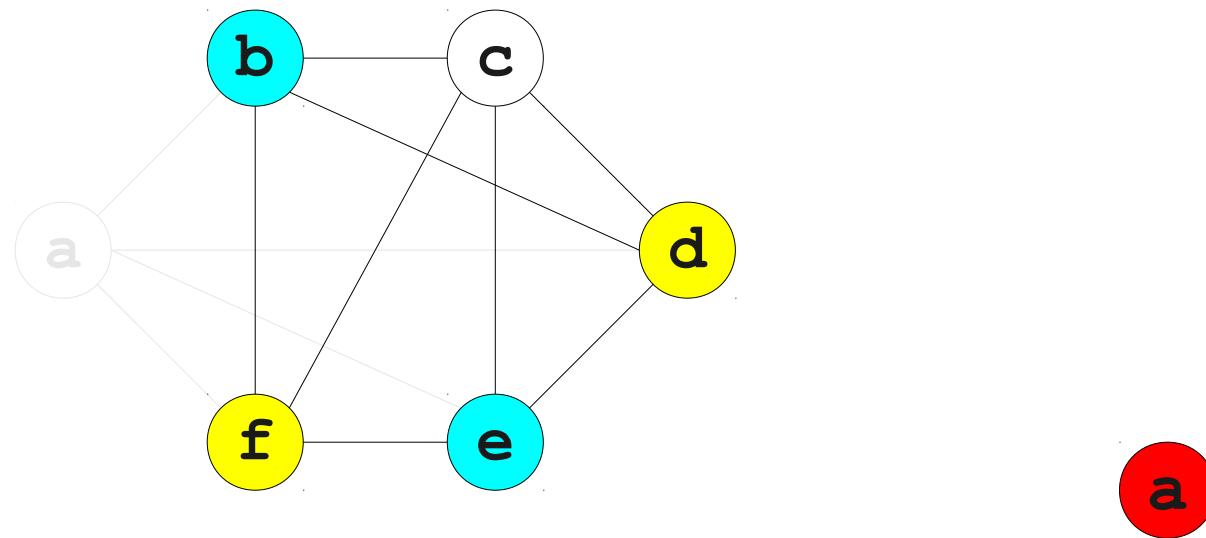
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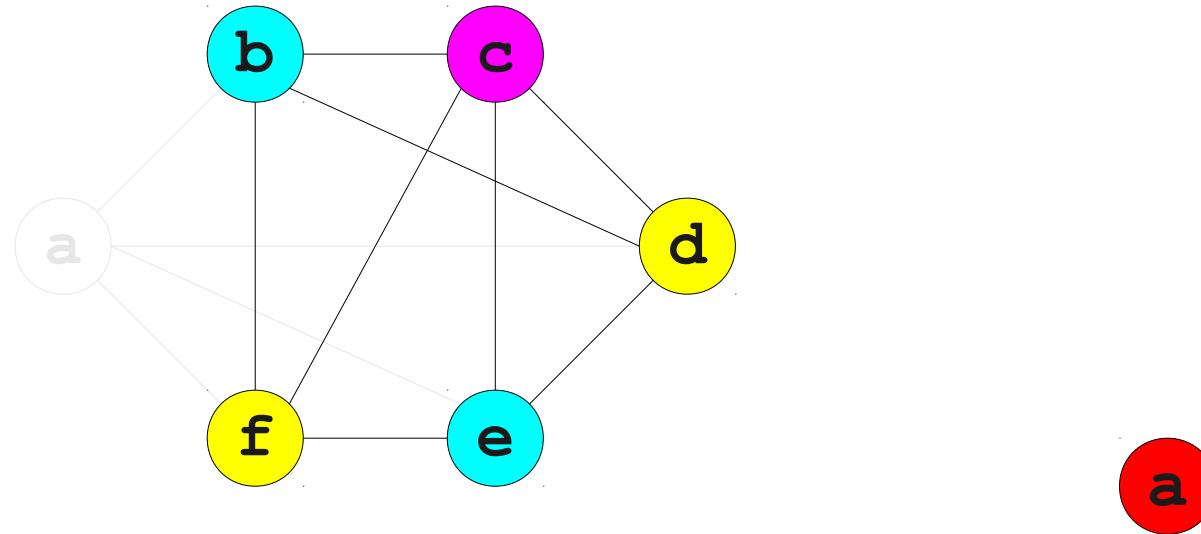
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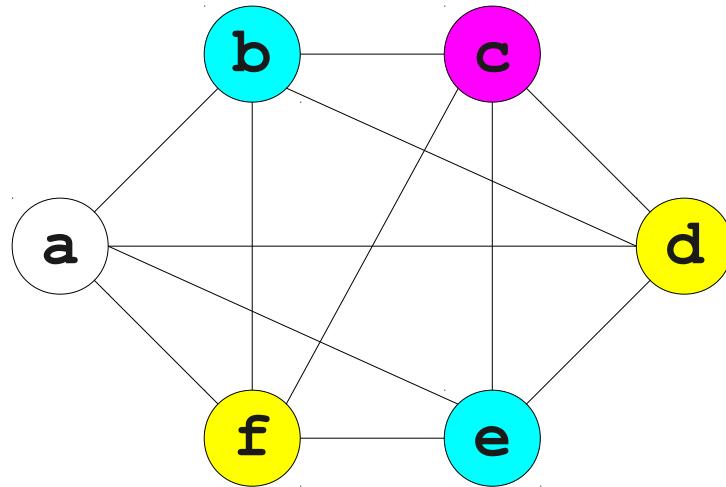
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Registers

R₀	R₁	R₂
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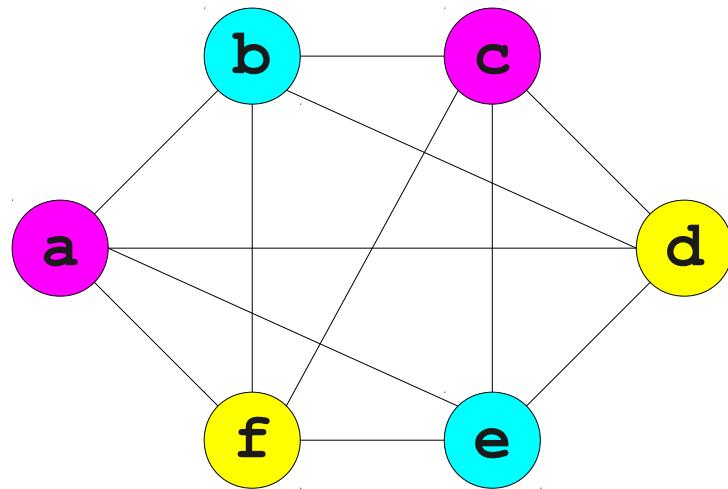
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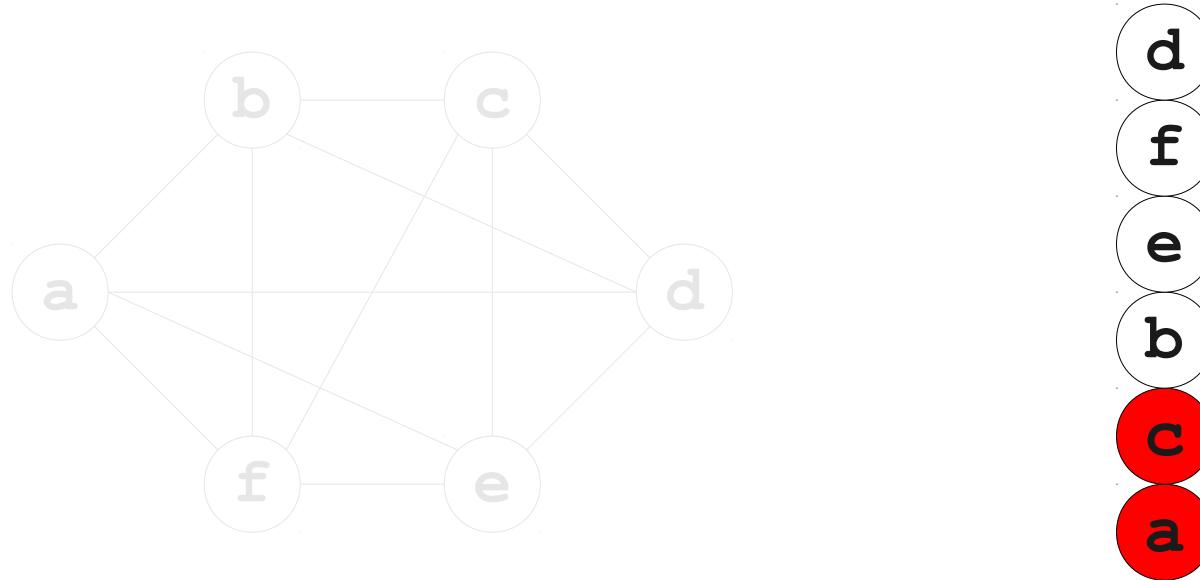
Another Example



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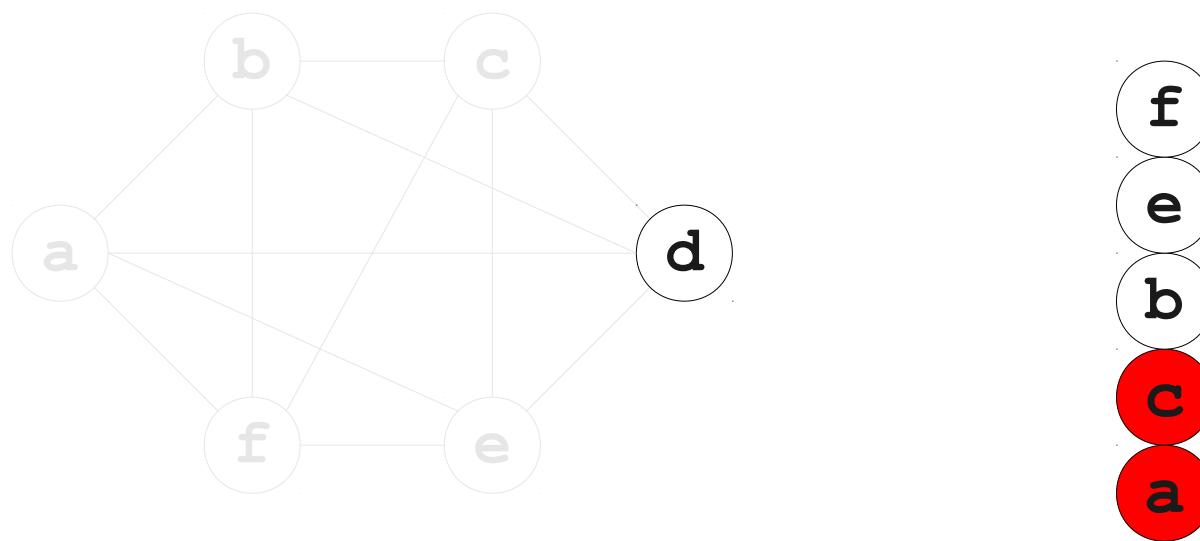
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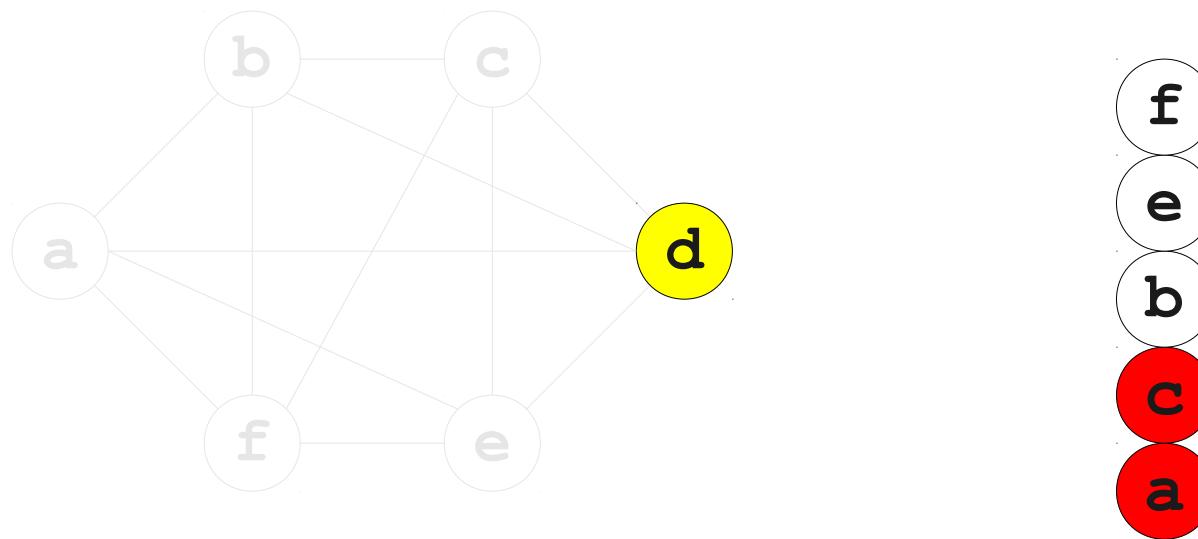
Another Example



Registers

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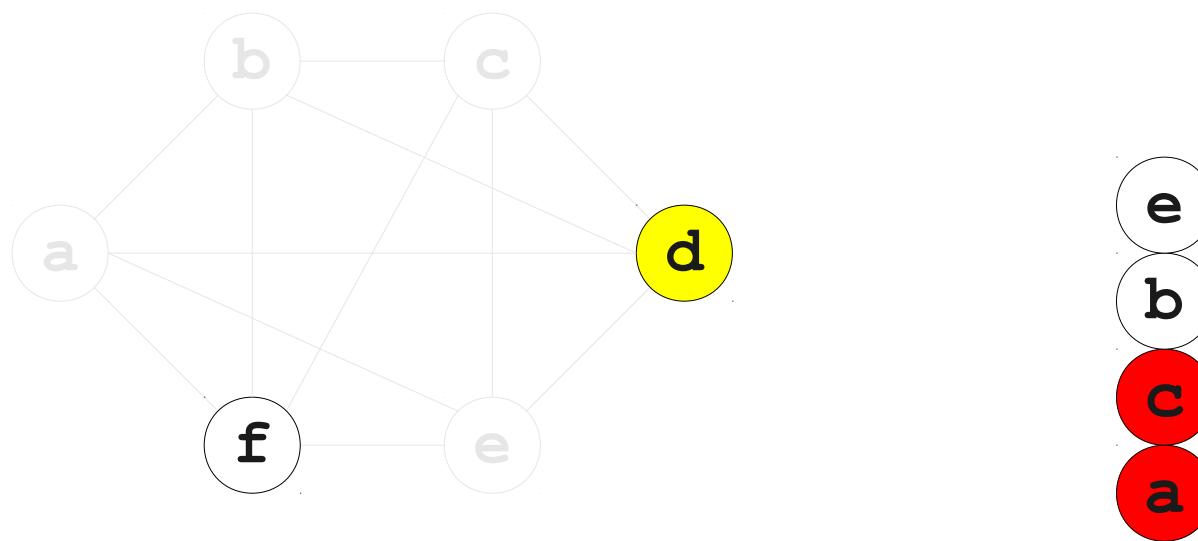
Another Example



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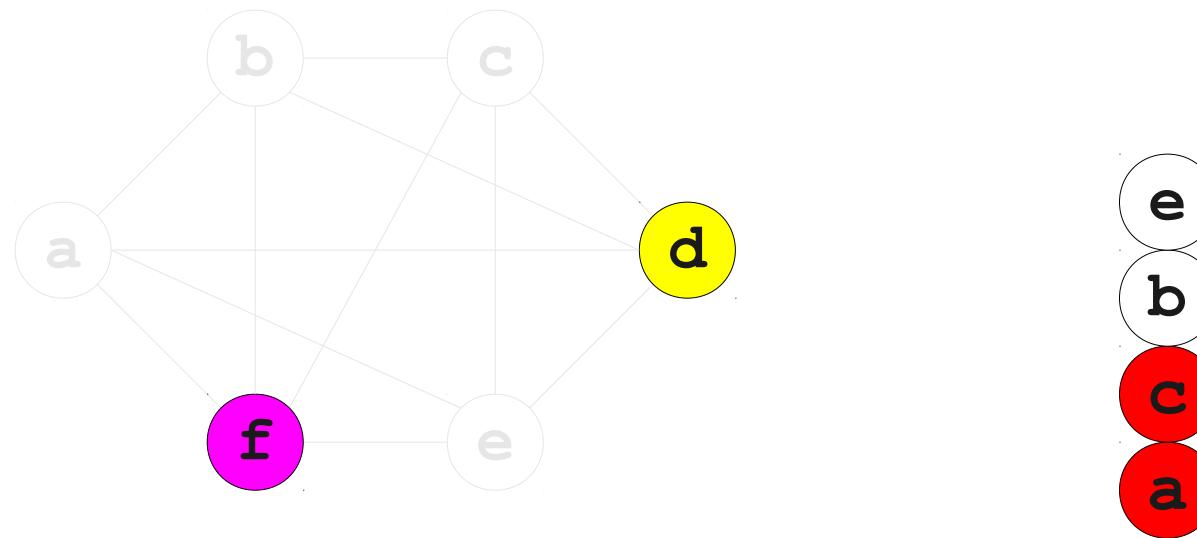
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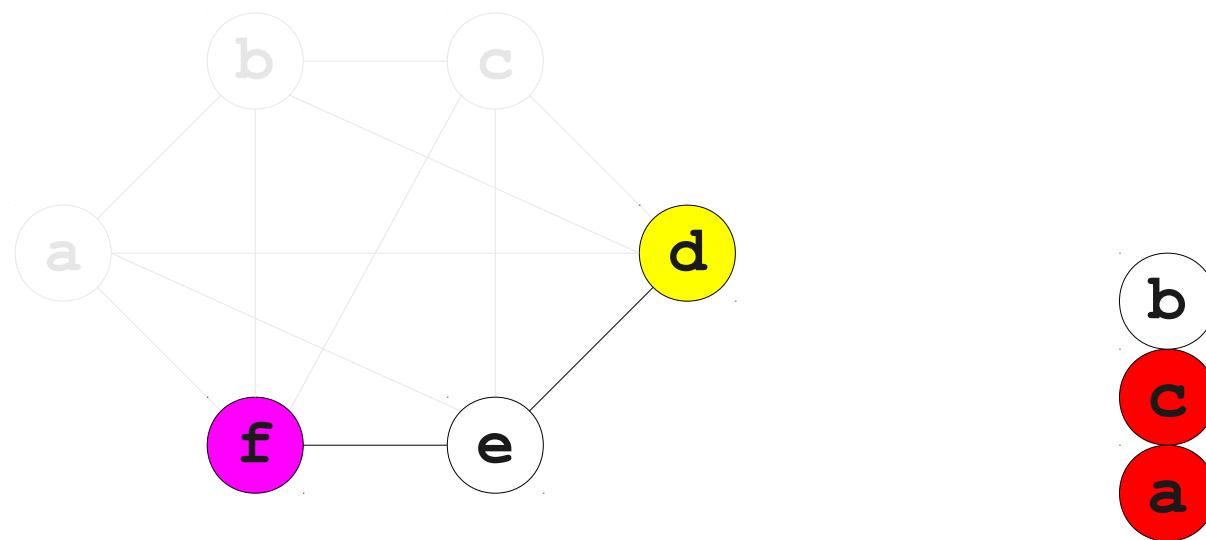
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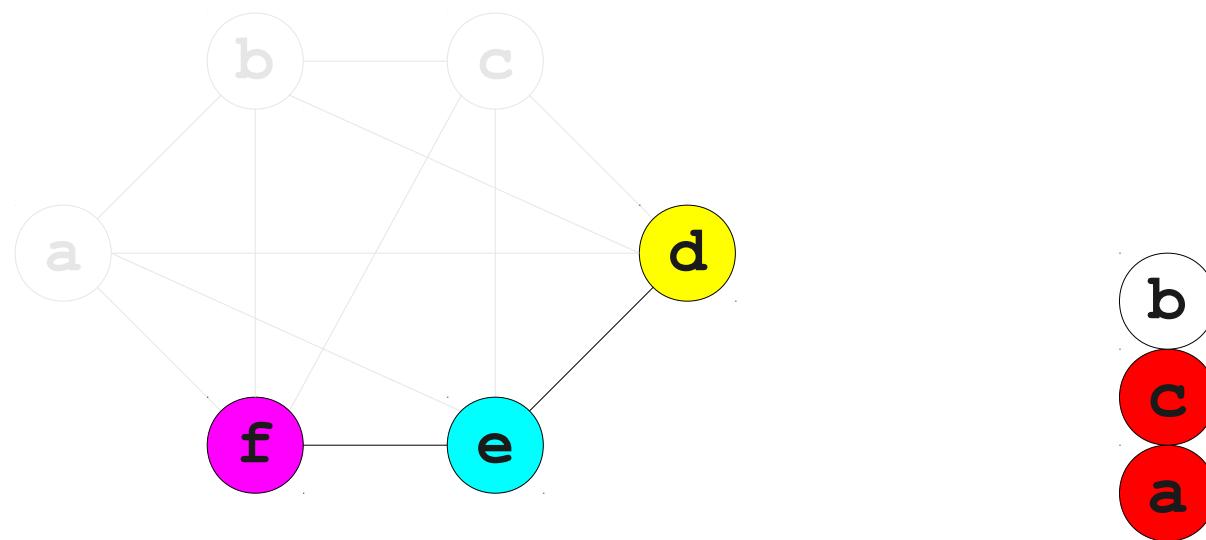
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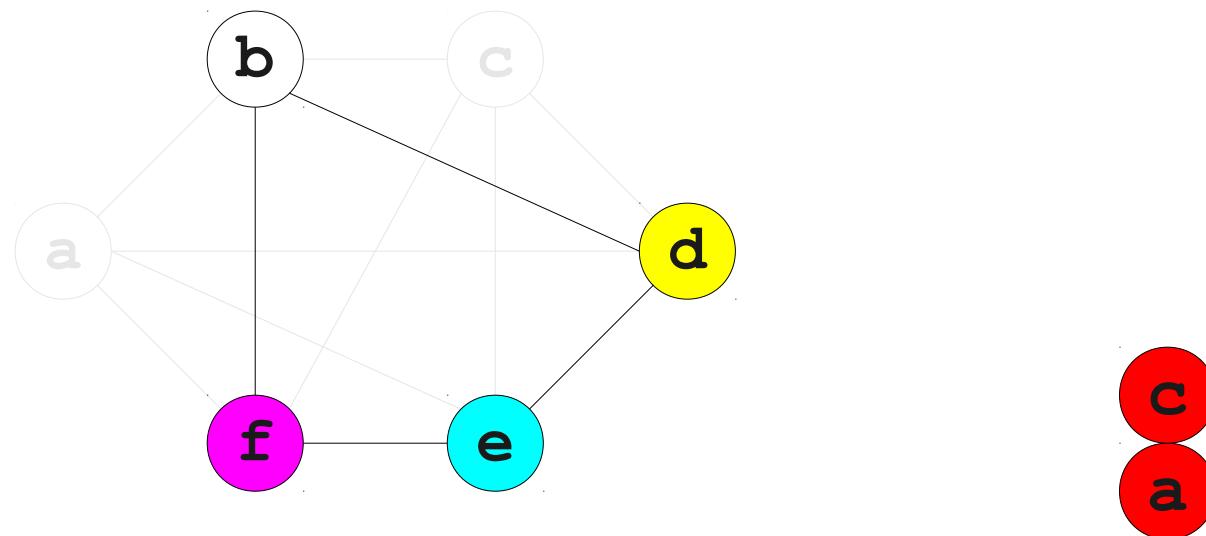
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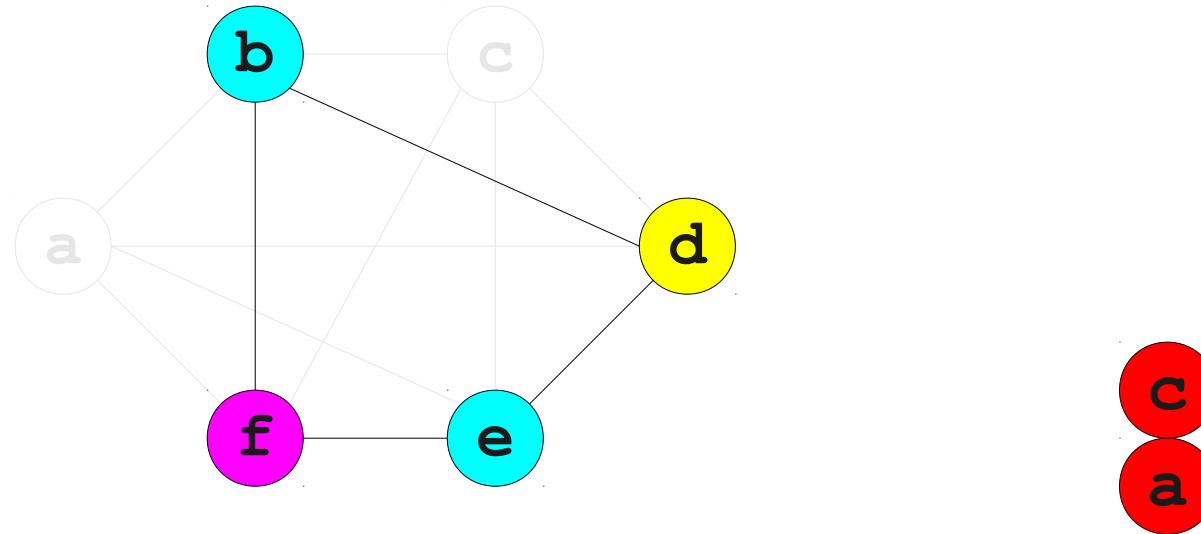
Another Example



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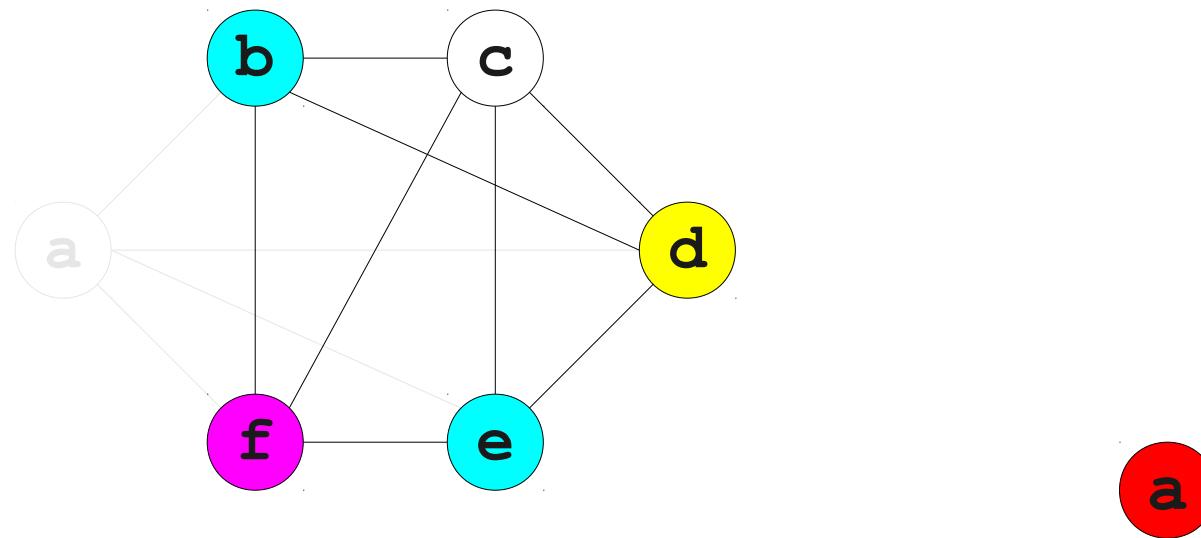
Another Example



Registers

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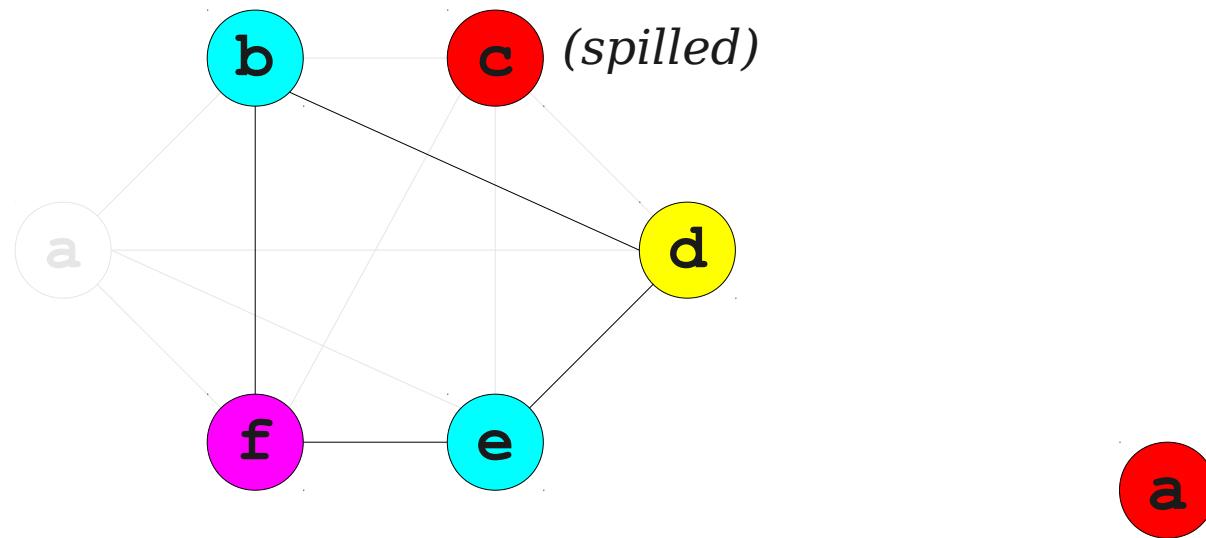
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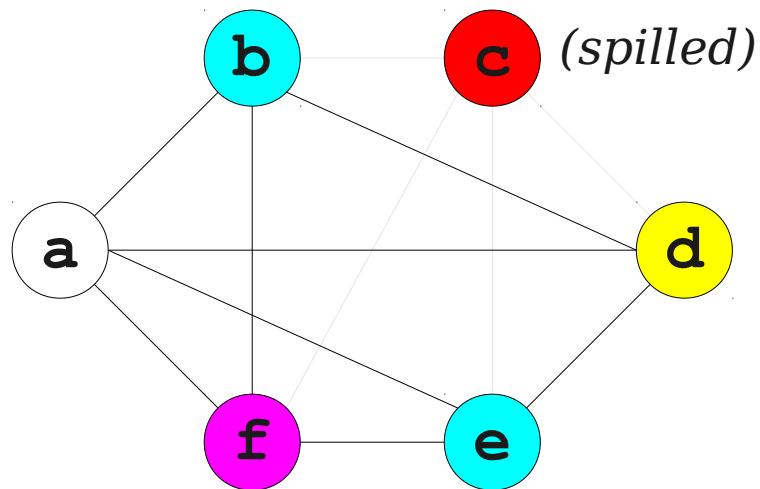
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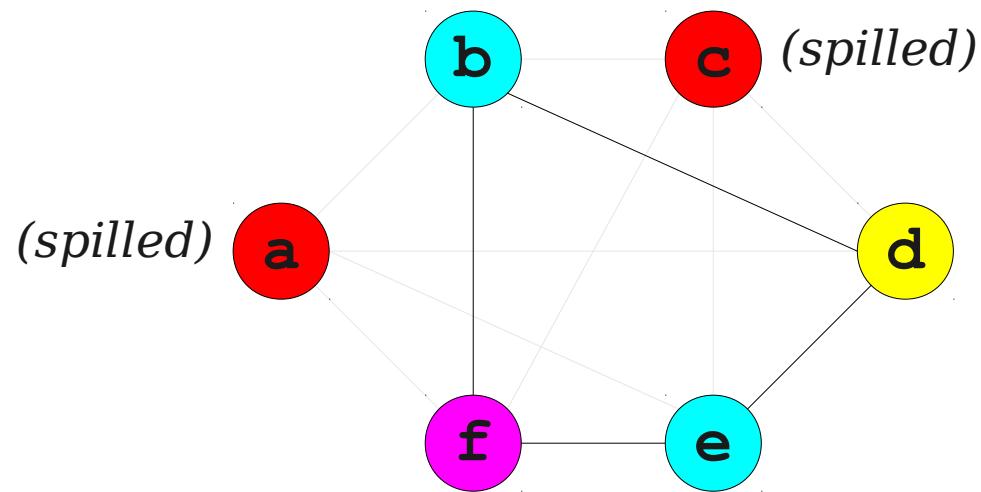
Another Example



Registers



Another Example



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Chaitin's Algorithm

Chaitin's algorithm is efficient ($O(|V| + |E|)$), simple to implement

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Chaitin's algorithm is efficient ($O(|V| + |E|)$), simple to implement

- How good the coloring is depends on the order we color the nodes to the graph
 - called the **elimination ordering**
- For every graph, there is a elimination ordering such that Chaitin's algorithm produces an optimal coloring
 - therefore finding this optimal elimination ordering for a general graph is NP-complete

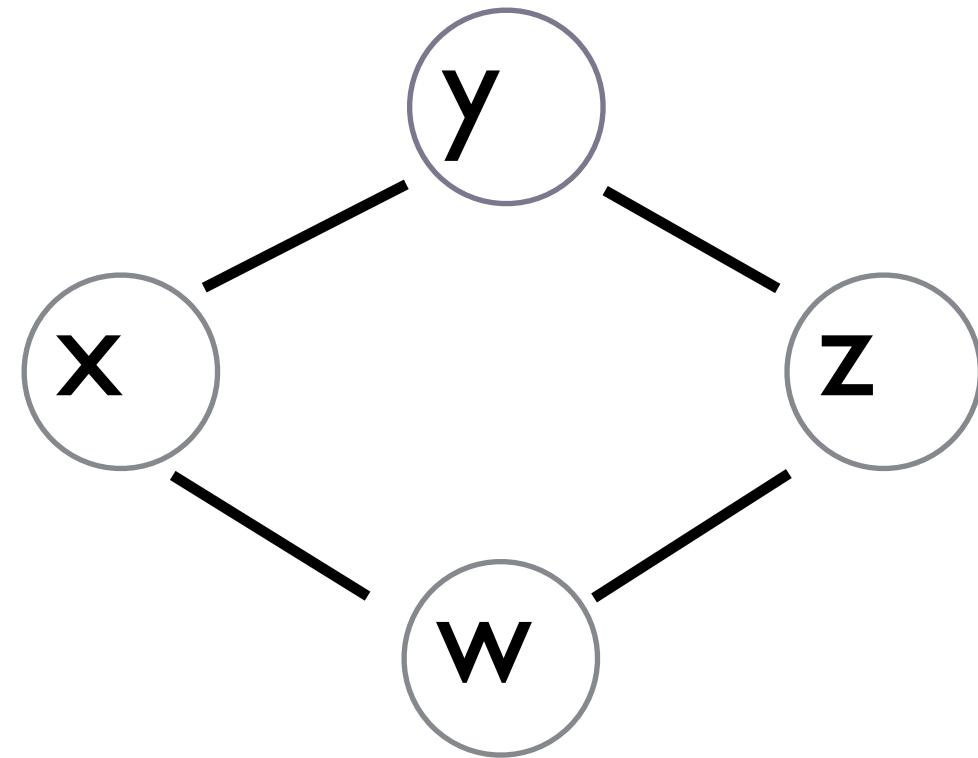
Graph Coloring SSA Programs

Hack et al, "Register Allocation for Programs in SSA-Form",
Compiler Construction 2006

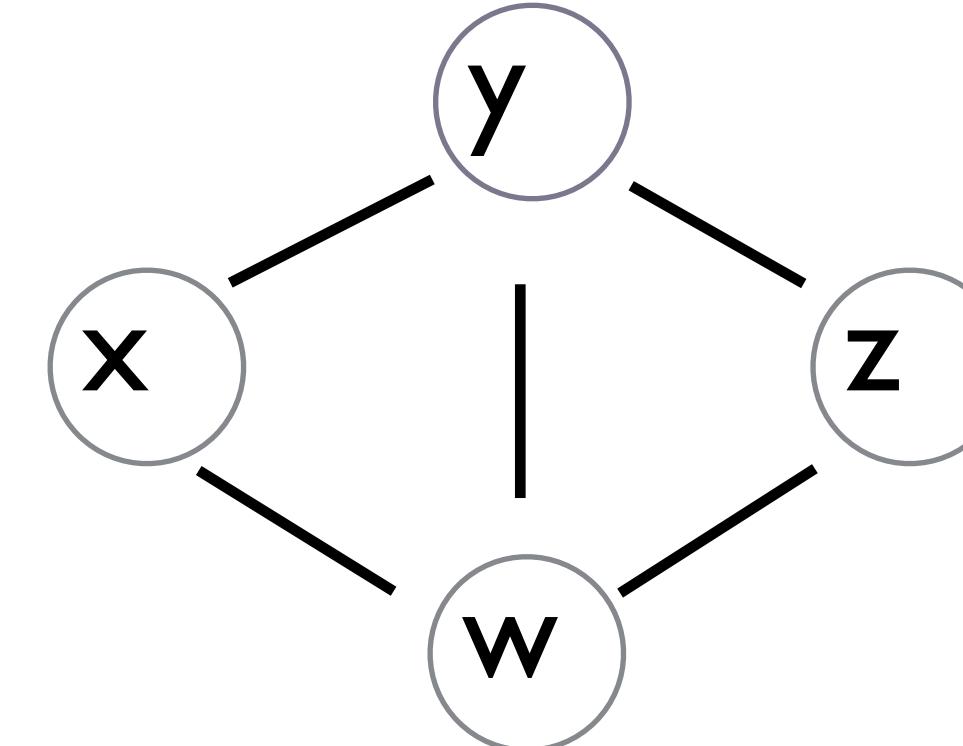
Graph Coloring SSA Programs

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- The interference graphs of an SSA program are all **chordal**
 - Every cycle ≥ 4 nodes has a **chord**



Not chordal



chordal

Coloring Chordal Graphs

Theorem: Every chordal graph has a **perfect elimination ordering**

Coloring Chordal Graphs

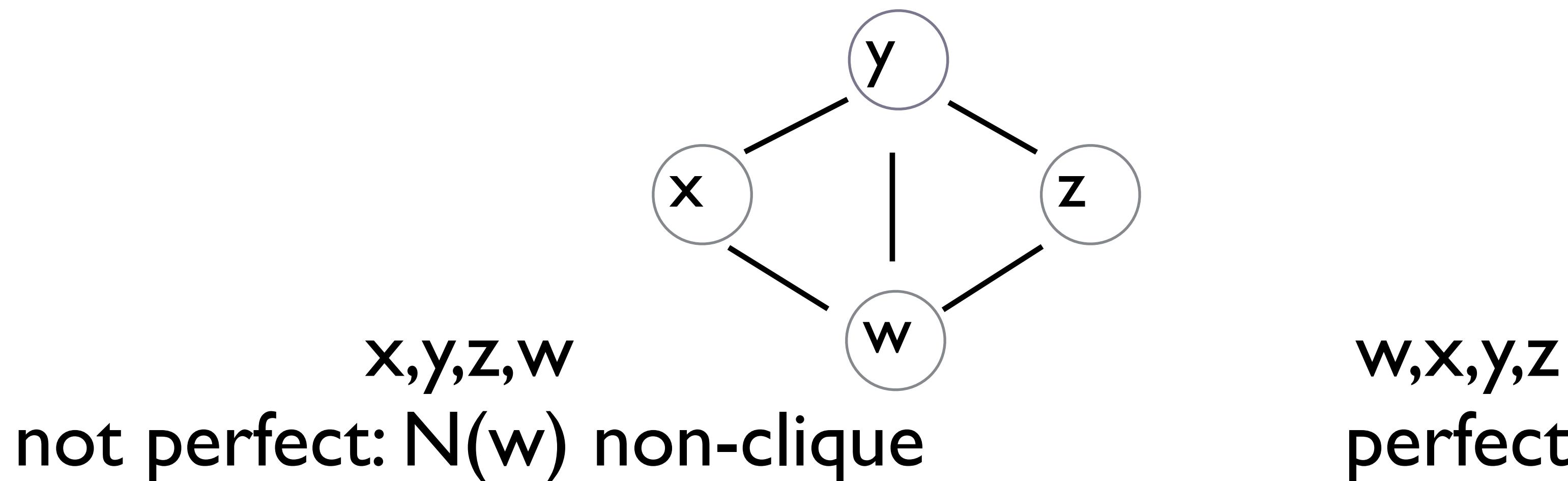
Theorem: Every chordal graph has a **perfect elimination ordering**

- a total ordering of nodes v_1, v_2, v_3, \dots such that for each v_i , v_i forms a clique with all its neighbors earlier in the order
- Chaitin's algo produces an optimal coloring if we use a PEO

Coloring Chordal Graphs

Theorem: A graph is chordal iff it has a **perfect elimination ordering (PEO)**

- a total ordering of nodes v_1, v_2, v_3, \dots such that for each v_i , v_i forms a clique with all its neighbors earlier in the order
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Coloring Chordal Graphs

Theorem: A graph is chordal iff it has a **perfect elimination ordering (PEO)**

Theorem: A graph is chordal iff it is the intersection graph of a group of subtrees of a tree

- In an SSA program, each variable's liveness is a subtree of the AST
- The interference graph is exactly the intersection graph of those subtrees
- Therefore, the interference graph of an SSA program is chordal!

SSA Interference Graphs are Chordal!

Every SSA Interference Graph is chordal

Chaitin's algorithm computes optimal coloring given a PEO

So we can color SSA interference graphs if we can find a PEO

SSA programs have a perfect elimination ordering that is easy to compute:

"in-scope" or "dominance" relation

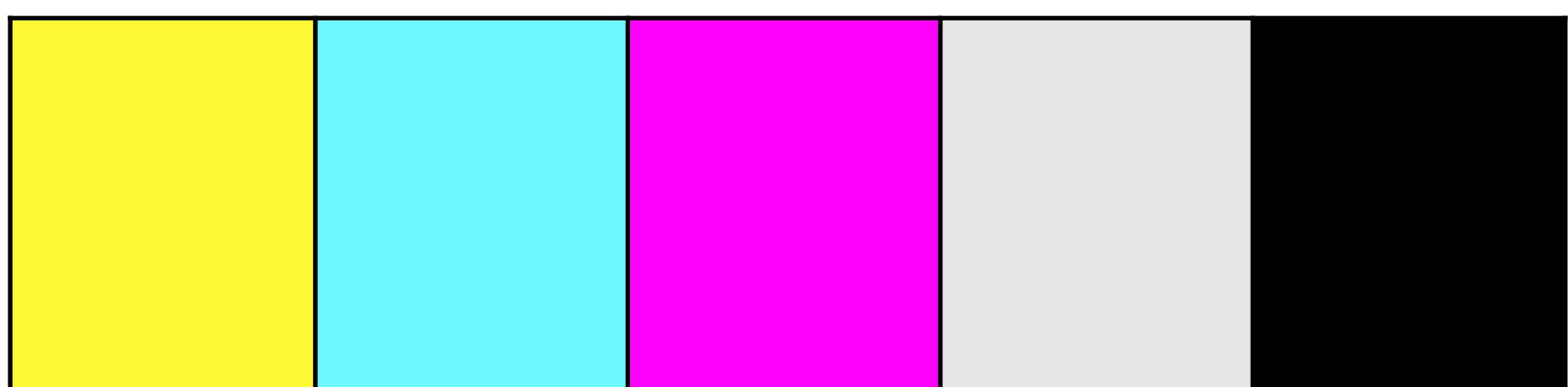
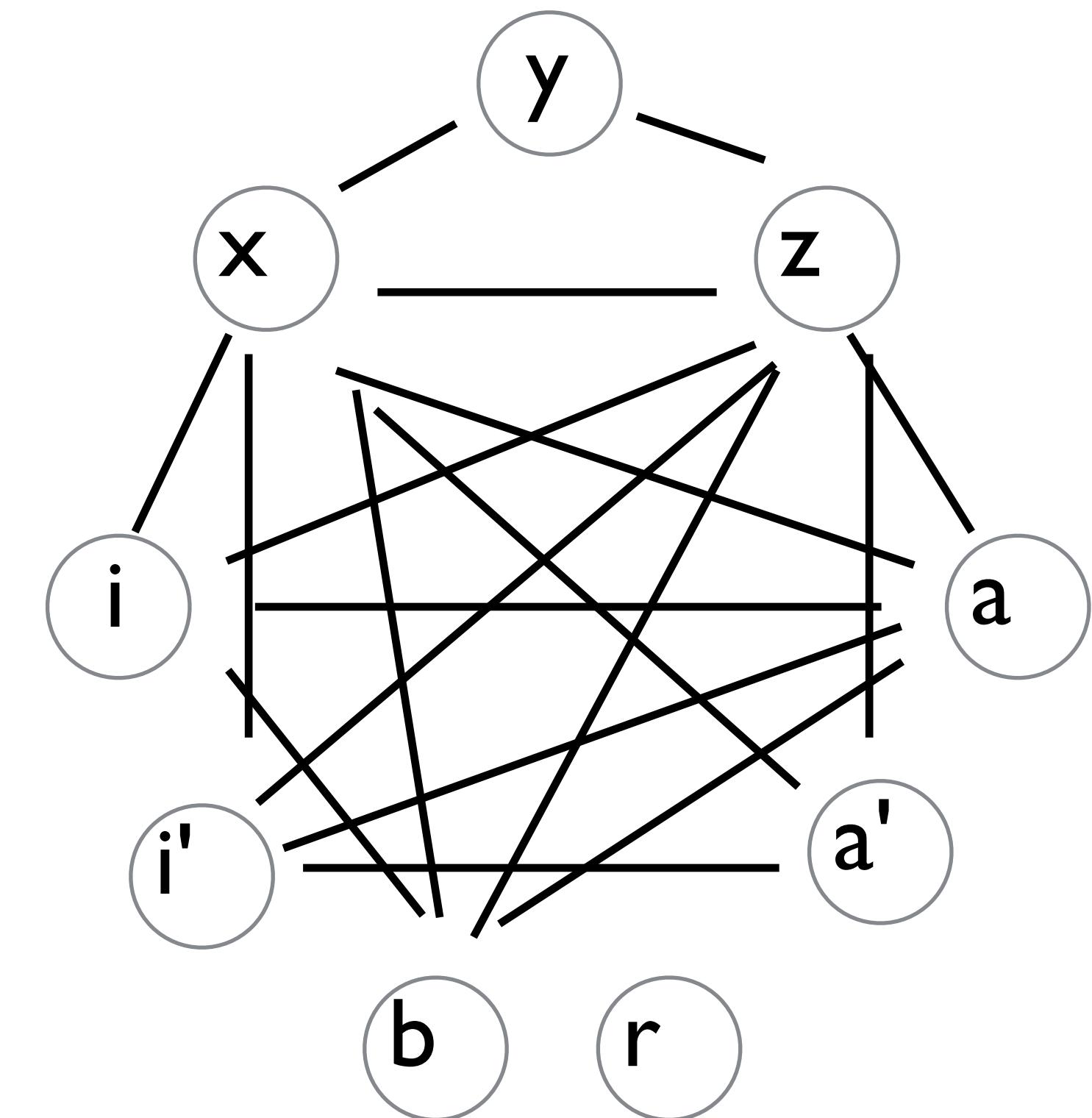
a variable x dominates y if x is in scope when y is defined
(includes simultaneous binding)

- x 's definition is "closer to the root" of the AST than y
- easy to compute: pre-order traversal of the nodes

SSA Program

```
f(x,y,z):  
loop(i,a):  
    thn():  
        r = a * z  
        ret r  
    els():  
        i' = i - 1  
        a' = a + x  
        br loop(i', a')  
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```

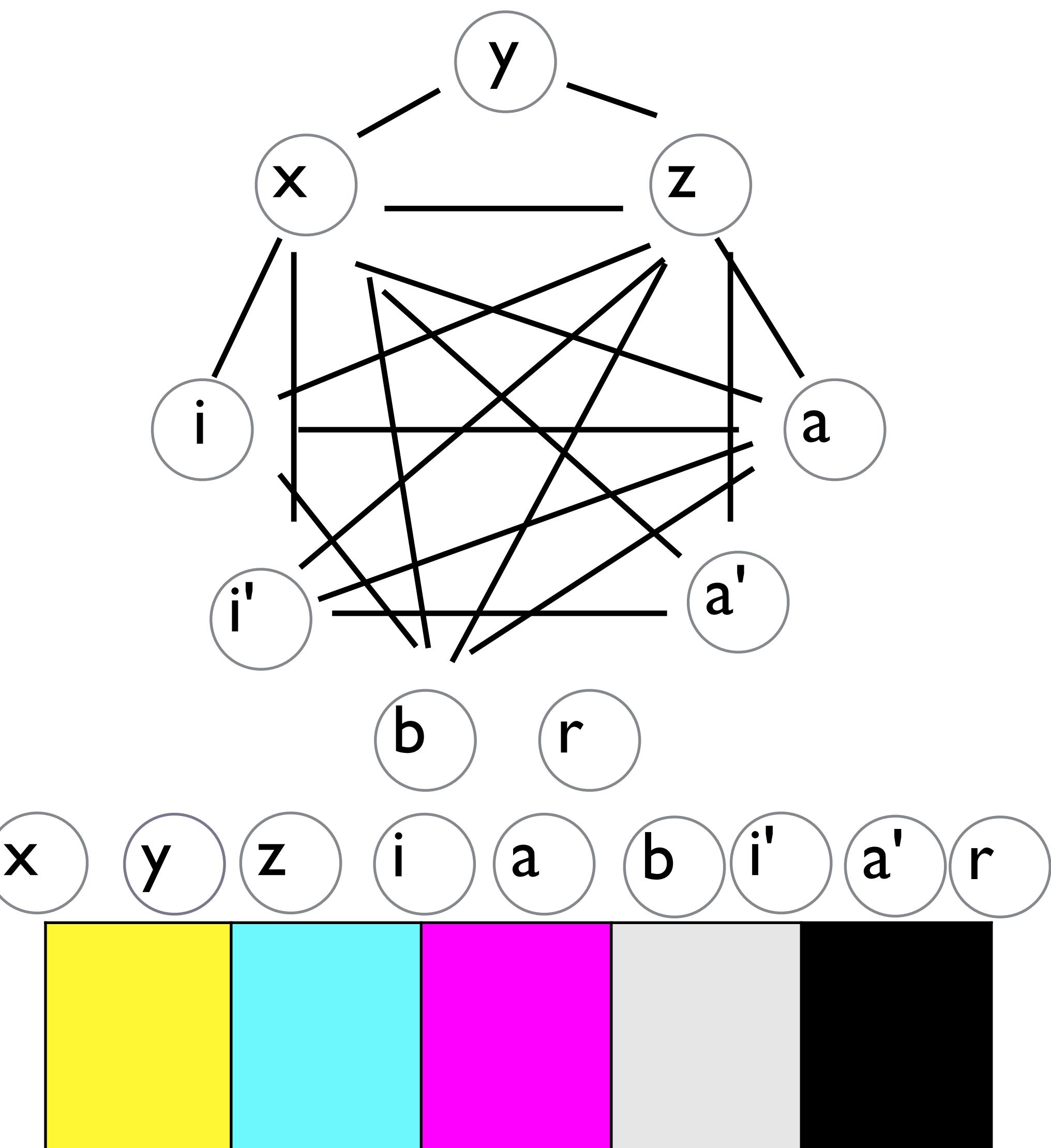
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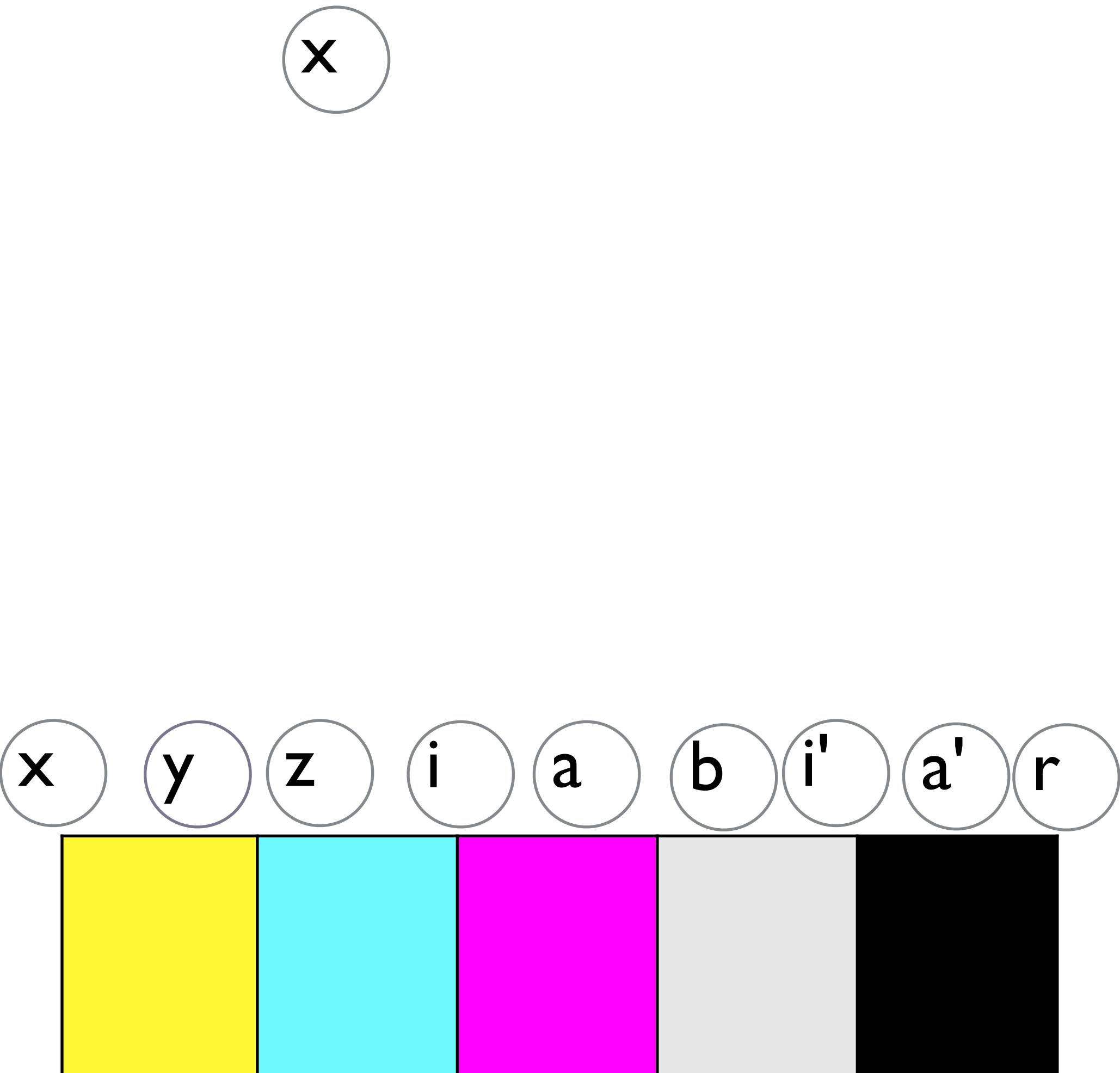
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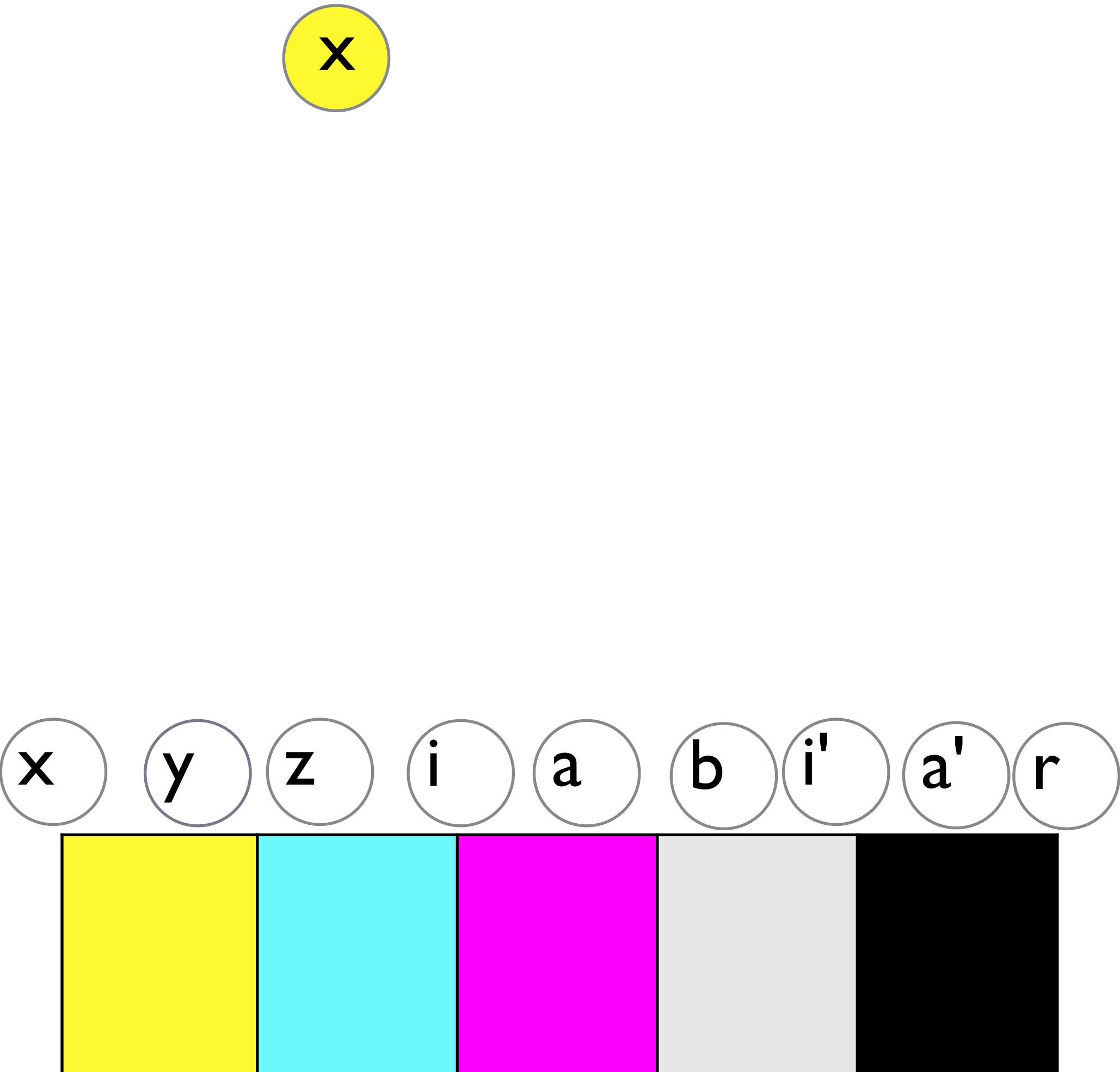
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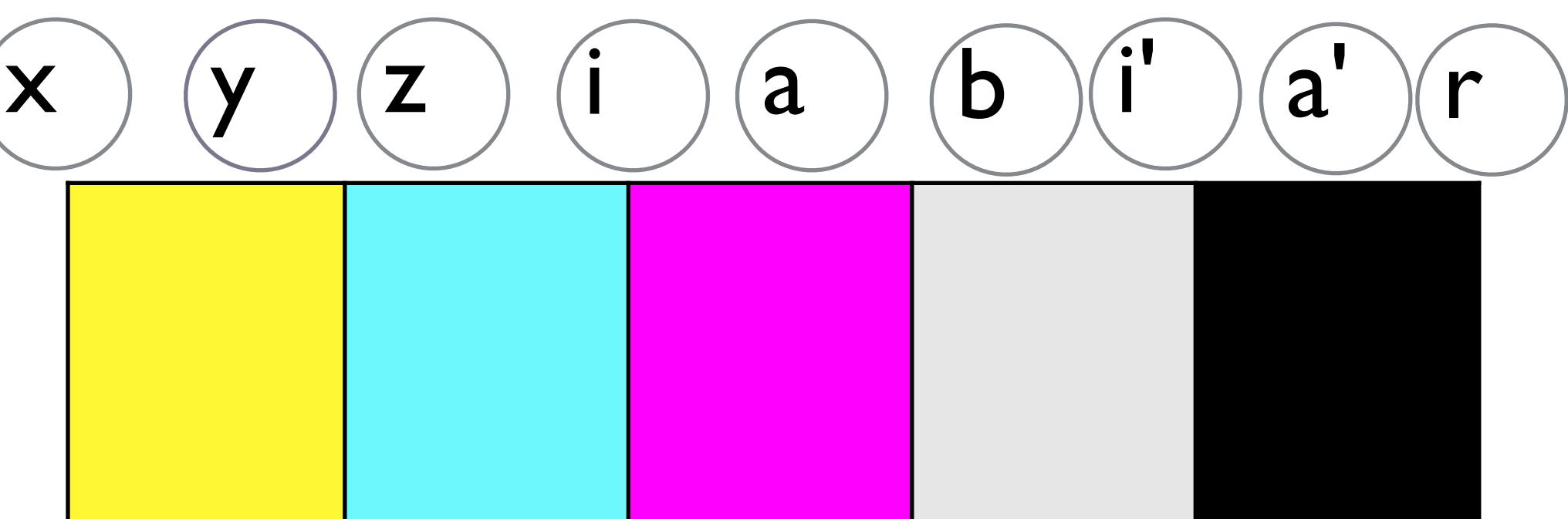
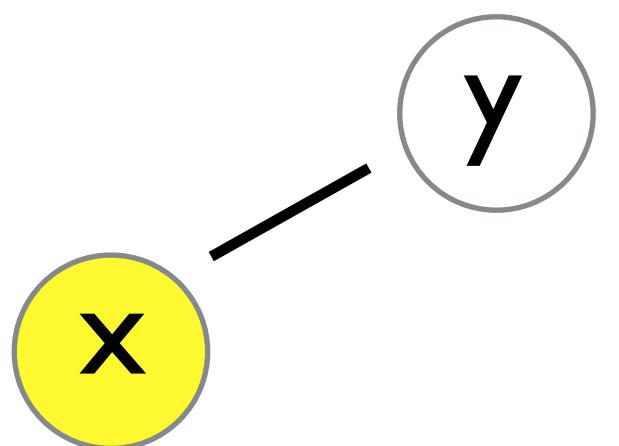
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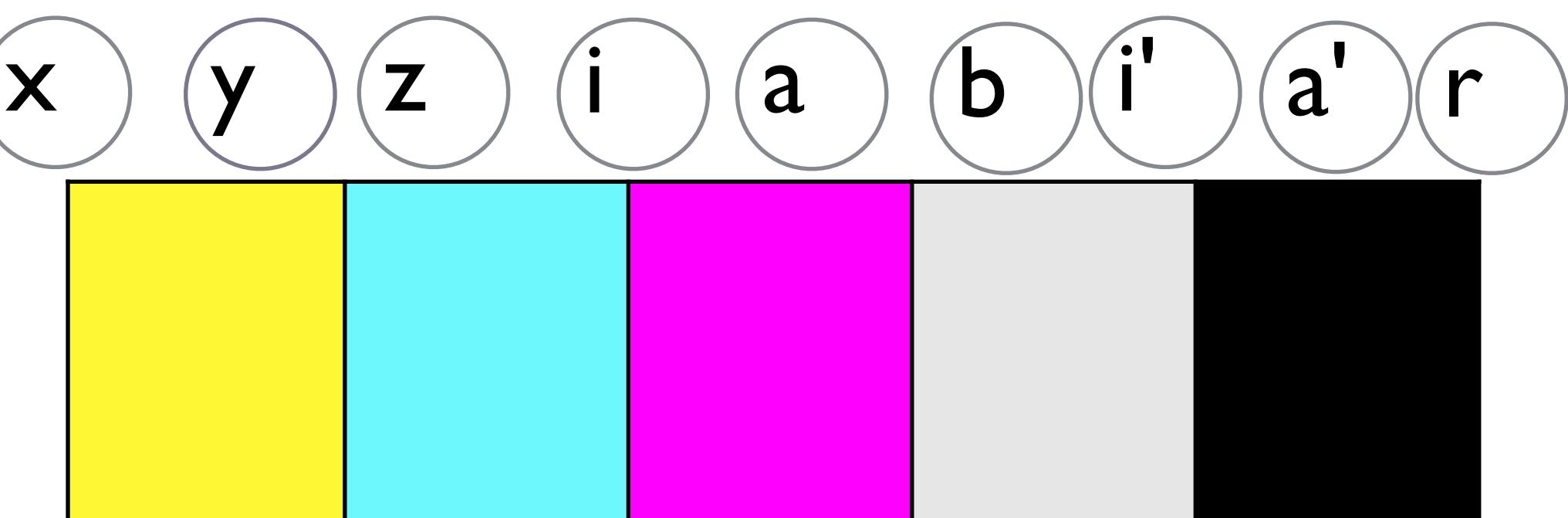
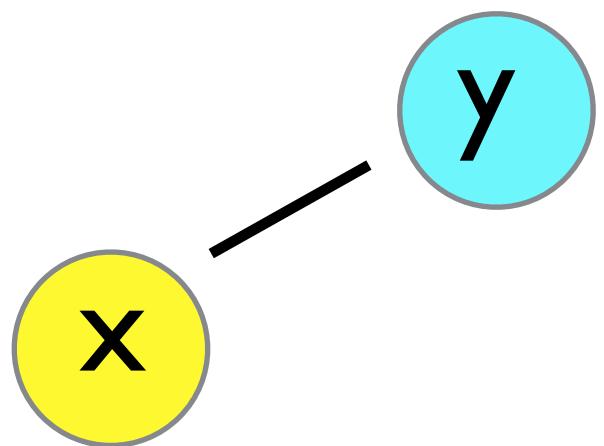
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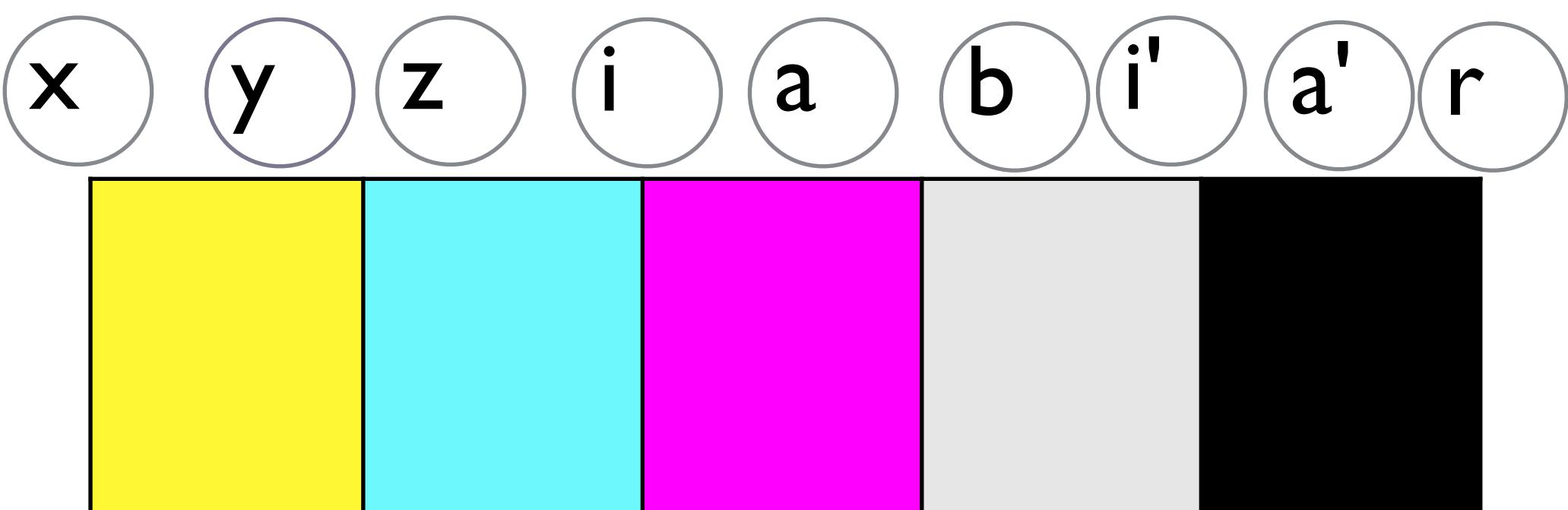
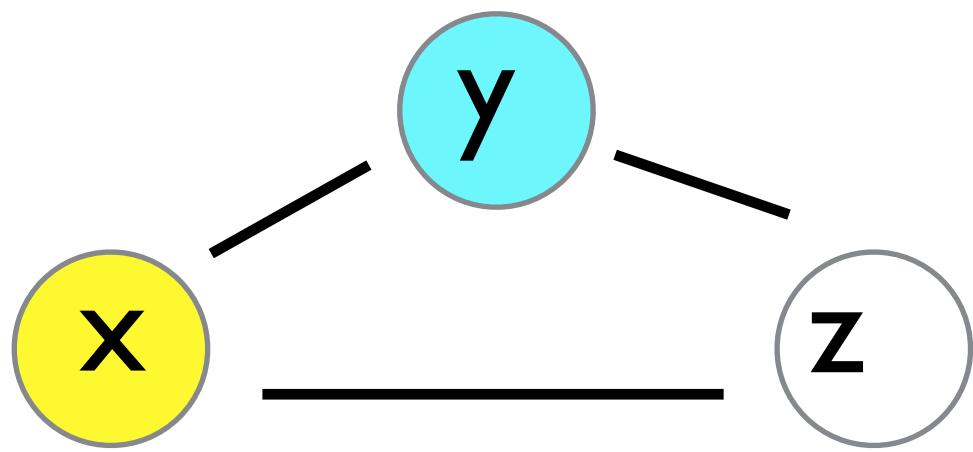
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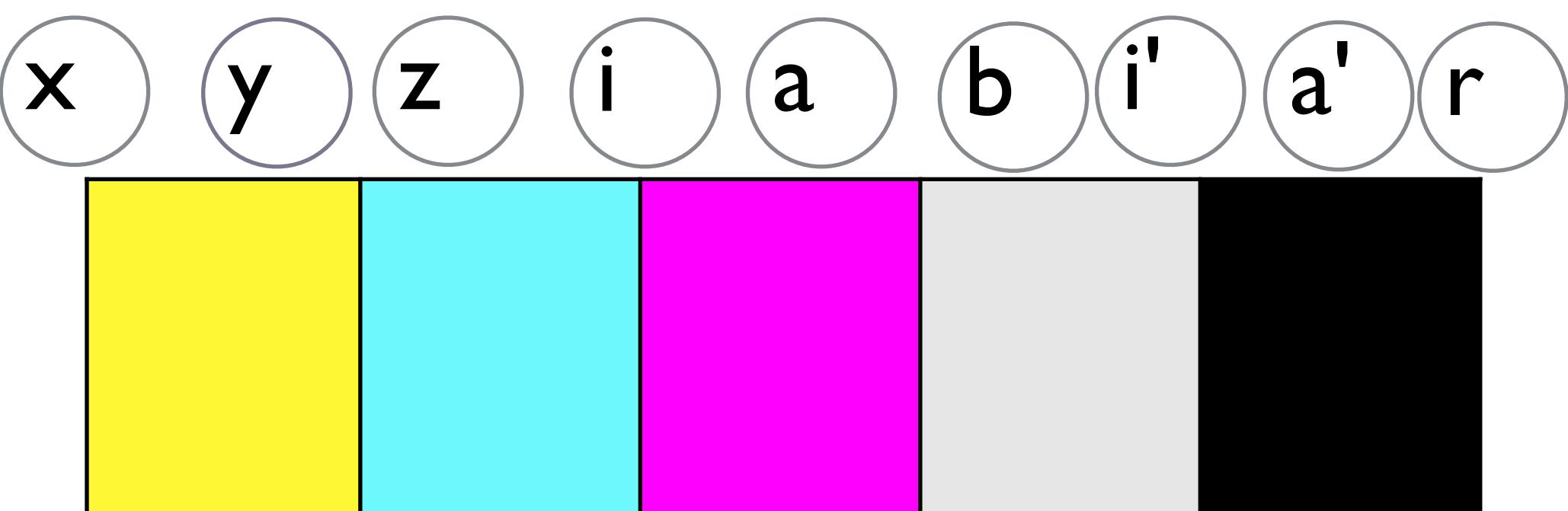
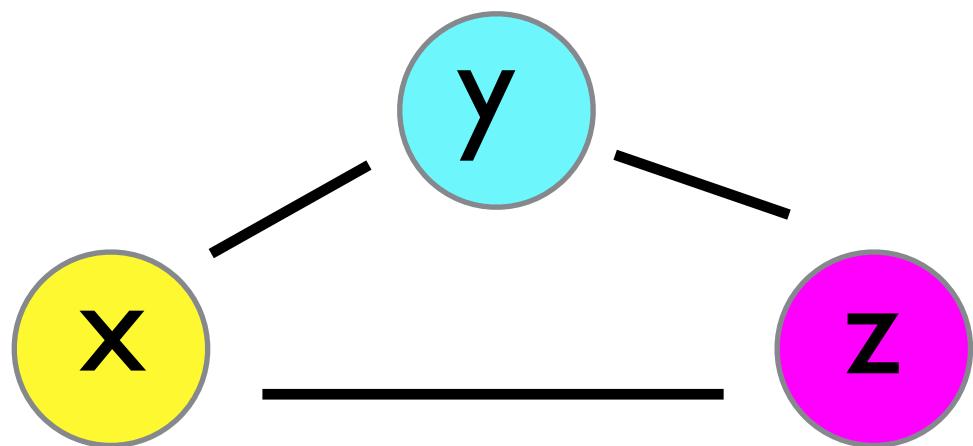
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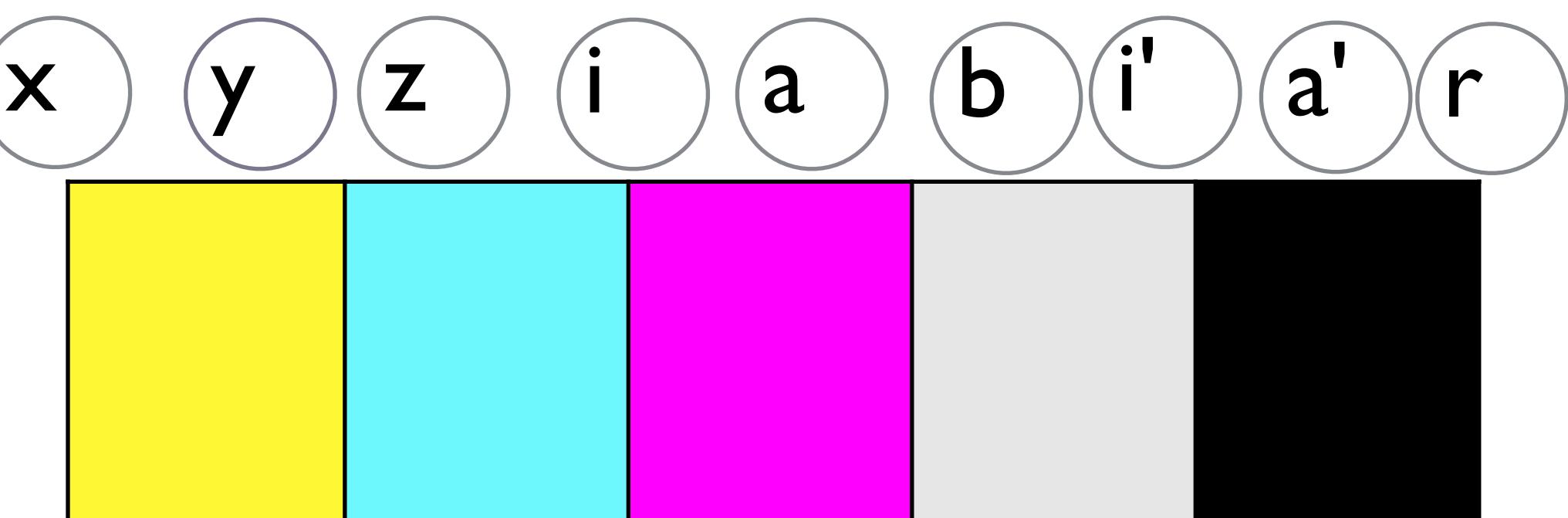
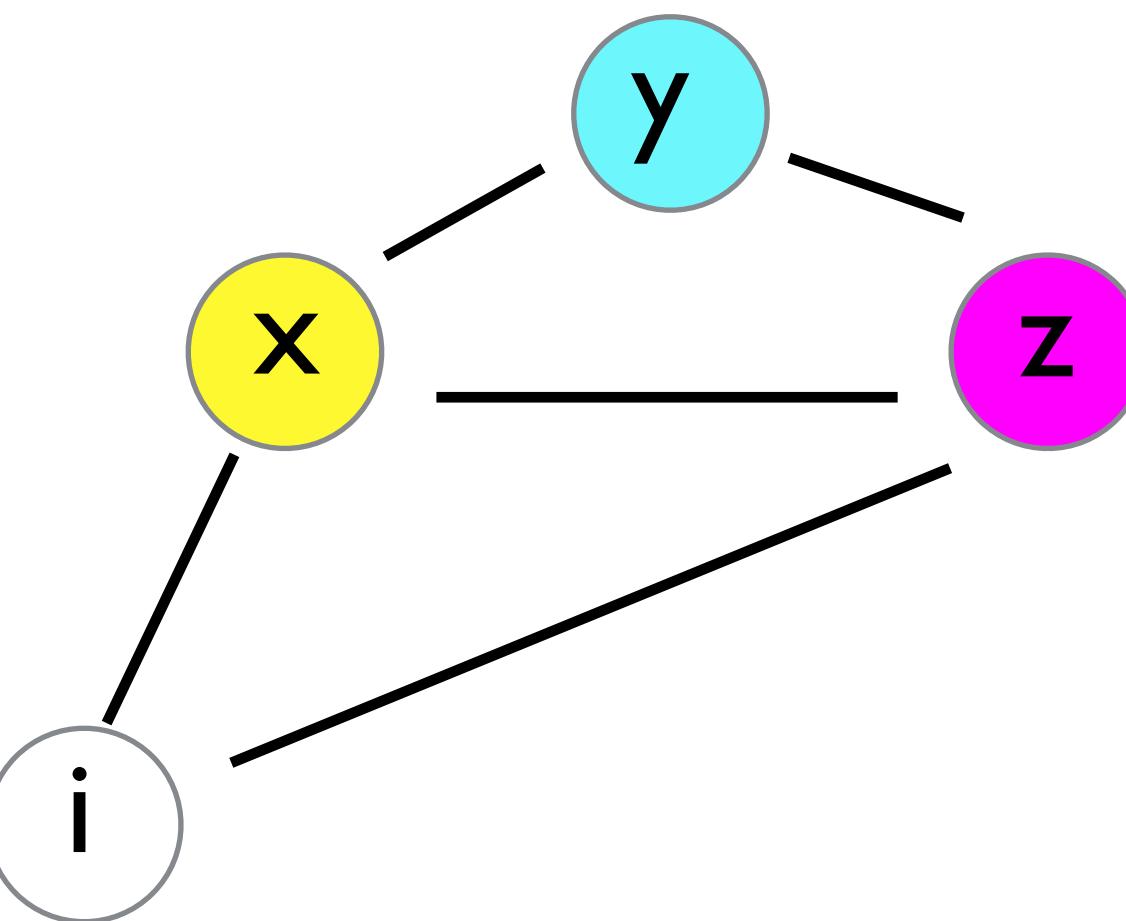
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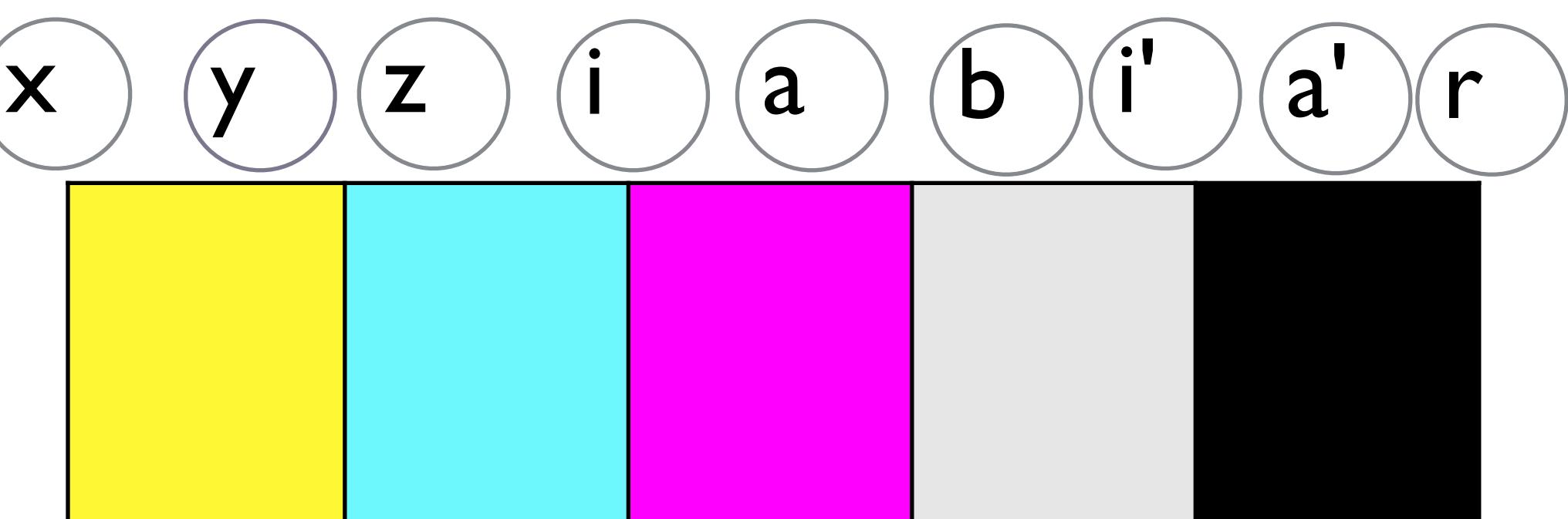
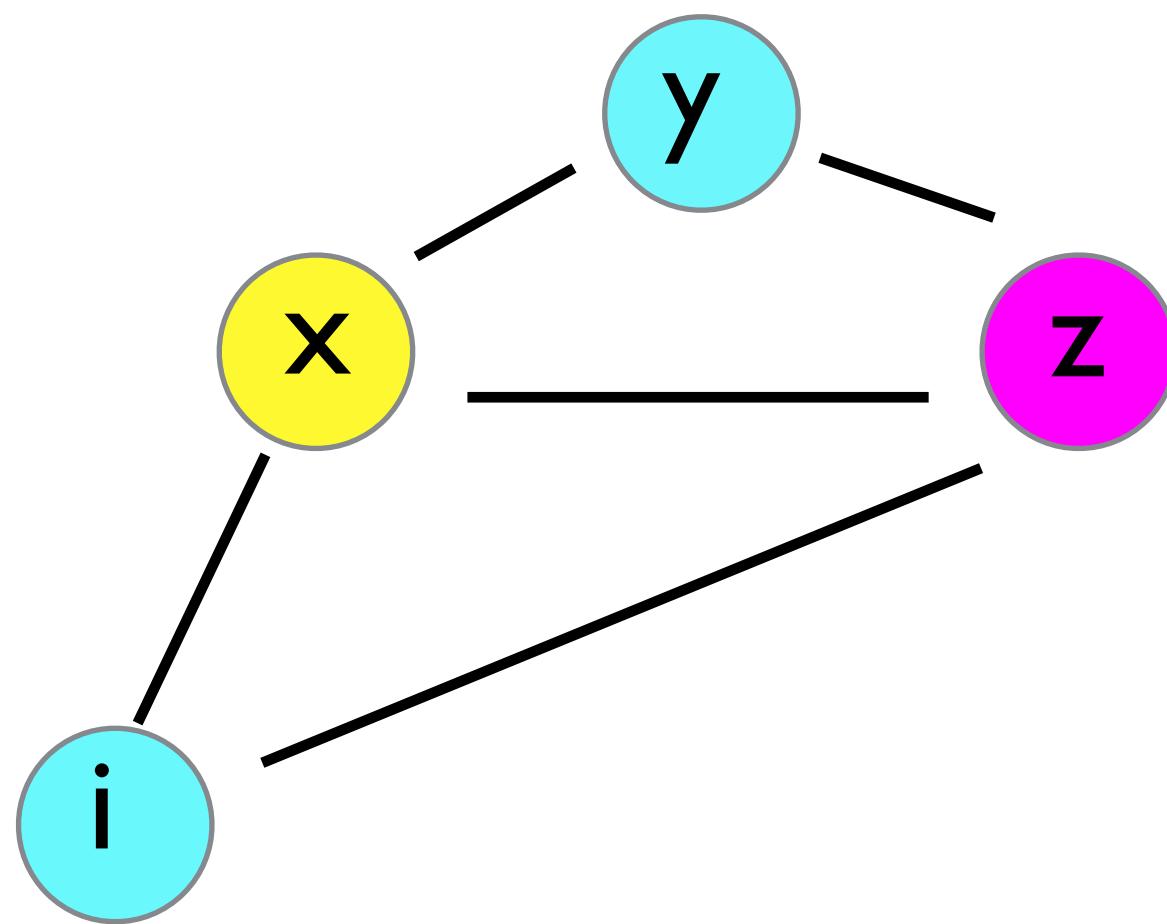
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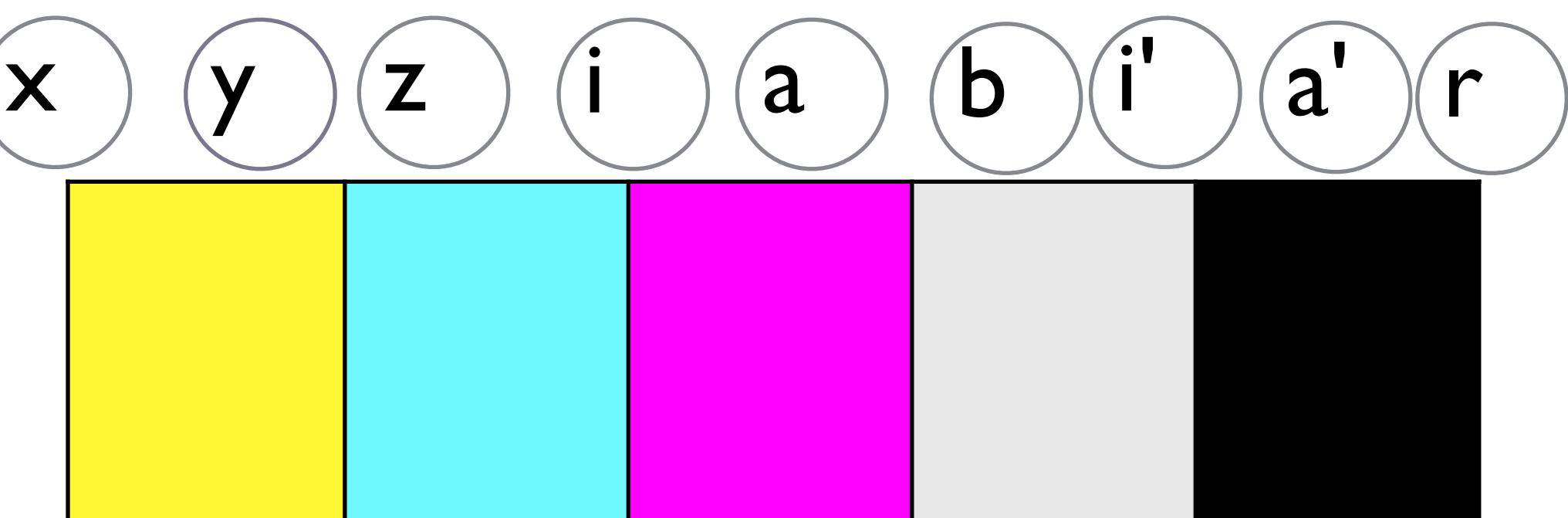
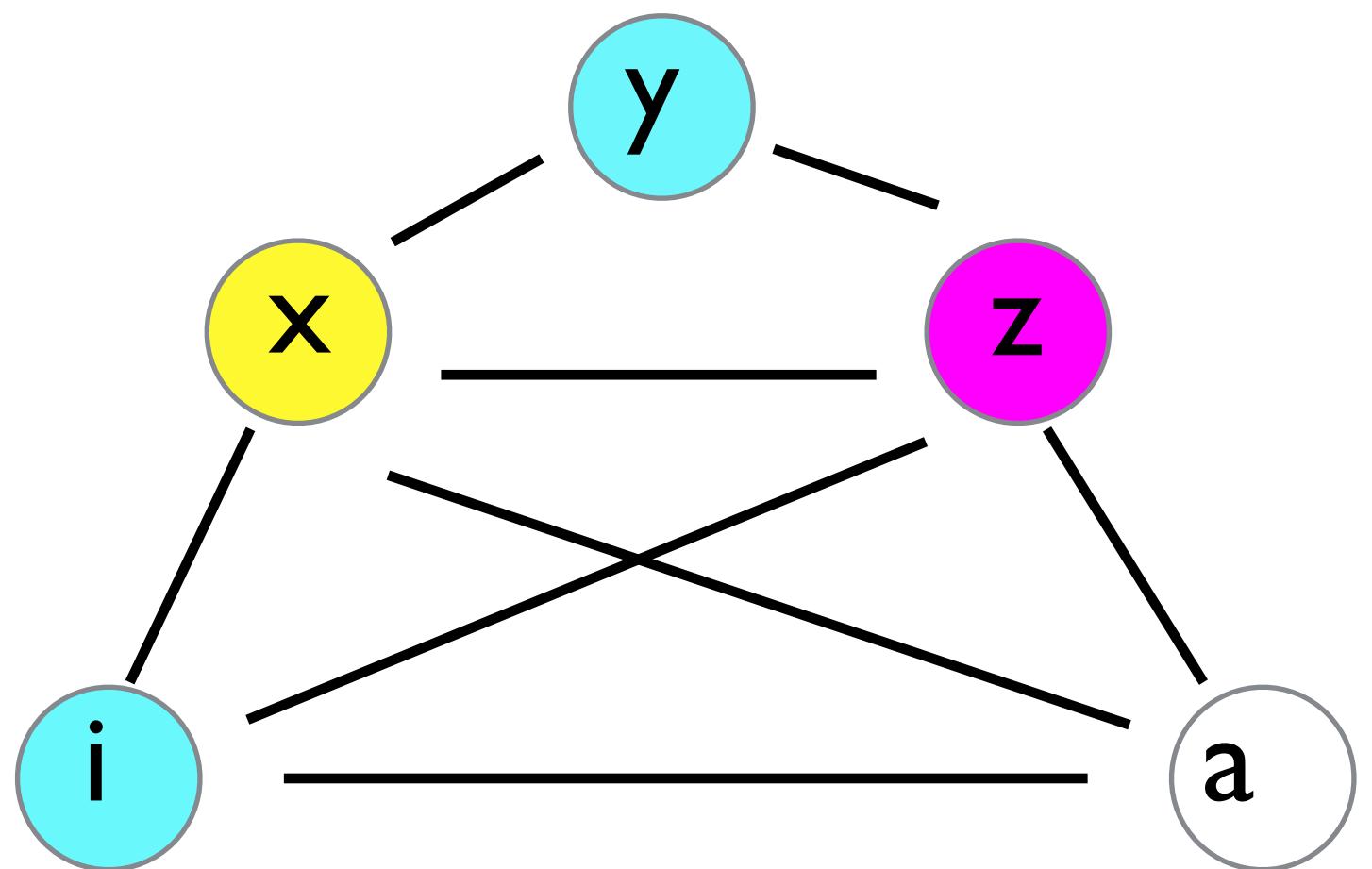
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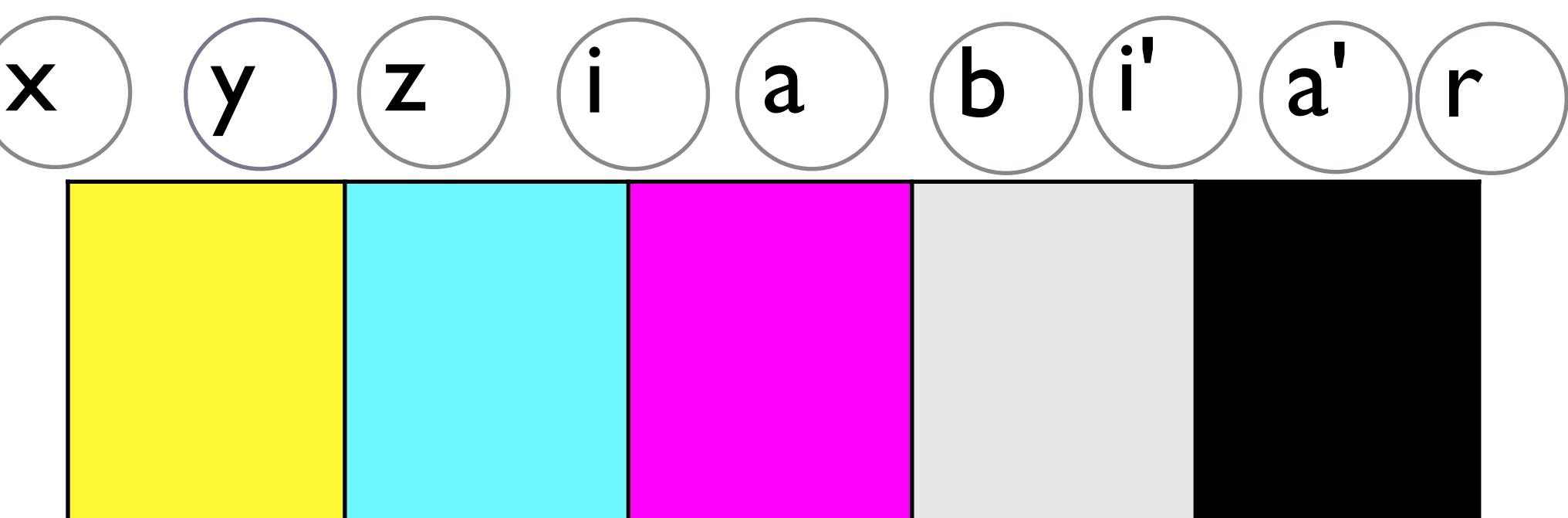
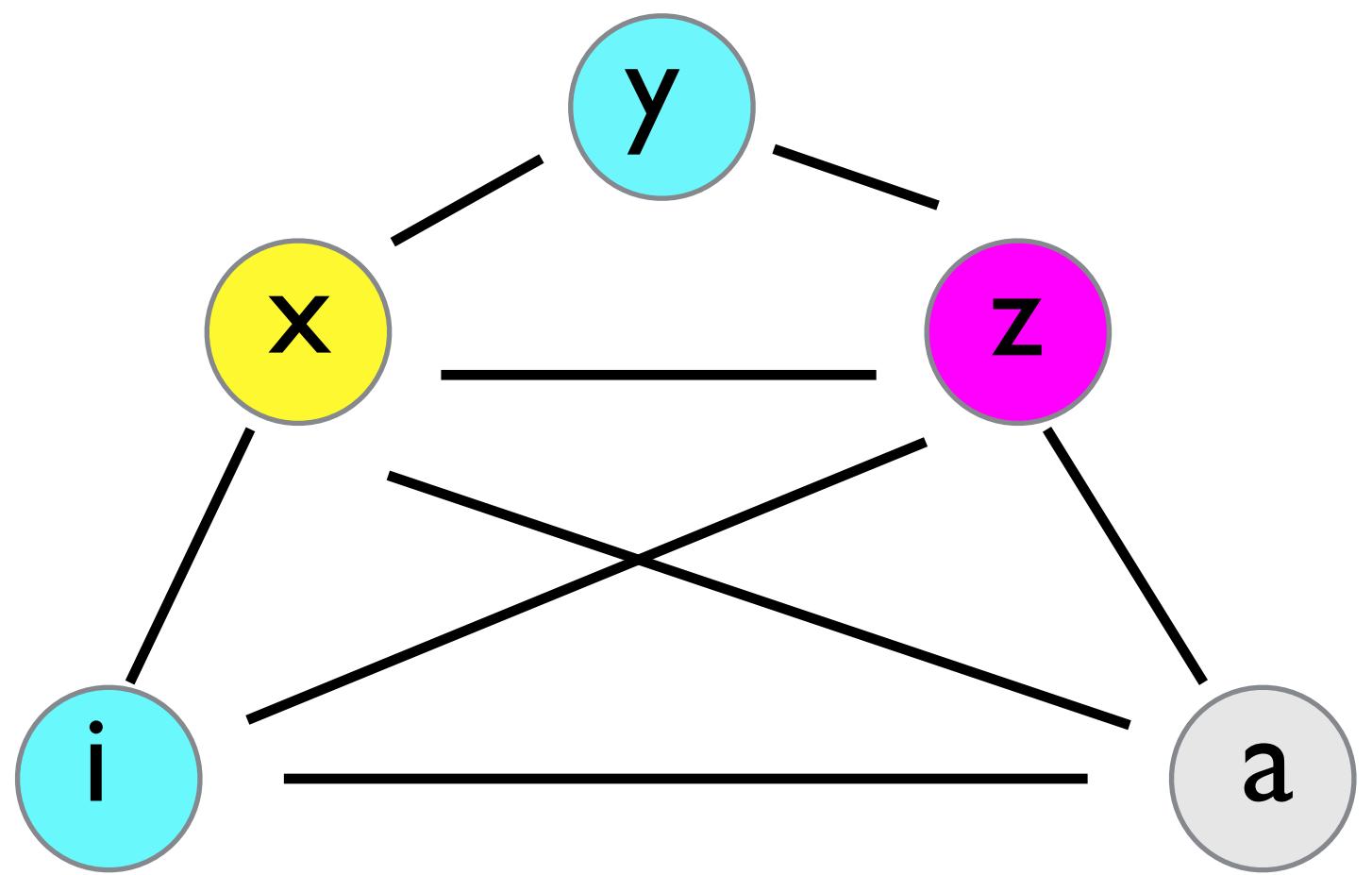
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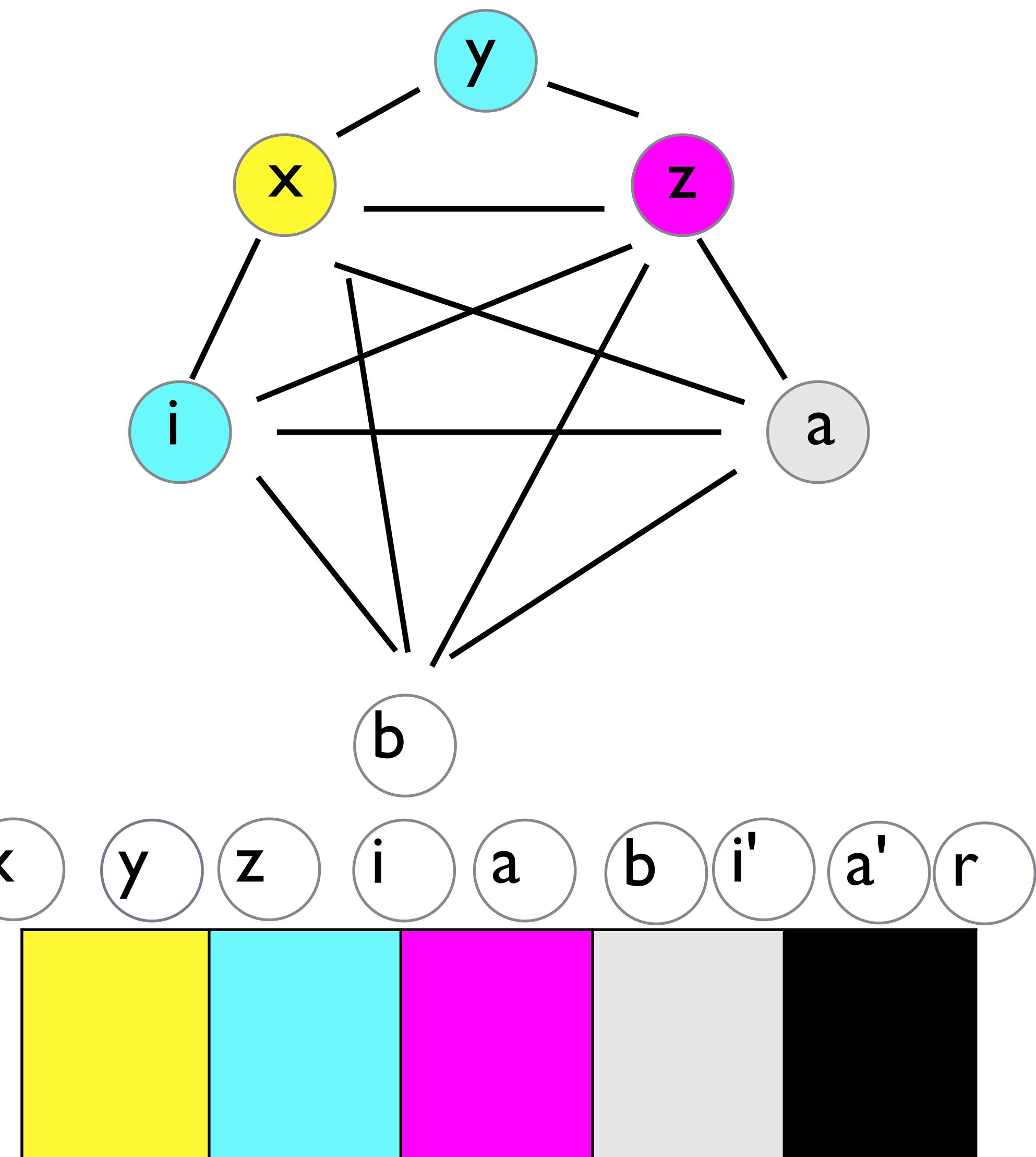
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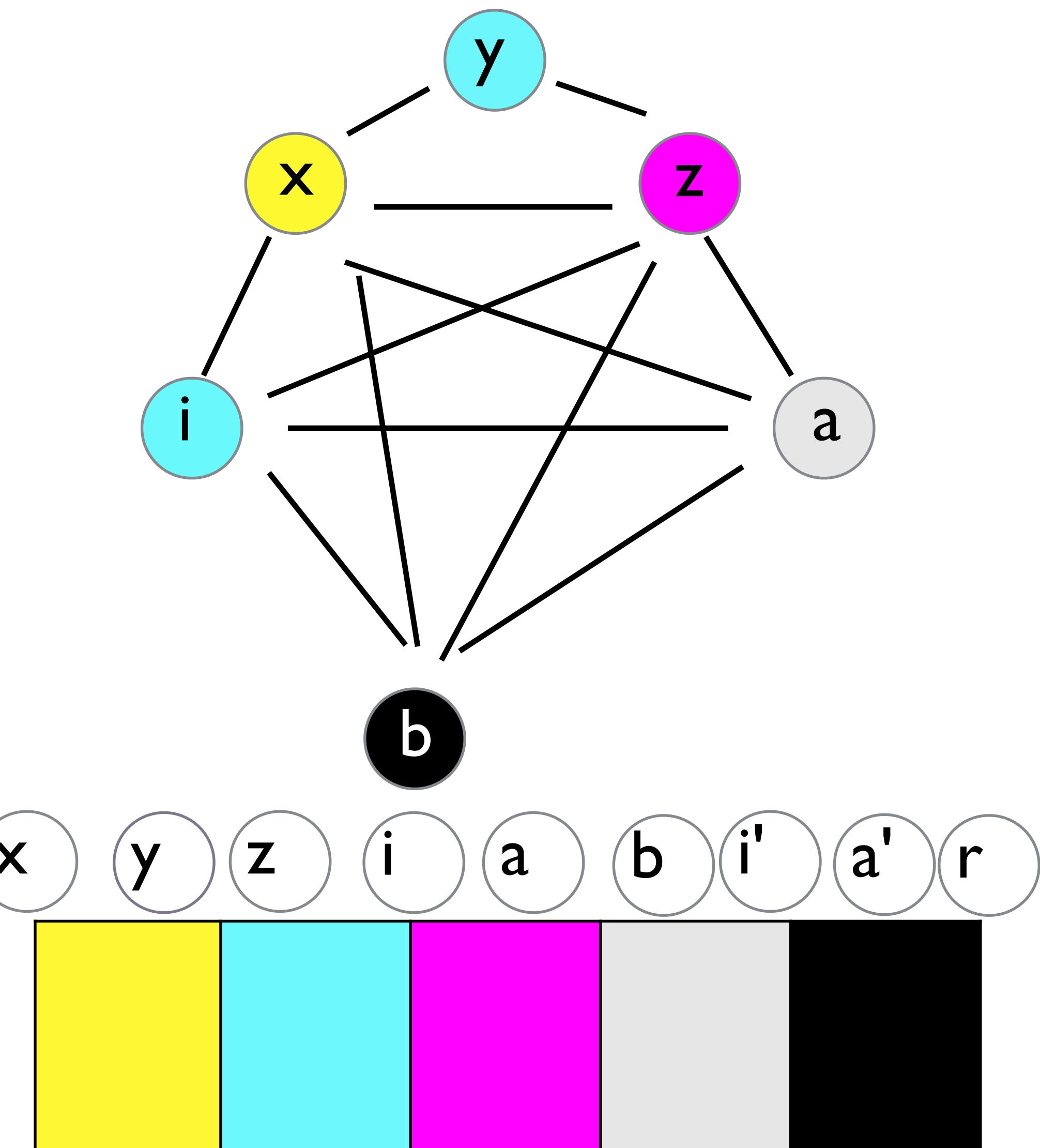
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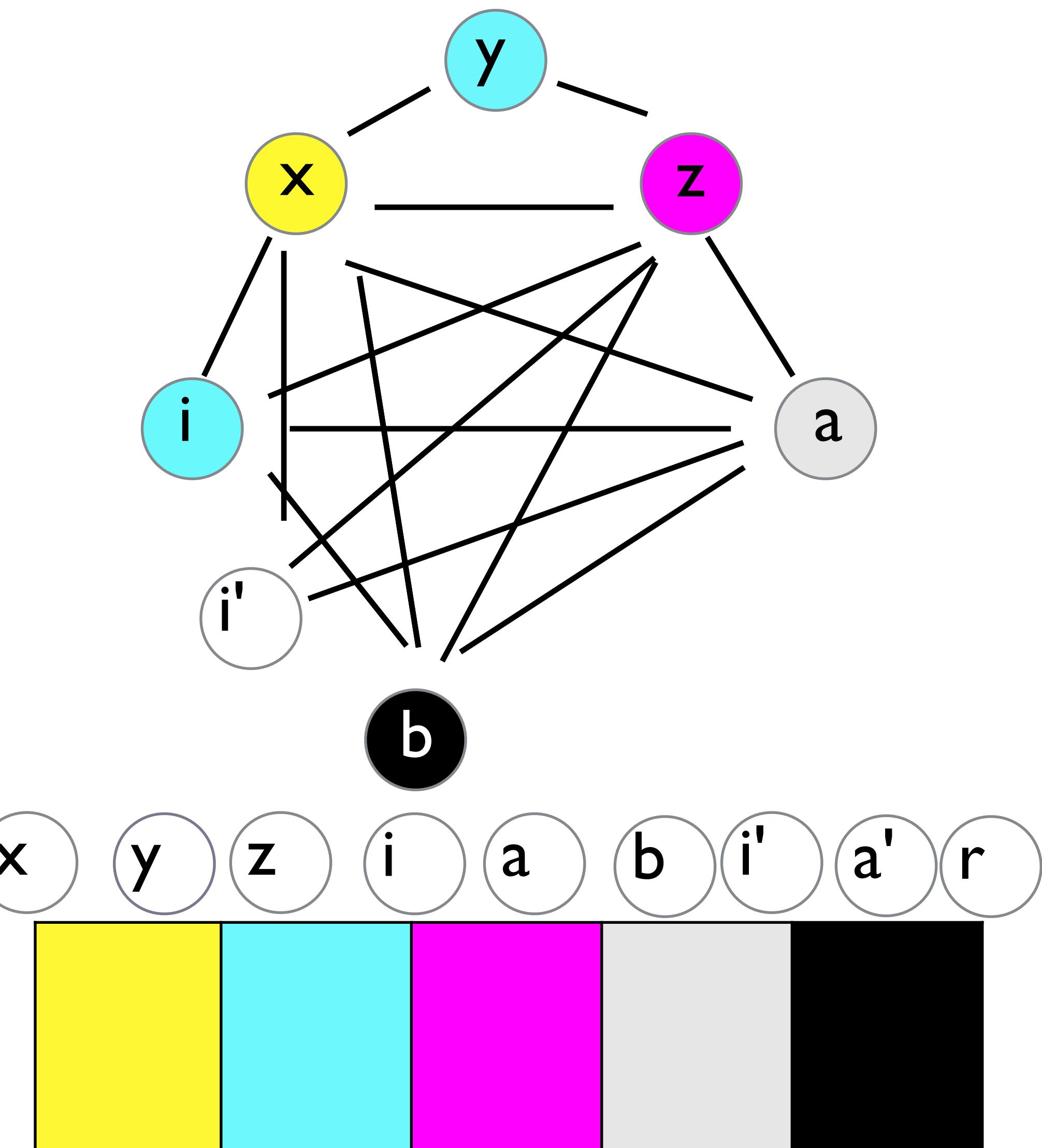
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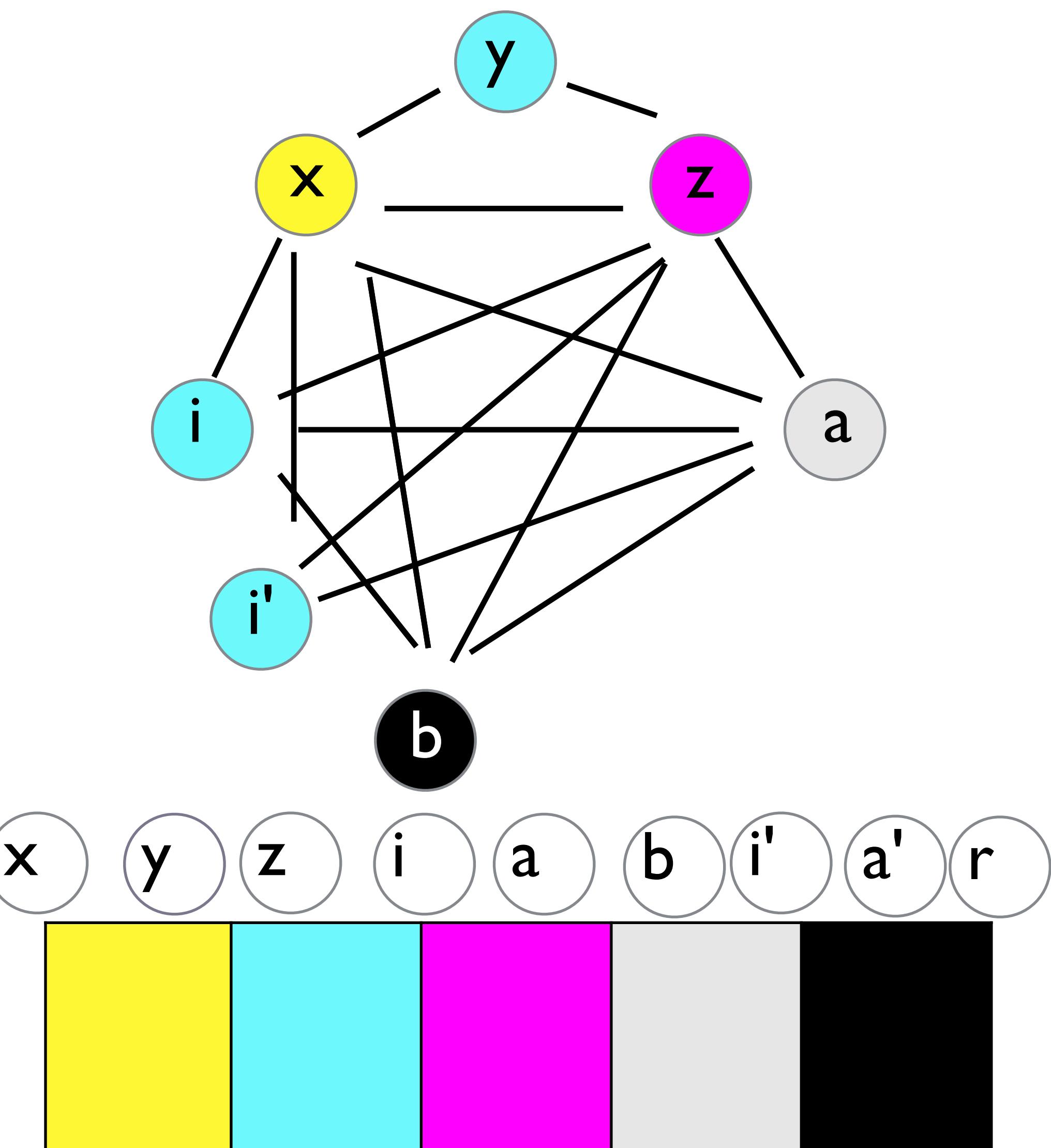
Interference Graph



SSA Program

```
f(x,y,z):  
  loop(i,a):  
    thn():  
      r = a * z  
      ret r  
    els():  
      i' = i - 1  
      a' = a + x  
      br loop(i', a')  
    b = i == 0  
    cbr b thn() els()  
  br loop(y, 0)
```

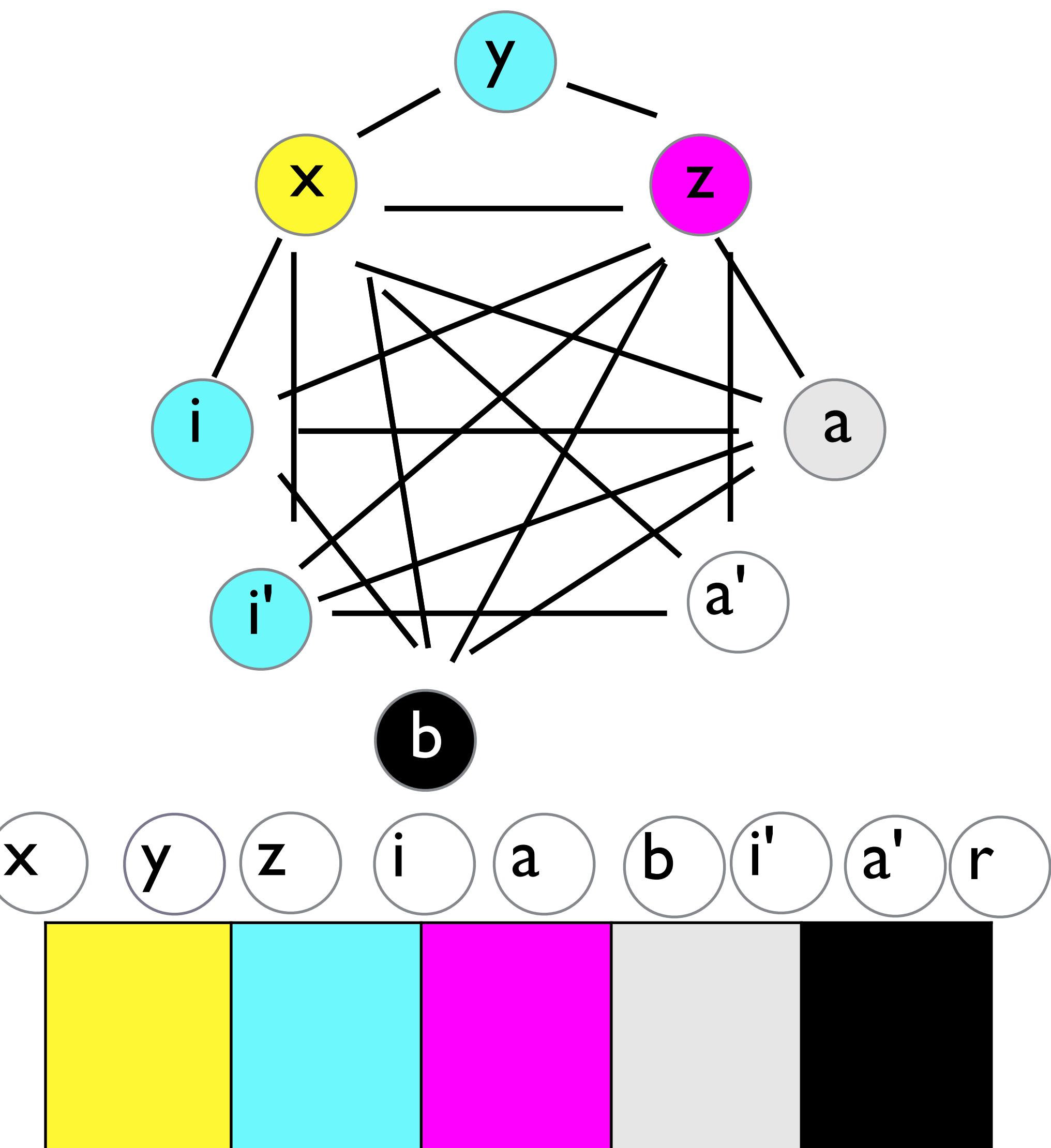
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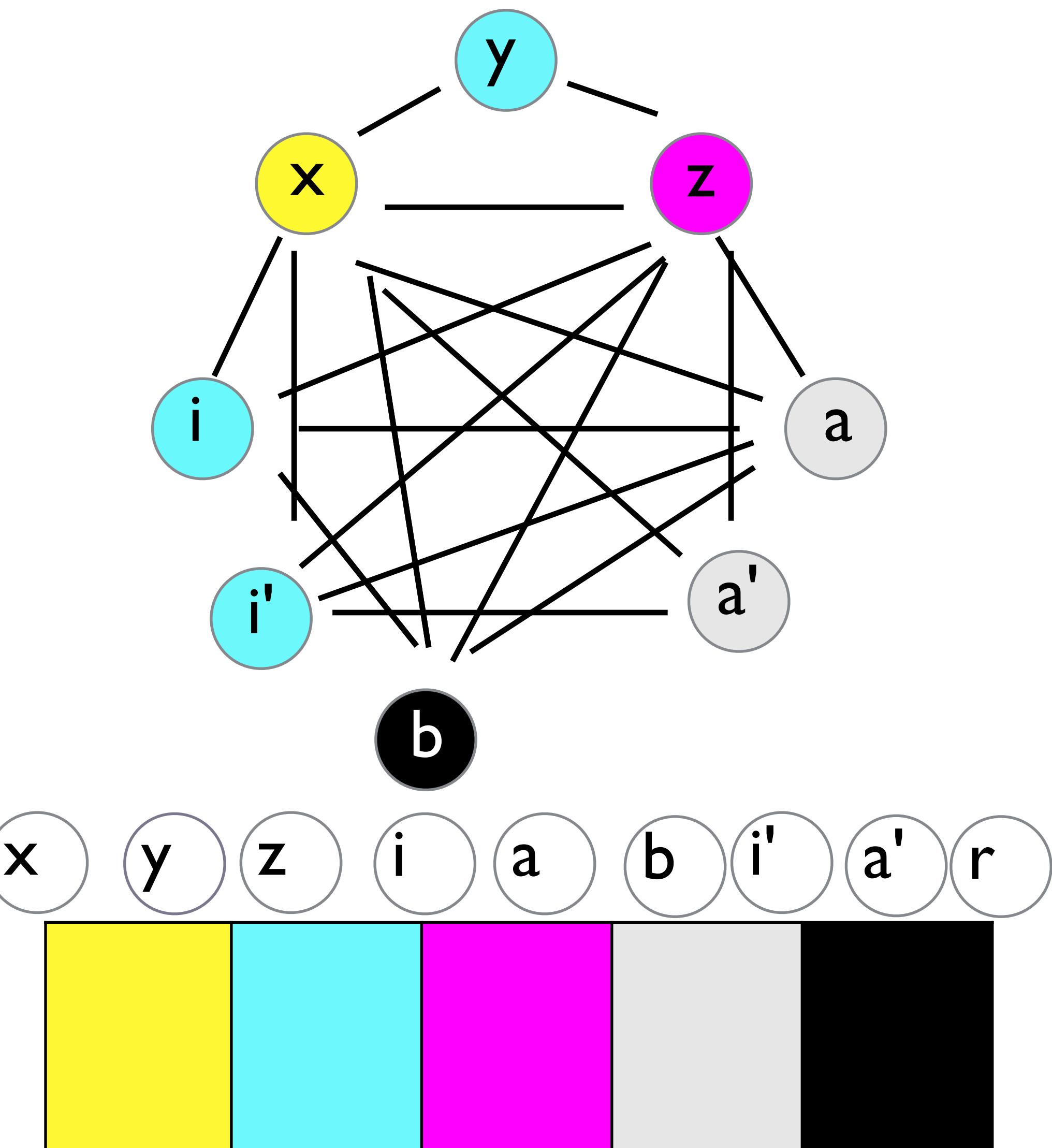
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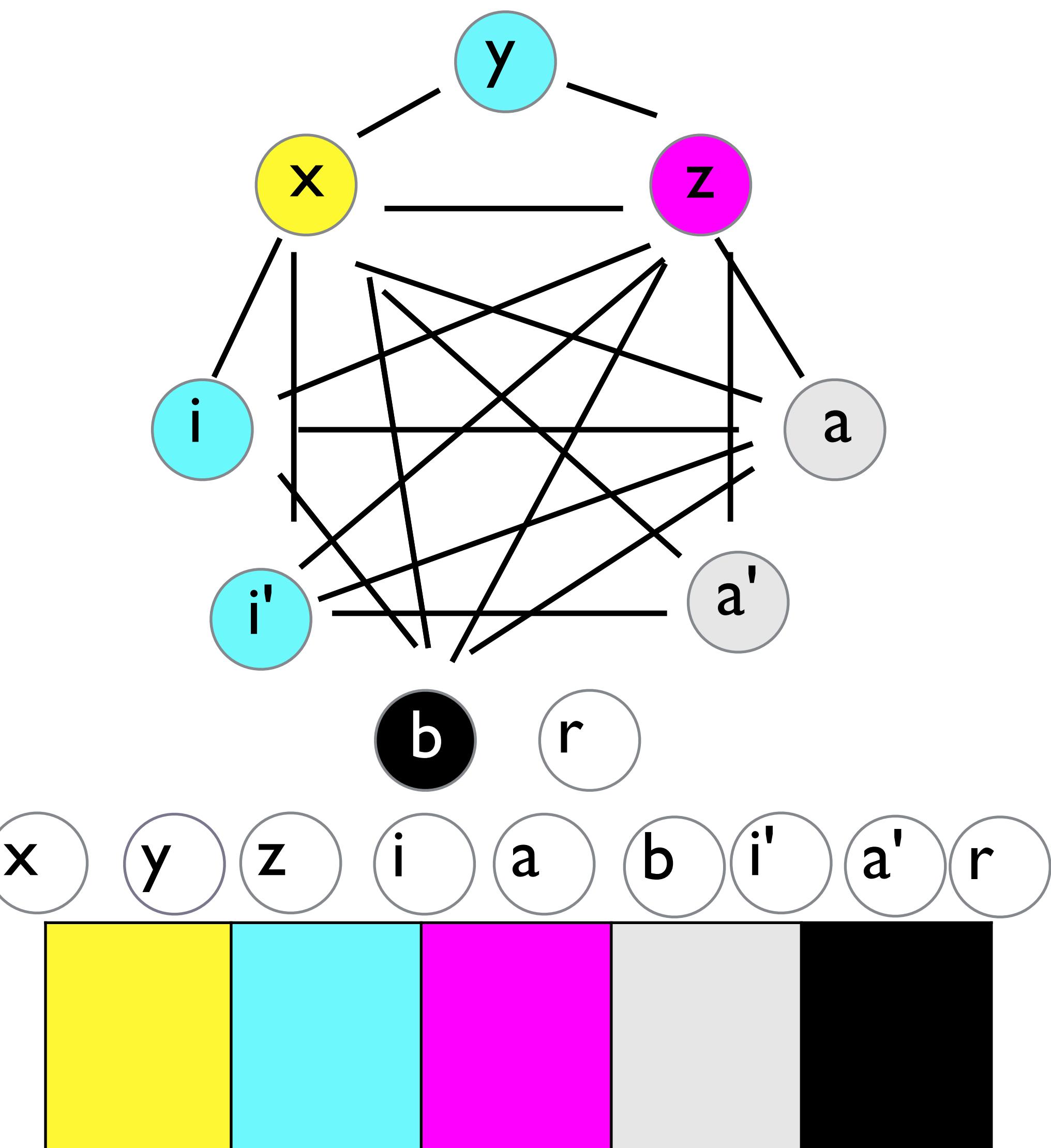
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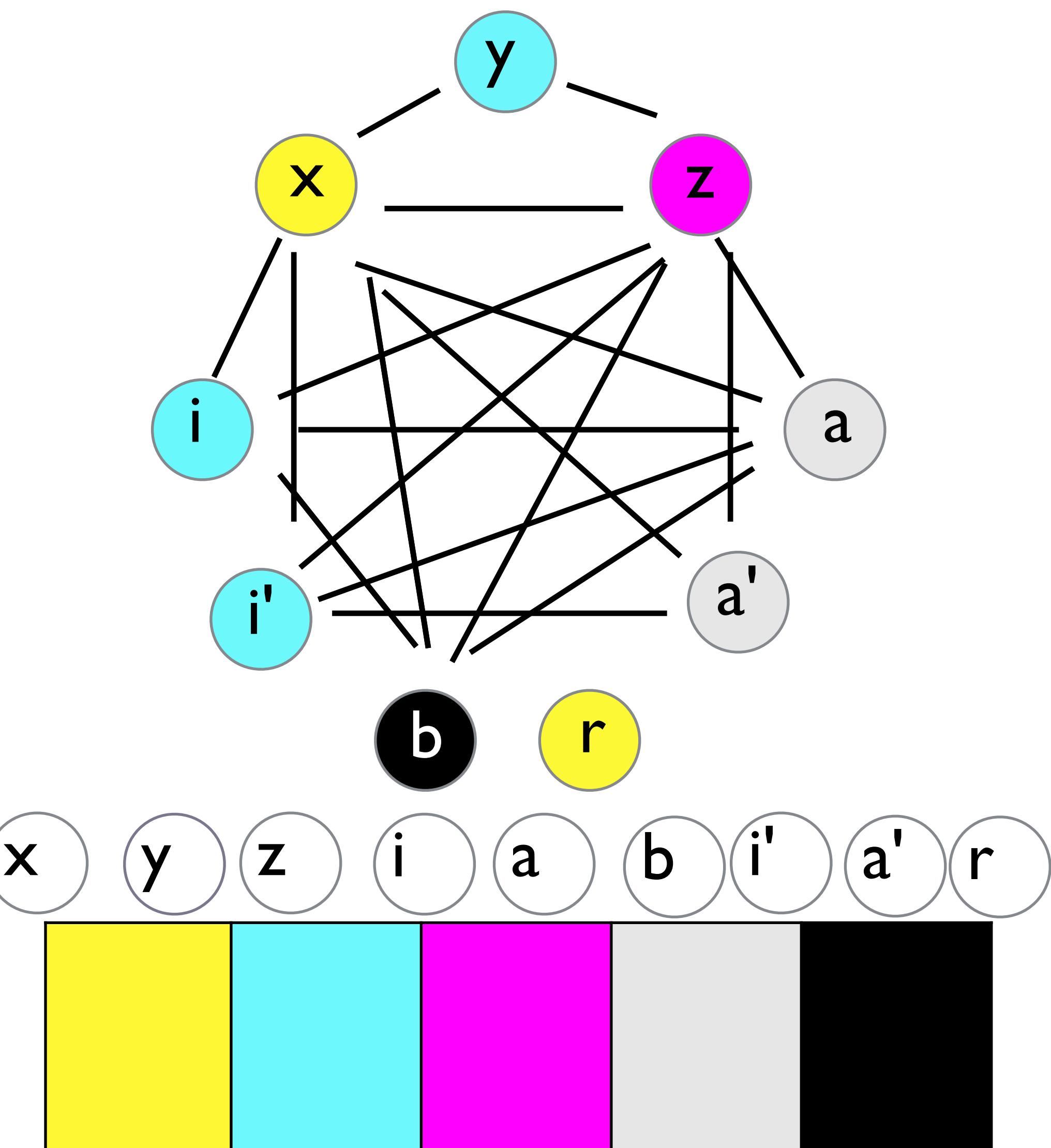
Interference Graph



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br loop(y, 0)
```

Interference Graph



Incorporating Register Allocation

Perform register allocation on all the **blocks** of our SSA program. Treat function blocks specially.

Effects on Codegen

Store results directly in the output register

$x = y - z$

`mov rx, ry`

`sub rx, rz`

Does this always work? Need to be careful if output register is the same as one of the input registers (e.g., $rx = rz$)

Can either use a scratch register, or op-specific tricks:

`sub rx, ry`

`imul rx, -1`

Spilling

If a variable is picked to be spilled:

- When it is assigned to, store the result in memory
- When it is used, access memory

One issue:

$$x = y + z$$

If all x, y, z are spilled, cannot implement this without a register.

Easy solution: reserve one scratch register for this purpose:

`mov r, [rsp - off(y)]`

`add r, [rsp - off(z)]`

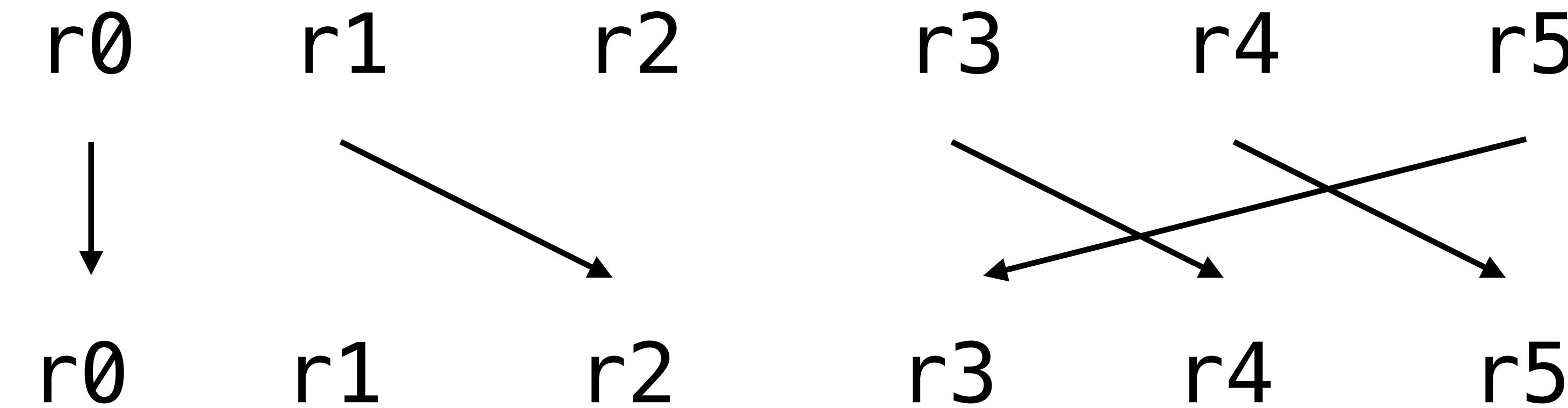
`mov [rsp - off(x)], r`

Implementing Branch with Arguments

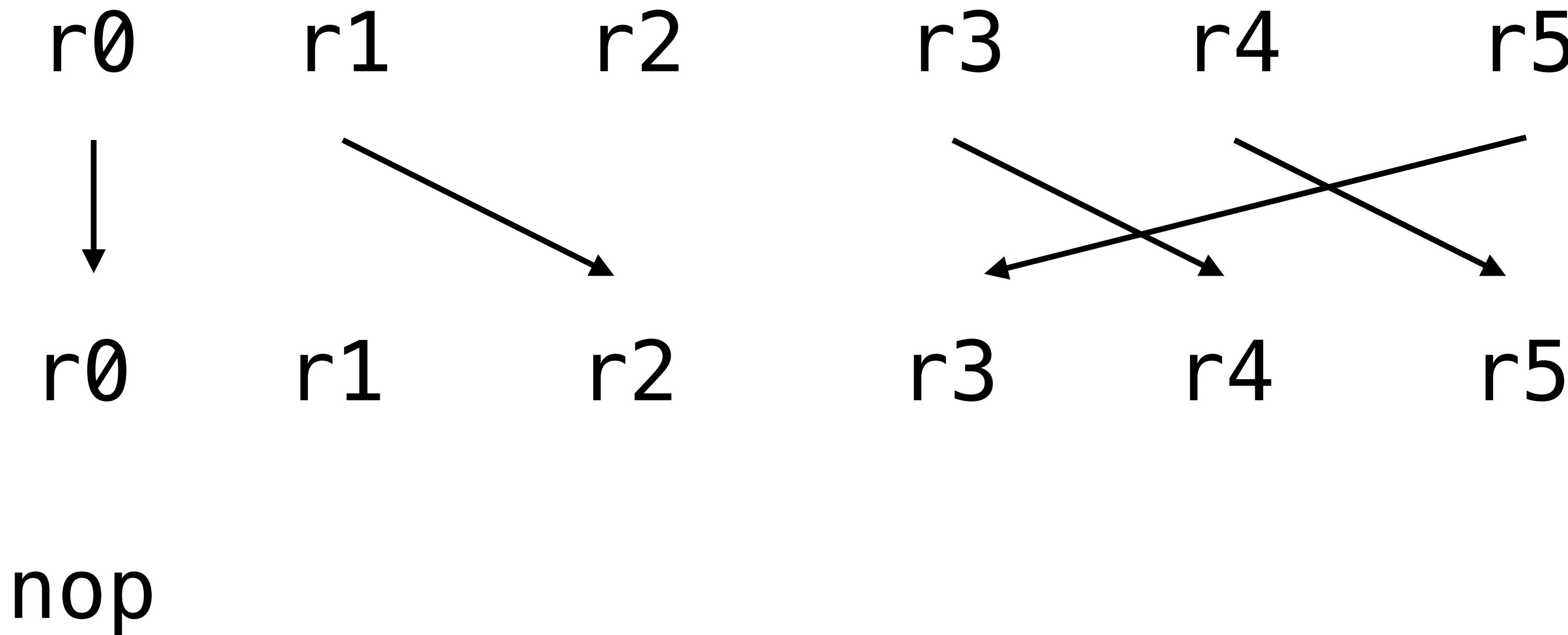
```
f(a,b,c): ...
...
br f(x,y,z)
               mov r_x, r_a
               mov r_y, r_b
               mov r_z, r_z
               jmp f
```

what if a,b,c registers and
x,y,z registers overlap?

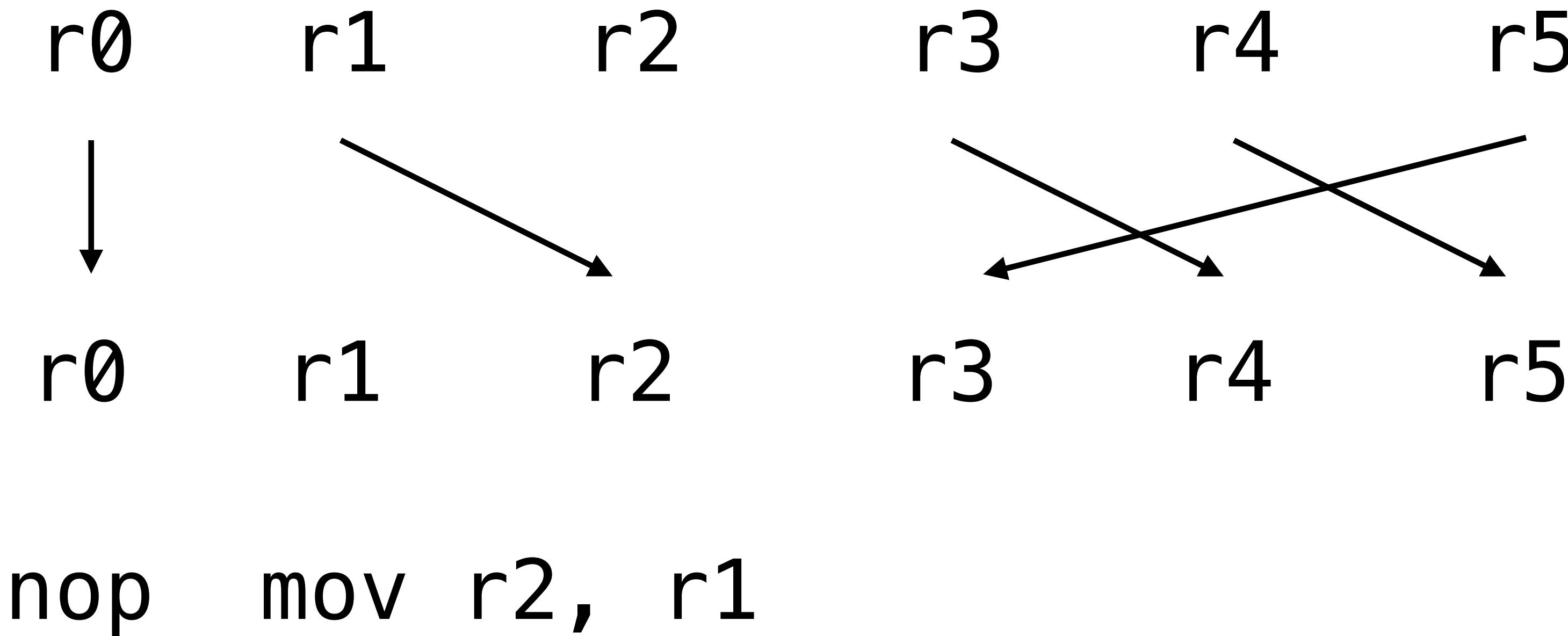
Implementing Branch with Arguments



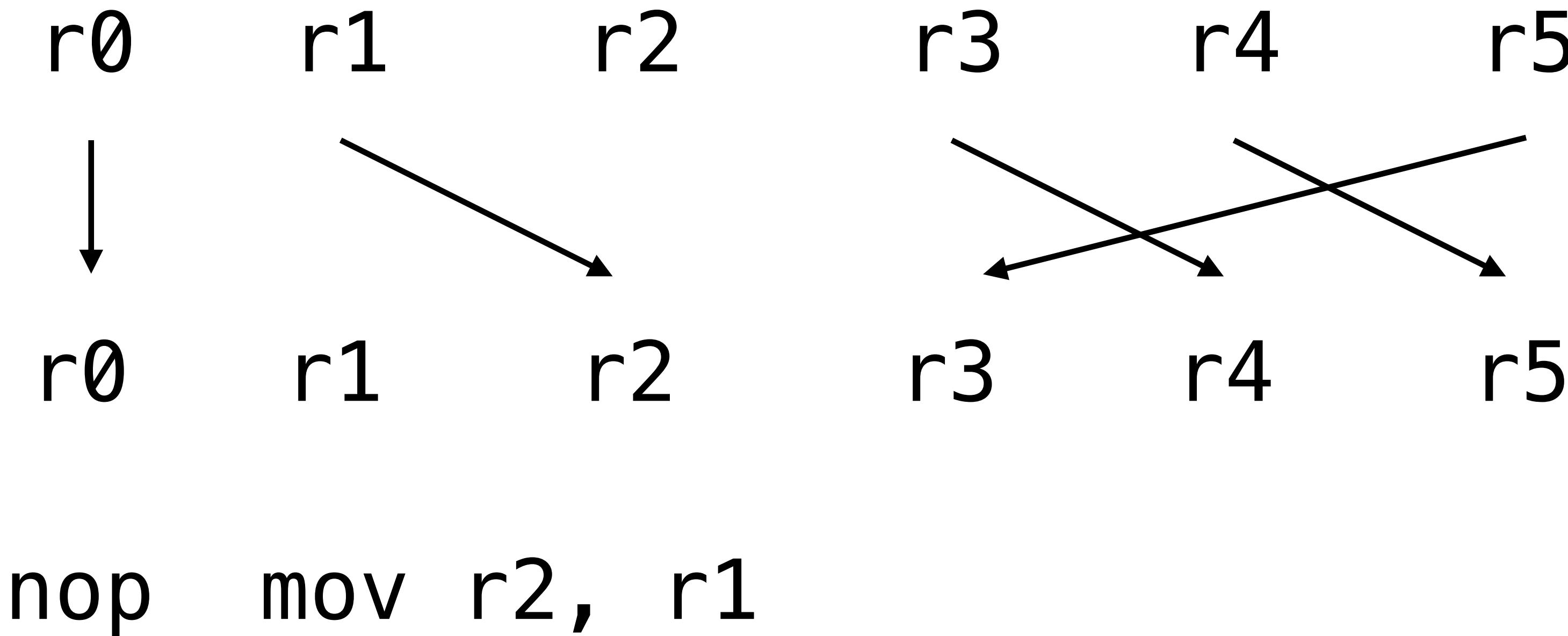
Implementing Branch with Arguments



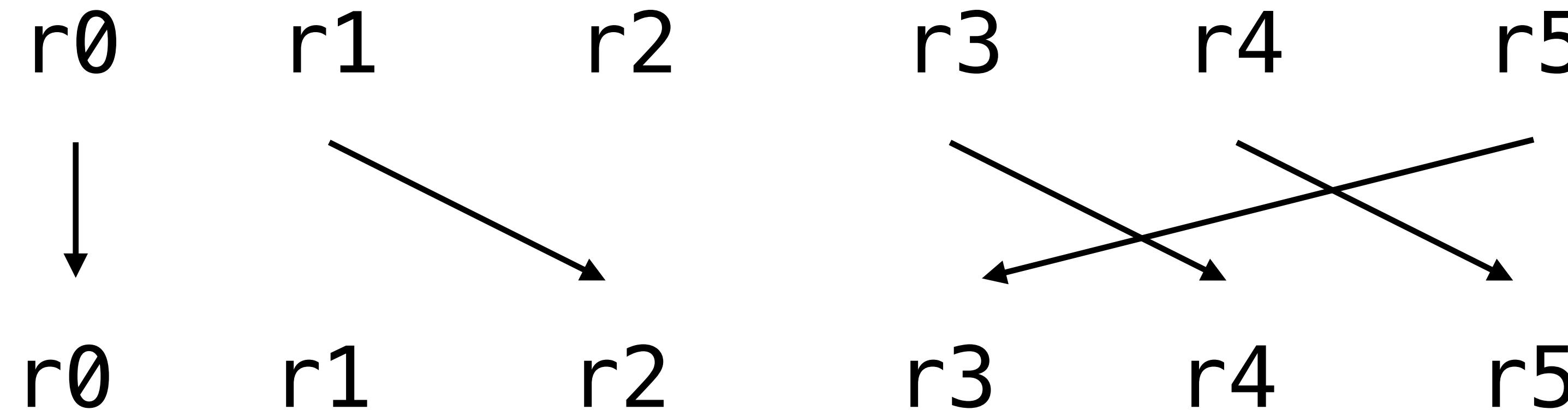
Implementing Branch with Arguments



Implementing Branch with Arguments



Implementing Branch with Arguments



nop	mov r2, r1	xchg r3, r4 xchg r3, r5
-----	------------	----------------------------

xchg is like mov, but **exchanges** the values without need for an extra register.
Faster than xor swapping

SSA reg allocation is polytime, but minimizing the resulting number of movs/xchg is NP hard

Register Allocation vs Calling Conventions

Now that we are using registers we need to take care to respect treatment of registers in the calling conventions we use.

Register Allocation vs Calling Conventions

Now that we are using registers we need to take care to respect treatment of registers in the calling conventions we use.

In System V AMD 64 Calling convention, registers are divided into two classes:

- **volatile** aka **caller-save**: when you make a call, the value of these registers may change when the callee returns
- **non-volatile** aka **callee-save**: when you make a call, the value of these registers will be the same when the callee returns

Volatile/Caller Save registers

volatile aka **caller-save**

x = ...

y = **f(z)**

z = **x** + **y**

if **x** is stored in a volatile register, its value may be overwritten by the function **f**.

- Simple solution: save all live volatiles to the stack before a call, restore after the call
- Better solution: add nodes to interference graph for volatile registers, add conflicts at every non-tail call

Non-volatile/Callee Save registers

```
y = ...  
z = x + y  
ret z
```

if **y** is stored in a **non-volatile** register, the value of the register must be **saved** on entry and **restored** when we return

- Solution: save all used non-volatiles to the stack at the beginning of every global function def, restore them before every return/external tail call
- Start spilled variables **after** the saved non-volatile registers

Implementing function blocks

```
fun f_fun(a,b,c,...):  
    br f_tail(a,b,c,...)
```

1. Save all used non-volatiles/callee-save registers
2. Treat f_fun's args are pre-determined by the calling convention, otherwise similar to any branch with args

```
mov [rsp - 8], rbx
```

```
mov [rsp - 16], rbp
```

```
...
```

```
; br f_tail(...)
```

Implementing ret

ret x

- Move x into rax
- Restore non-volatile/callee-saves

mov rax, loc(x)

mov rbx, [rsp - 8]

mov rbp, [rsp - 16]

...