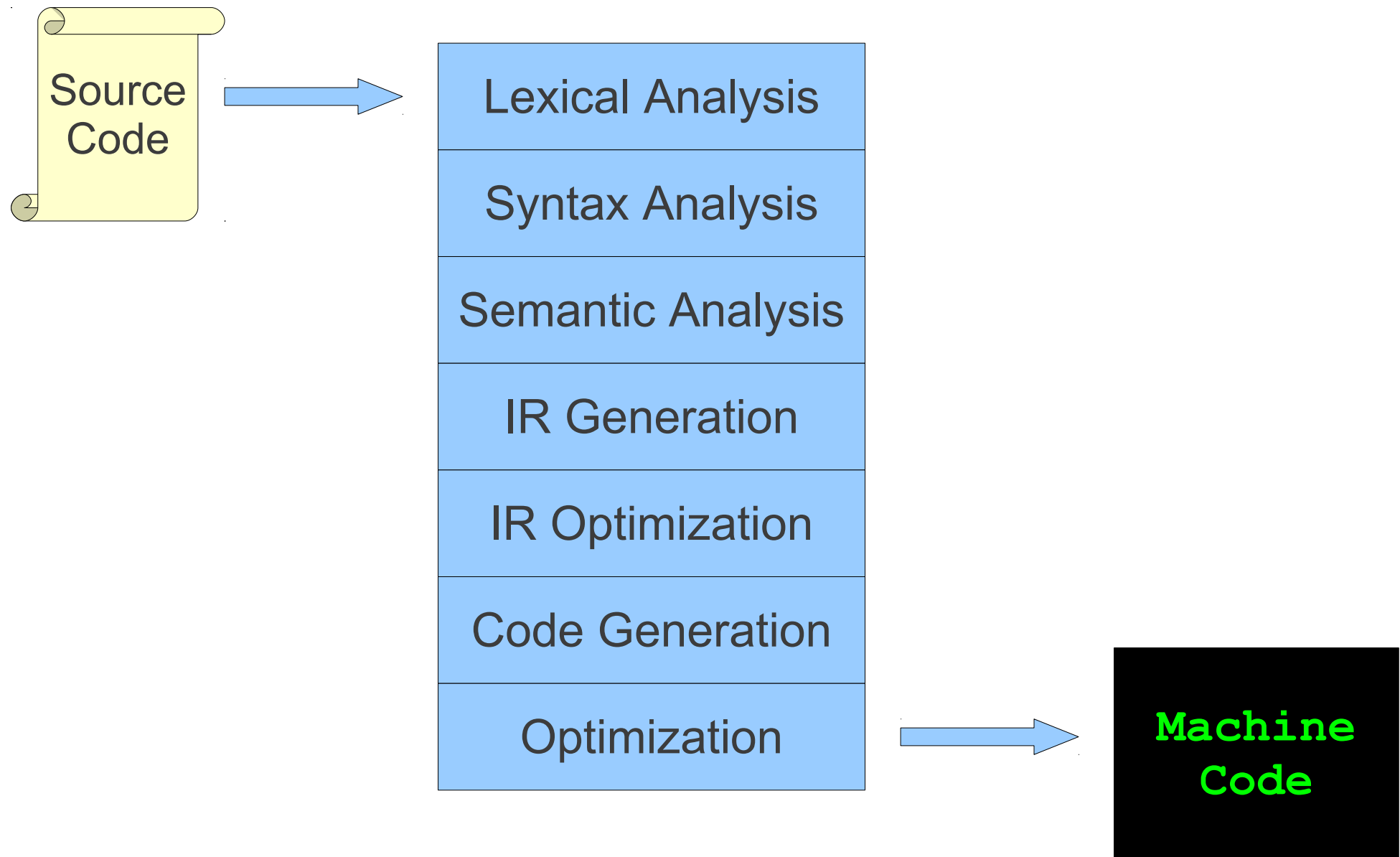


# Lexical Analysis

Dec 6, 2021

# Previously on EECS 483...

## Structure of a modern compiler



```
while (y < z) {  
    int x = a + b;  
    y += x;  
}
```

Lexical Analysis

Syntax Analysis

Semantic Analysis

IR Generation

IR Optimization

Code Generation

Optimization

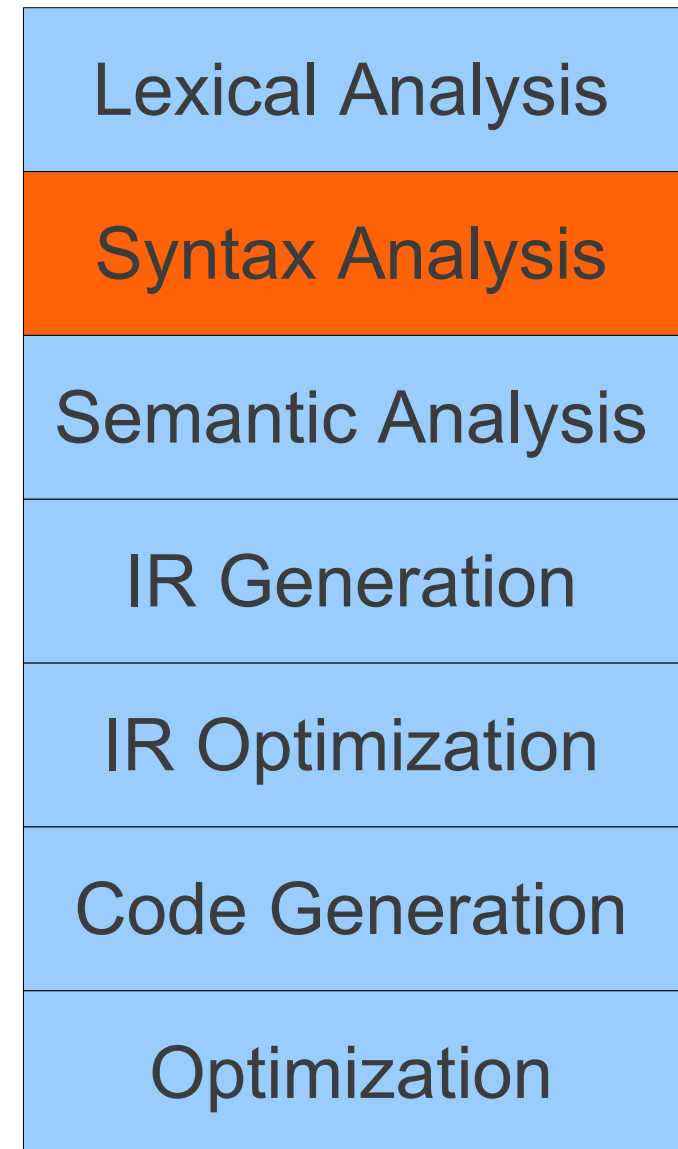
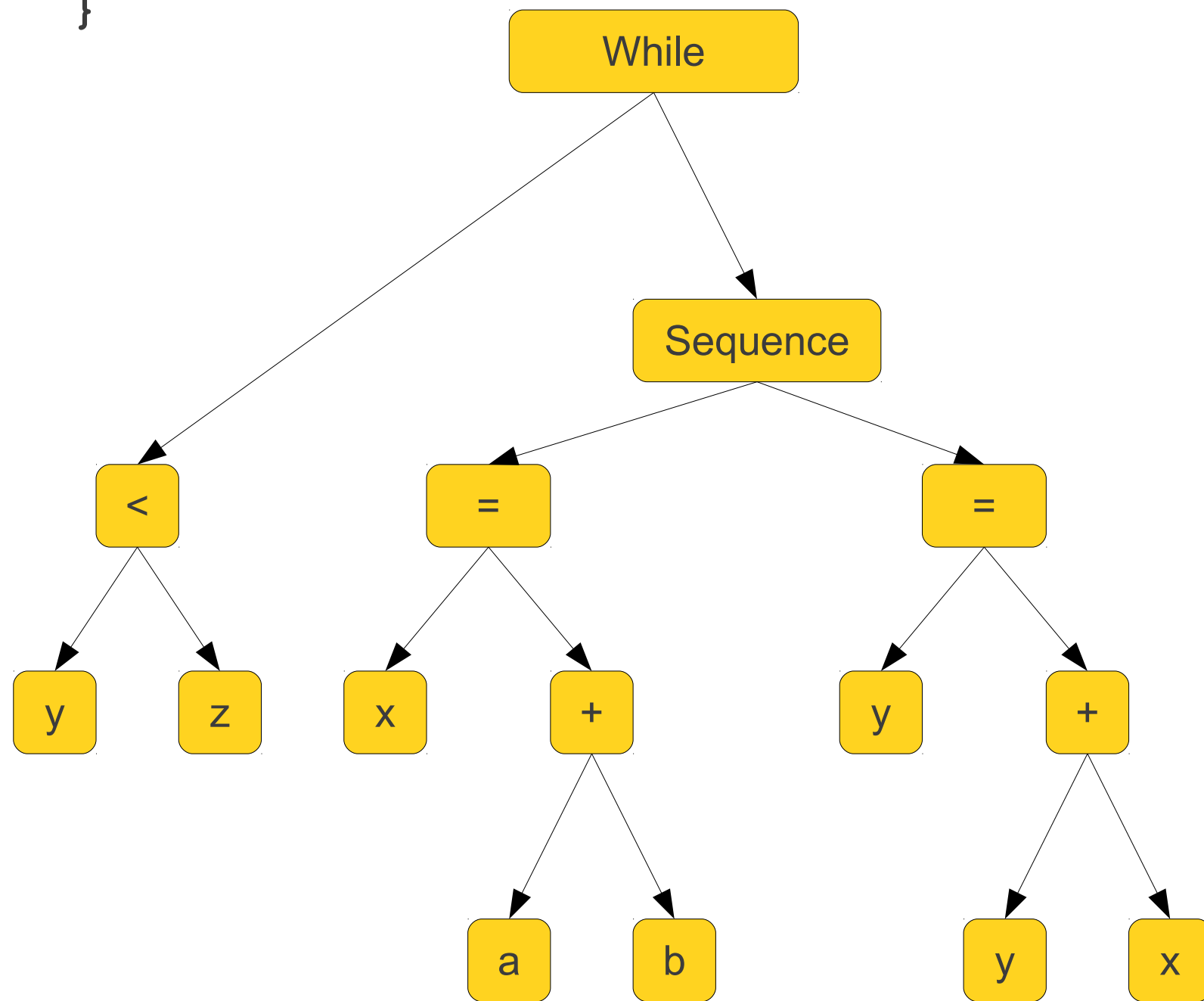
```
while (y < z) {  
    int x = a + b;  
    y += x;  
}
```

```
T_While  
T_LeftParen  
T_Identifier y  
T_Less  
T_Identifier z  
T_RightParen  
T_OpenBrace  
T_Int  
T_Identifier x  
T_Assign  
T_Identifier a  
T_Plus  
T_Identifier b  
T_Semicolon  
T_Identifier y  
T_PlusAssign  
T_Identifier x  
T_Semicolon  
T_CloseBrace
```

Lexical Analysis
Syntax Analysis
Semantic Analysis
IR Generation
IR Optimization
Code Generation
Optimization

**Lexical analysis (Scanning):** Group sequence of characters into lexemes – smallest meaningful entity in a language (keywords, identifiers, constants)

```
while (y < z) {  
    int x = a + b;  
    y += x;  
}
```



**Syntax analysis (Parsing):** Convert a linear structure – sequence of tokens – to a hierarchical tree-like structure - abstract syntax tree (AST)

# Goal of Lexical Analysis

Breaking the program down into words or “tokens”

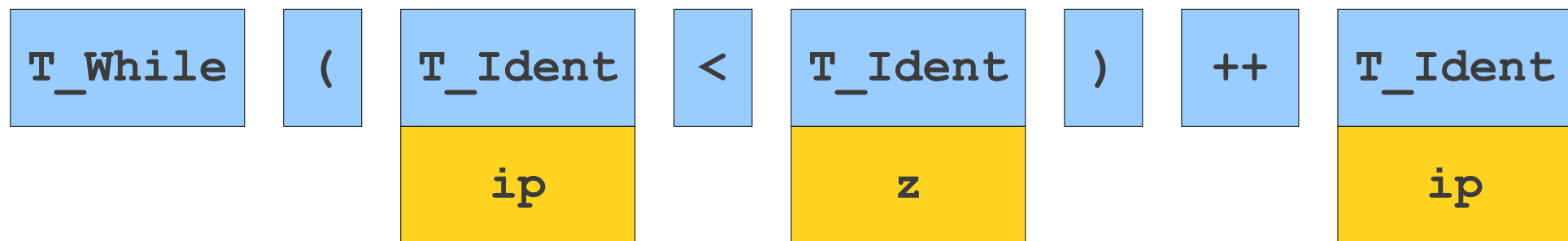
Input: code (character stream)

w	h	i	l	e		(	i	p		<		z	)	\n	\t	+	+	i	p	;
---	---	---	---	---	--	---	---	---	--	---	--	---	---	----	----	---	---	---	---	---

```
while (ip < z)
    ++ip;
```

# Goal of Lexical Analysis

Output: Token Stream



w	h	i	l	e		(	i	p		<		z	)	\n	\t	+	+	i	p	;
---	---	---	---	---	--	---	---	---	--	---	--	---	---	----	----	---	---	---	---	---

```
while (ip < z)
    ++ip;
```

# What's a token?

- What's a lexical unit of code?



w	h	i	l	e		(	1	3	7		<		i	)	\n	\t	+	+	i	;
---	---	---	---	---	--	---	---	---	---	--	---	--	---	---	----	----	---	---	---	---

What is my name ?

# Token Type

w	h	i	l	e		(	1	3	7		<		i	)	\n	\t	+	+	i	;
---	---	---	---	---	--	---	---	---	---	--	---	--	---	---	----	----	---	---	---	---

- Keyword: `for int if else while`
- Punctuation: `( ) { } ;`
- Operand: `+ - ++`
- Relation: `< > =`
- Identifier: (variable name, function name) `foo`  
`foo_2`
- Integer, float point, string: `2345 2.0 "hello world"`
- Whitespace, comment `/* this code is awesome */`

# Scanning a Source File

w	h	i	l	e		(	1	3	7		<		i	)	\n	\t	+	+	i	;
---	---	---	---	---	--	---	---	---	---	--	---	--	---	---	----	----	---	---	---	---

# Scanning a Source File

w	h	i	l	e		(	1	3	7		<		i	)	\n	\t	+	+	i	;
---	---	---	---	---	--	---	---	---	---	--	---	--	---	---	----	----	---	---	---	---

# Scanning a Source File

w	h	i	l	e		(	1	3	7		<		i	)	\n	\t	+	+	i	;
---	---	---	---	---	--	---	---	---	---	--	---	--	---	---	----	----	---	---	---	---

# Scanning a Source File

w	h	i	l	e		(	1	3	7		<		i	)	\n	\t	+	+	i	;
---	---	---	---	---	--	---	---	---	---	--	---	--	---	---	----	----	---	---	---	---

# Scanning a Source File

w	h	i	l	e		(	1	3	7		<		i	)	\n	\t	+	+	i	;
---	---	---	---	---	--	---	---	---	---	--	---	--	---	---	----	----	---	---	---	---

# Scanning a Source File

w	h	i	l	e		(	1	3	7		<		i	)	\n	\t	+	+	i	;
---	---	---	---	---	--	---	---	---	---	--	---	--	---	---	----	----	---	---	---	---



# Scanning a Source File

w	h	i	l	e		(	1	3	7		<		i	)	\n	\t	+	+	i	;
---	---	---	---	---	--	---	---	---	---	--	---	--	---	---	----	----	---	---	---	---

# Scanning a Source File

w	h	i	l	e		(	1	3	7		<		i	)	\n	\t	+	+	i	;
---	---	---	---	---	--	---	---	---	---	--	---	--	---	---	----	----	---	---	---	---

**Lexeme**: the piece of the original program from which we made the token

T\_While

**Token**

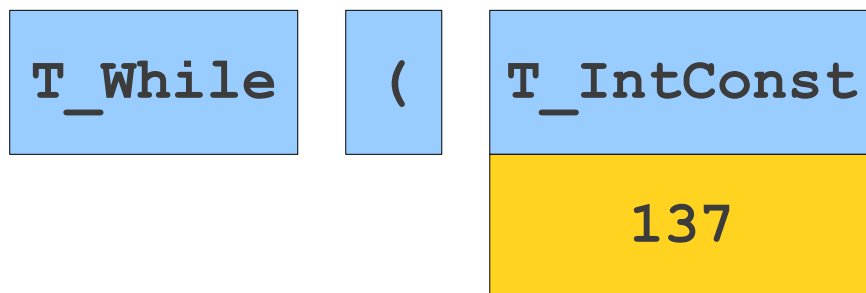
# Scanning a Source File

w	h	i	l	e		(	1	3	7		<		i	)	\n	\t	+	+	i	;
---	---	---	---	---	--	---	---	---	---	--	---	--	---	---	----	----	---	---	---	---

T_While	(	T_IntConst
		137

# Scanning a Source File

w	h	i	l	e		(	1	3	7		<		i	)	\n	\t	+	+	i	;
---	---	---	---	---	--	---	---	---	---	--	---	--	---	---	----	----	---	---	---	---



Some tokens can have **attributes** that store extra information about the token. Here we store which integer is represented.

# Lexical Analyzer

- Recognize substrings that correspond to tokens: **lexemes**
  - **Lexeme:** actual text of the token
- For each lexeme, identify token type
  - <Token type, attribute>
  - attribute: optional, extra information, often numeric value

# Challenges for Lexical Analyzer

- How do we determine which lexemes are associated with each token?
- When there are multiple ways we could scan the input, how do we know which one to pick?
  - if
  - ifc
- How do we address these concerns efficiently?

# Associate Lexemes to Tokens

- Tokens: categorize lexemes by what information they provide.
- Associate lexemes to token: Pattern matching
- How to describe patterns??

# Token: Lexemes

- Keyword: `for int if else while`

Finite possible  
lexemes

- Punctuation: `( ) { } ;`

- Operand: `+ - ++`

- Relation: `< > =`

Infinite  
possible  
lexemes

- Identifier: (variable name,function name) `foo`  
`foo_2`

- Integer, float point, string: `2345 2.0 "hello world"`

- Whitespace, comment `/* this code is awesome */`



- How do we describe which (potentially infinite) set of lexemes is associated with each token type?

# Formal Languages

- A **formal language** is a set of strings.
- Many infinite languages have finite descriptions:
  - Define the language using an automaton.
  - Define the language using a grammar.
  - Define the language using a regular expression.
- We can use these compact descriptions of the language to define sets of strings.

- What type of formal language should we use to describe tokens?

# Regular Expressions

- **Regular expressions** are a family of descriptions that can be used to capture certain languages (the *regular languages*).
- Often provide a compact and human-readable description of the language.
- Used as the basis for numerous software systems

# Atomic Regular Expressions

- The regular expressions we will use in this course begin with two simple building blocks.
- The symbol  $\epsilon$  is a regular expression matches the empty string.
- For any symbol  $a$ , the symbol  $a$  is a regular expression that just matches  $a$ .

# Compound Regular Expressions

- If  $R_1$  and  $R_2$  are regular expressions,  $\mathbf{R_1R_2}$  is a regular expression representing the **concatenation** of the languages of  $R_1$  and  $R_2$ .
- If  $R_1$  and  $R_2$  are regular expressions,  $\mathbf{R_1 \mid R_2}$  is a regular expression representing the **union** of  $R_1$  and  $R_2$ .
- If  $R$  is a regular expression,  $\mathbf{R^*}$  is a regular expression for the **Kleene closure** of  $R$ .
- If  $R$  is a regular expression,  $\mathbf{(R)}$  is a regular expression with the same meaning as  $R$ .

# Simple Regular Expressions

- Suppose the only characters are 0 and 1.
- Here is a regular expression for strings containing 00 as a substring:

**$(0 \mid 1)^*00(0 \mid 1)^*$**

# Simple Regular Expressions

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$(0 \mid 1)^*00(0 \mid 1)^*$

11011100101  
0000  
11111011110011111

# Simple Regular Expressions

- Suppose the only characters are 0 and 1.
- Here is a regular expression for strings containing 00 as a substring:

$(0 \mid 1)^*00(0 \mid 1)^*$

11011100101  
0000  
11111011110011111

# Applied Regular Expressions

- Suppose that our alphabet is all ASCII characters.
- A regular expression for even numbers is

?

# Applied Regular Expressions

- Suppose that our alphabet is all ASCII characters.
- A regular expression for even numbers is

**(+|-)?(0|1|2|3|4|5|6|7|8|9)\*(0|2|4|6|8)**

**42**  
**+1370**  
**-3248**  
**-9999912**

- More examples
  - Whitespace: `[ \t\n]+`
  - Integers: `[+\-]?[0-9]+`
  - Hex numbers: `0x[0-9a-f]+`
  - identifier
    - `[A-Za-z]([A-Za-z]|[0-9])*`

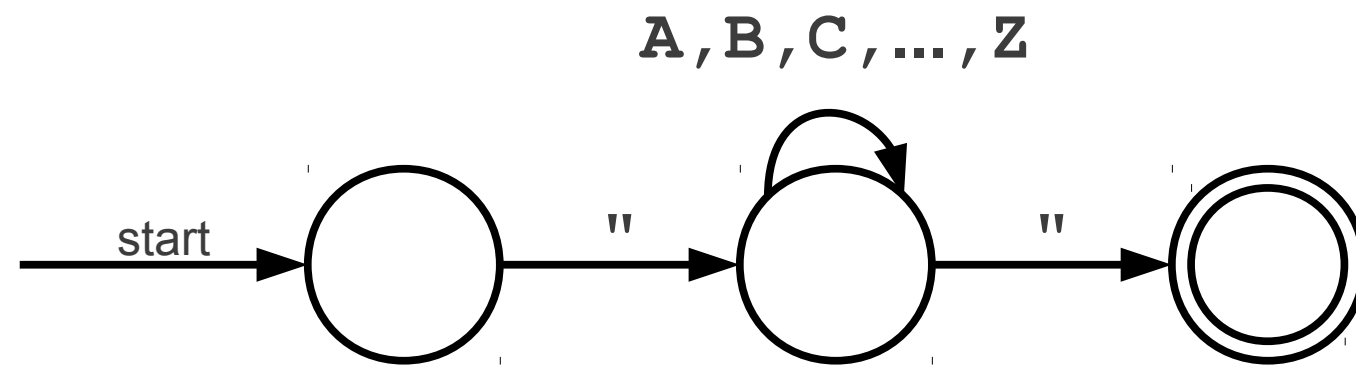
- Use regular expressions to describe token types
- How do we match regular expressions?

# Recognizing Regular Language

What is the machine that recognize regular language??

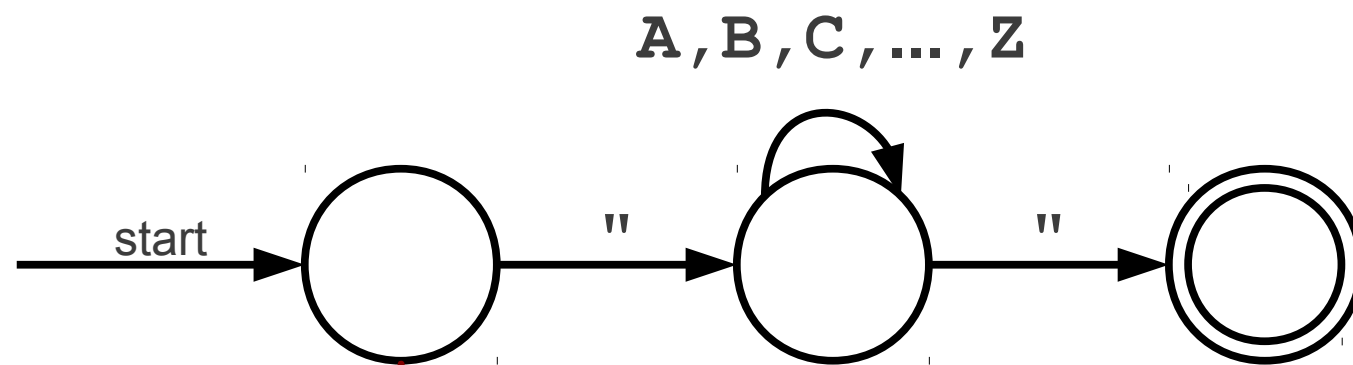
- Finite Automata
- DFA (Deterministic Finite Automata)
- NFA (Non-deterministic Finite Automata)

# A Simple Automaton



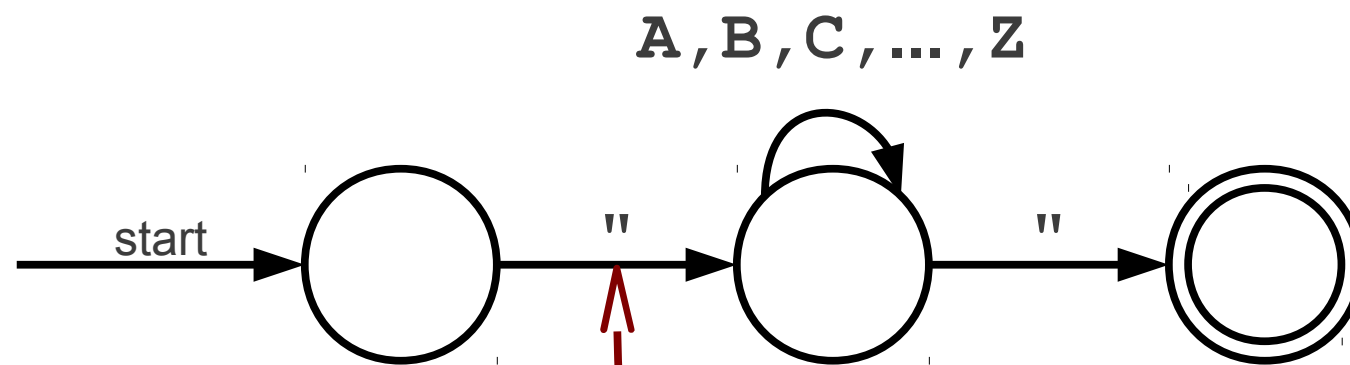


# A Simple Automaton



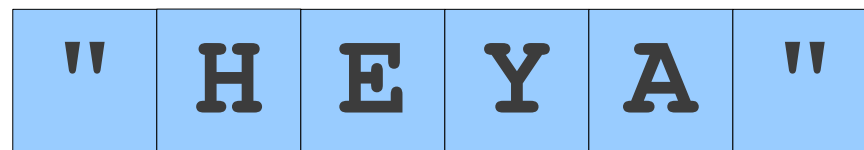
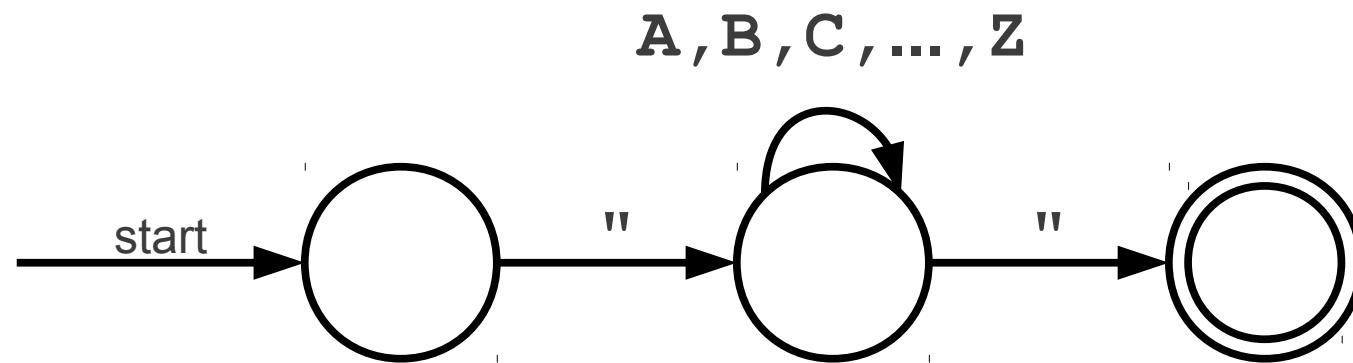
Each circle is a **state** of the automaton. The automaton's configuration is determined by what state(s) it is in.

# A Simple Automaton



These arrows are called **transitions**. The automaton changes which state(s) it is in by following transitions.

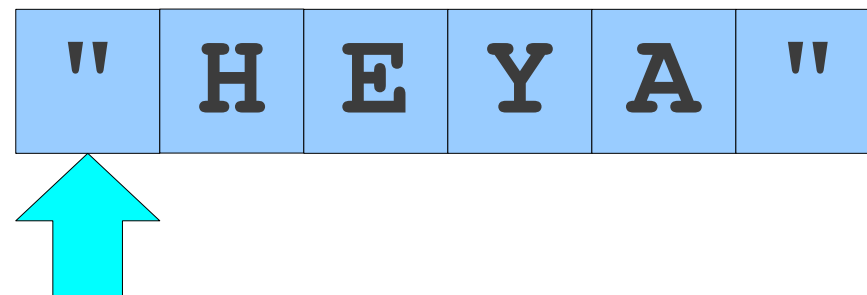
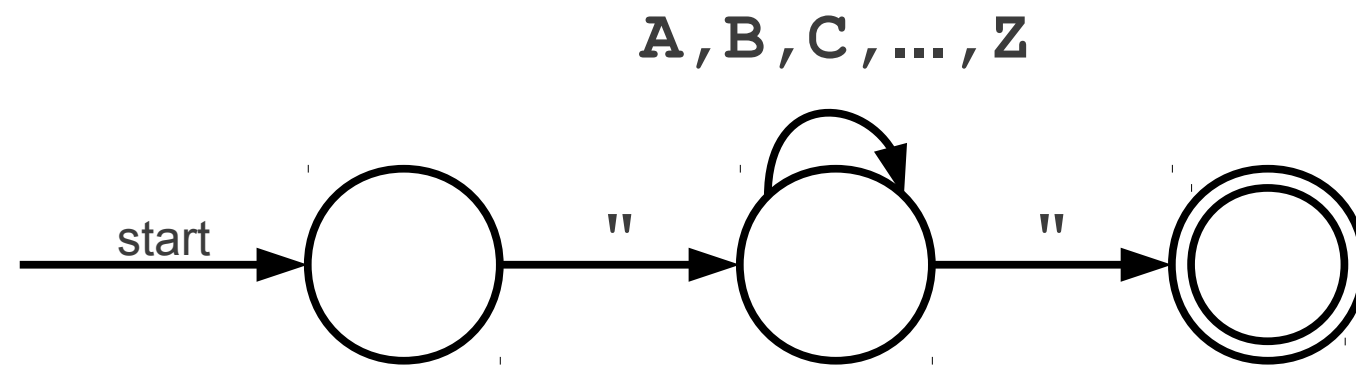
# A Simple Automaton



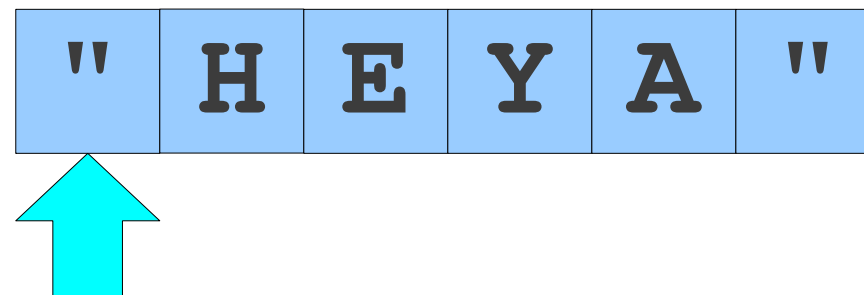
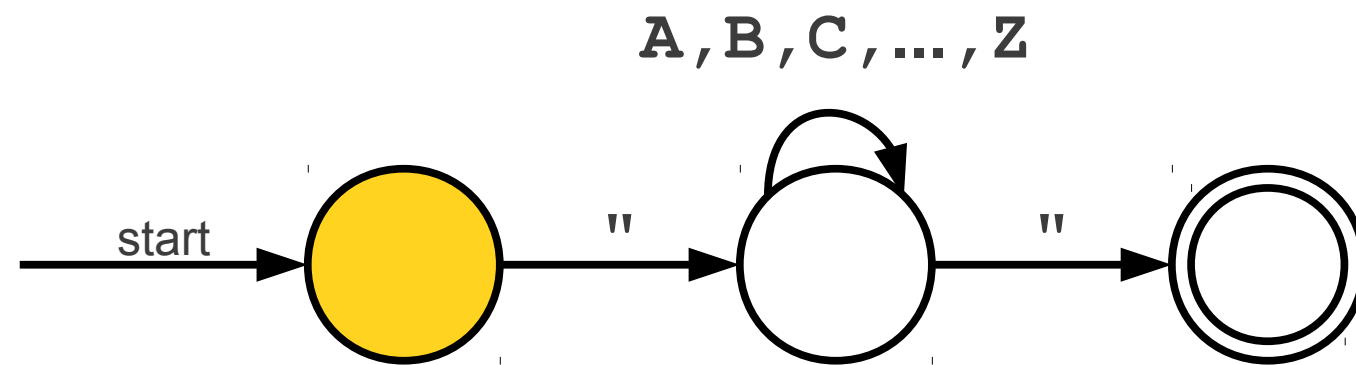
Finite Automata: Takes an input string and determines whether it's a valid sentence of a language

accept or reject

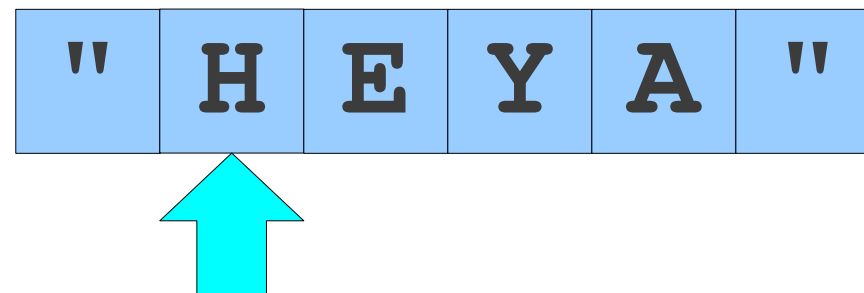
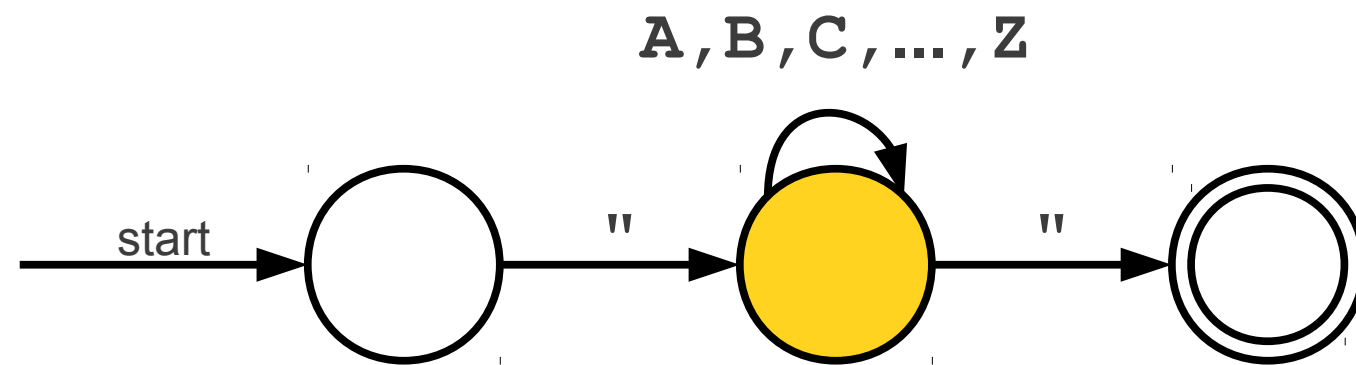
# A Simple Automaton



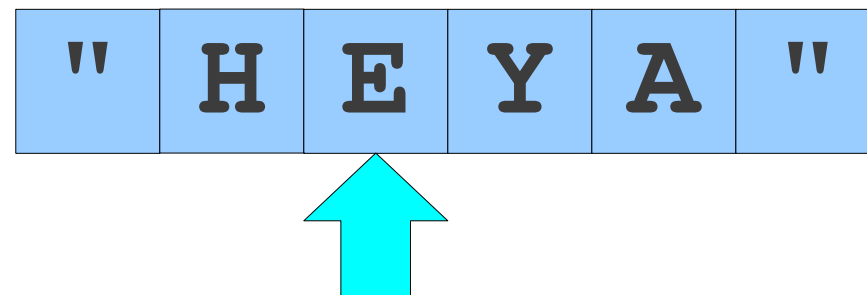
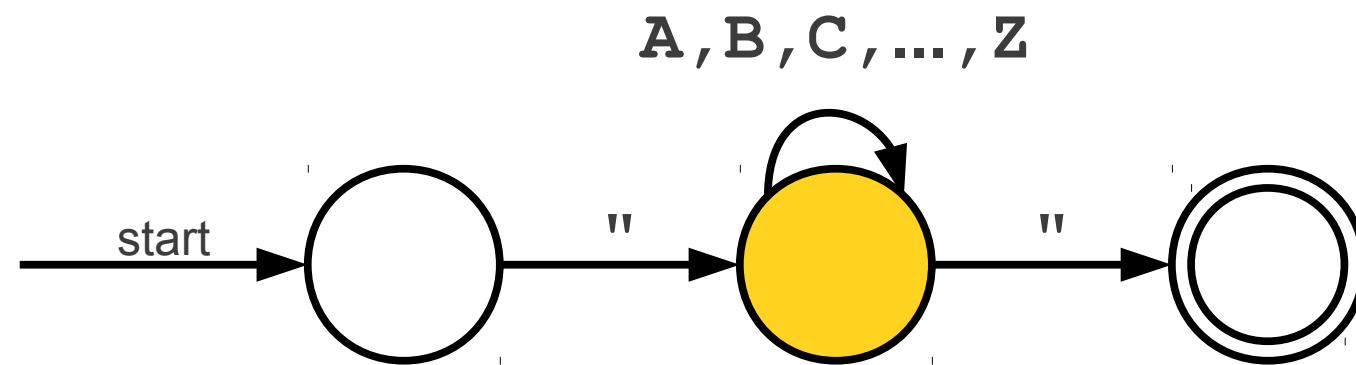
# A Simple Automaton



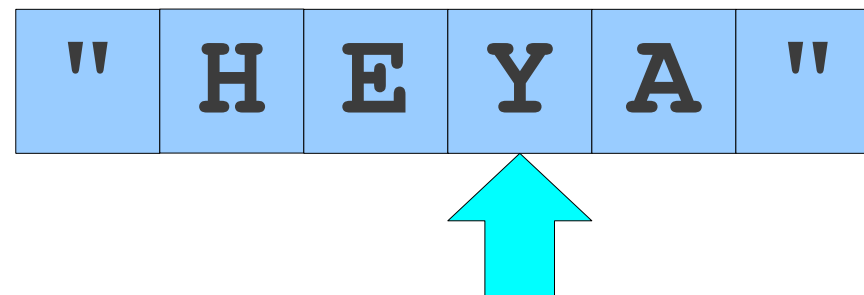
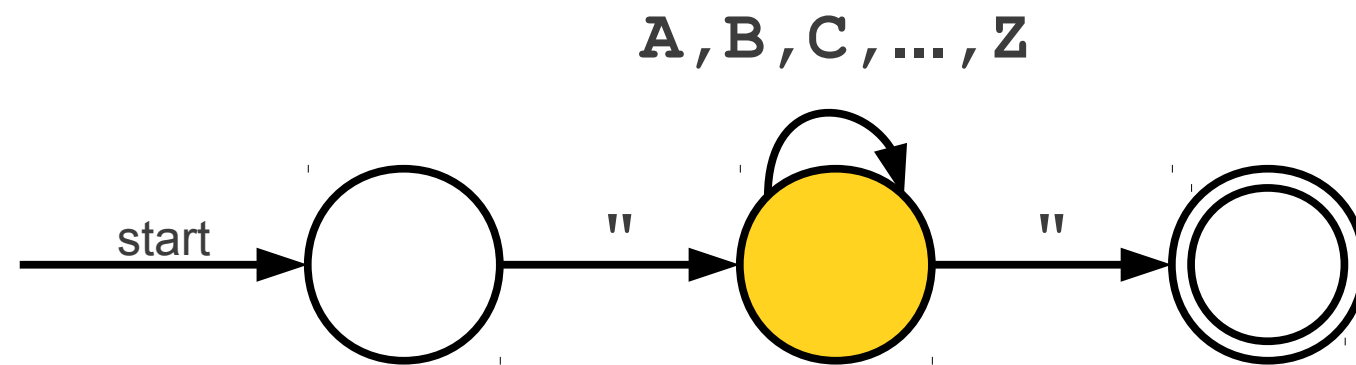
# A Simple Automaton



# A Simple Automaton

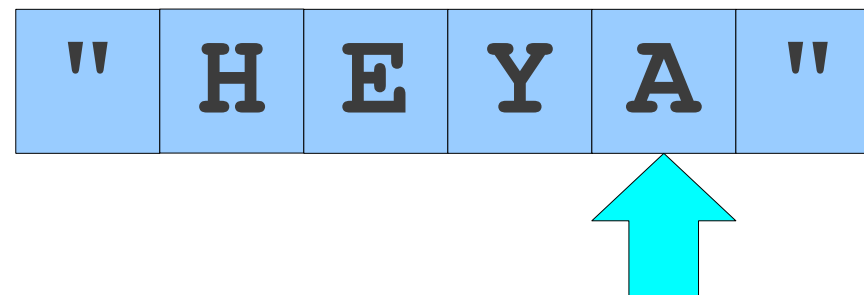
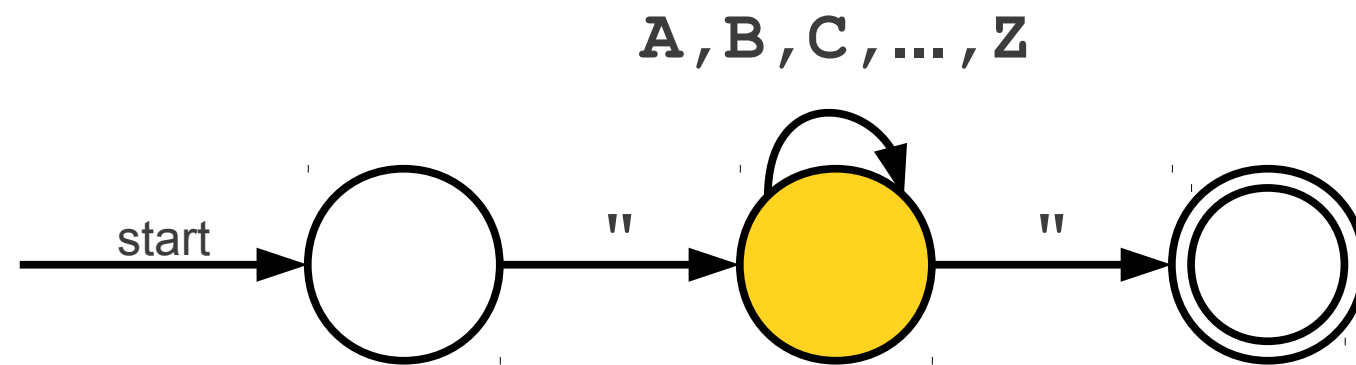


# A Simple Automaton

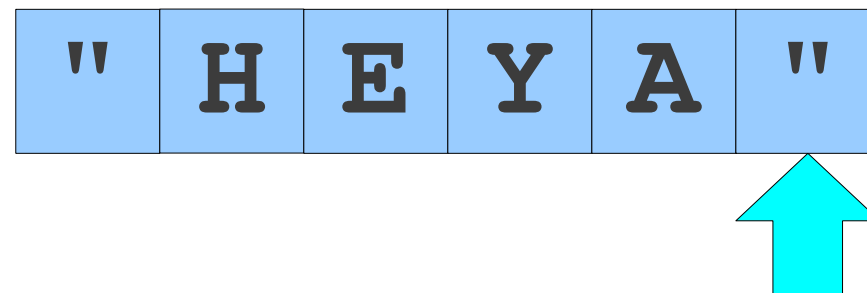
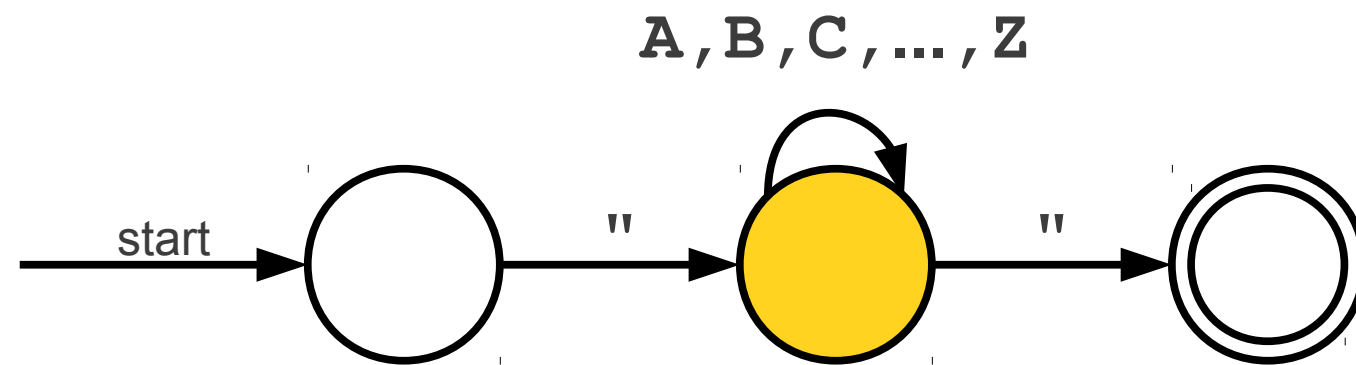




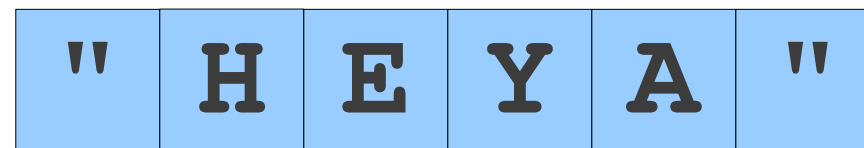
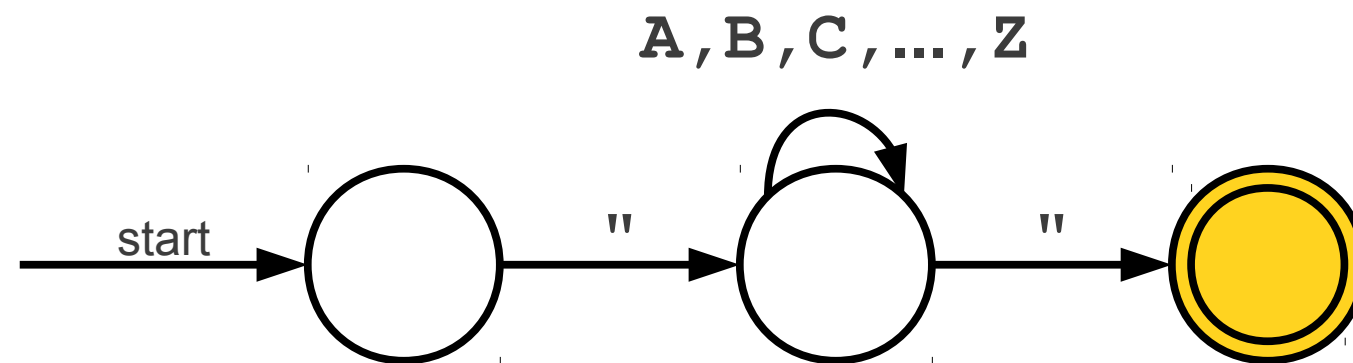
# A Simple Automaton



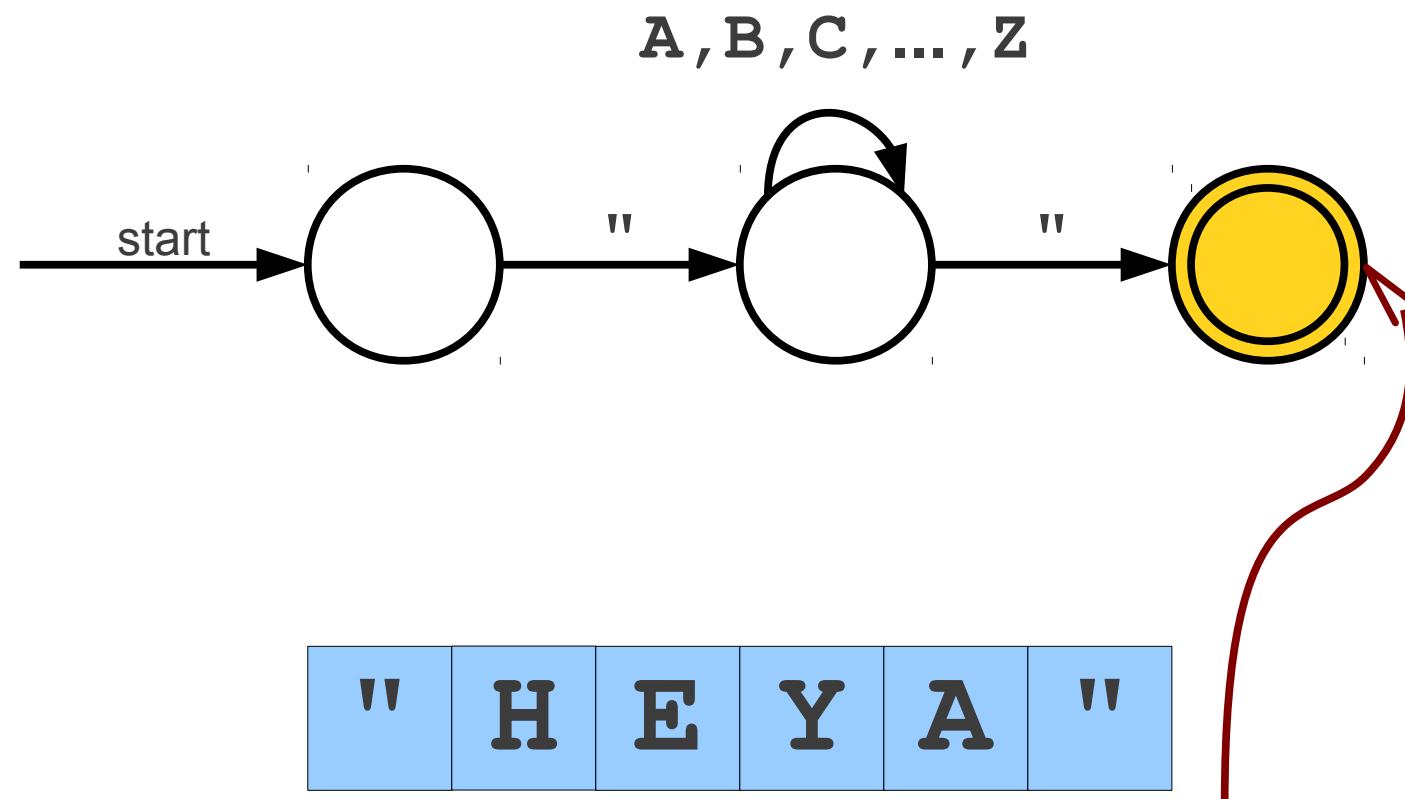
# A Simple Automaton



# A Simple Automaton

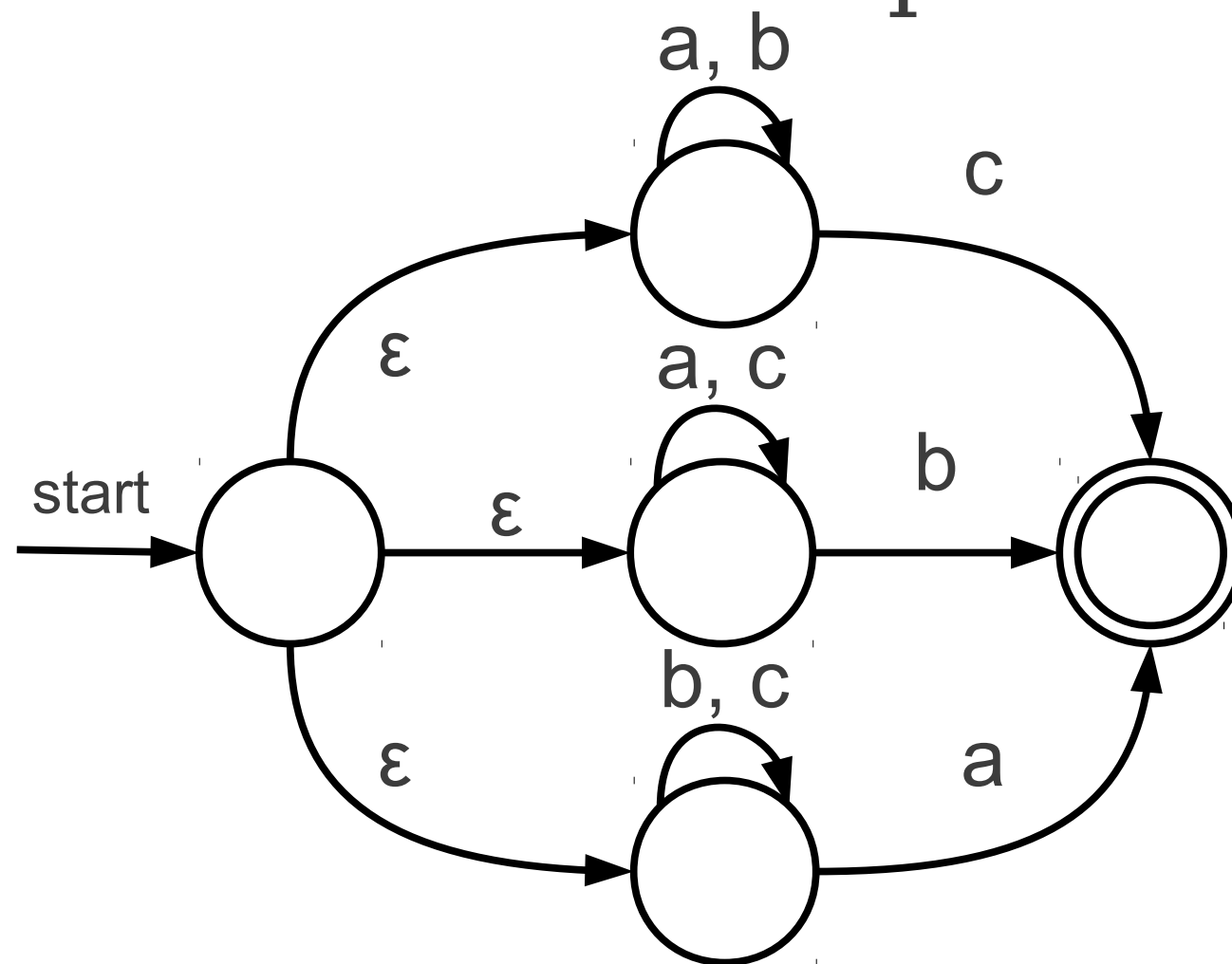


# A Simple Automaton

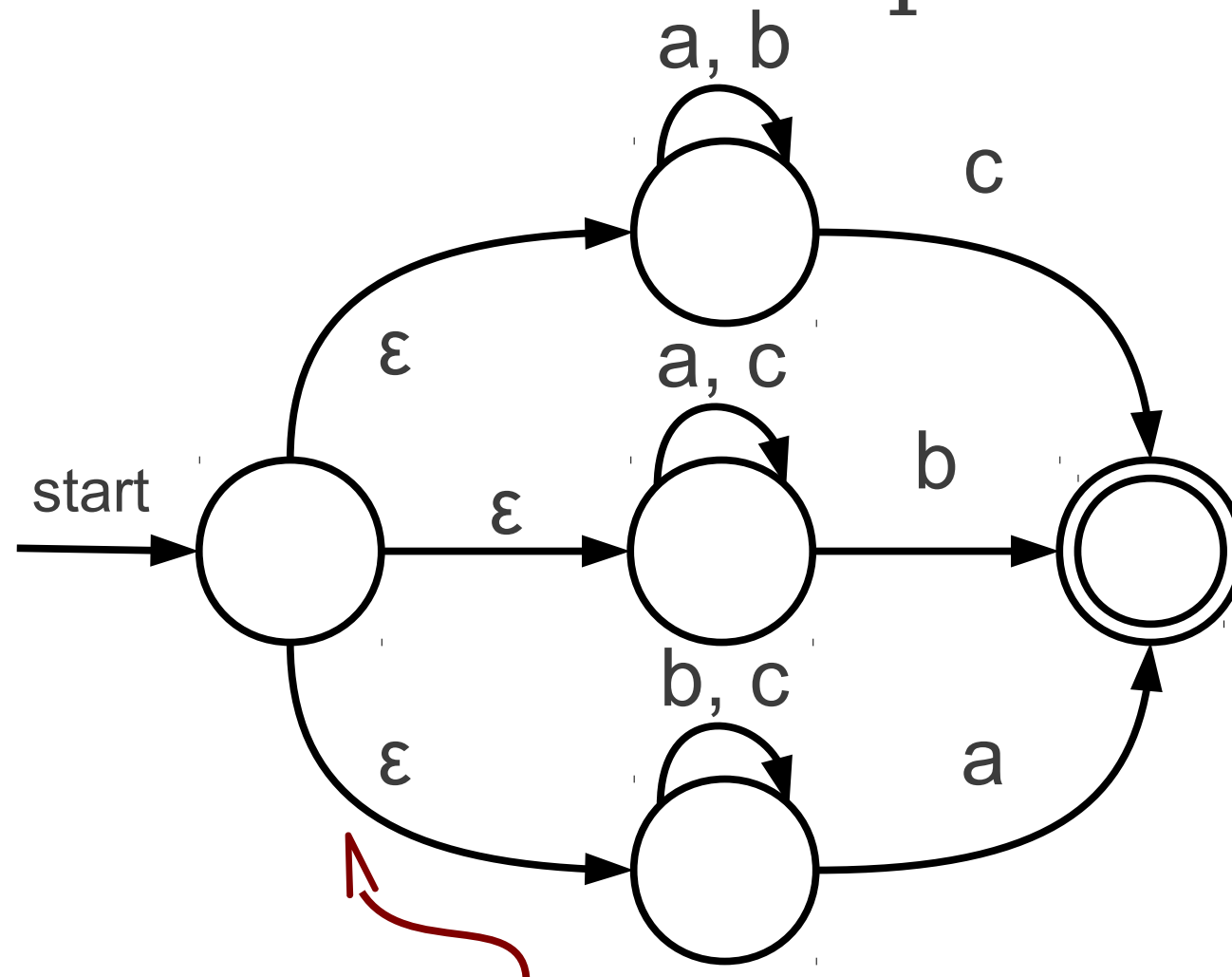


The double circle indicates that this state is an **accepting state**. The automaton accepts the string if it ends in an accepting state.

# An Even More Complex Automaton

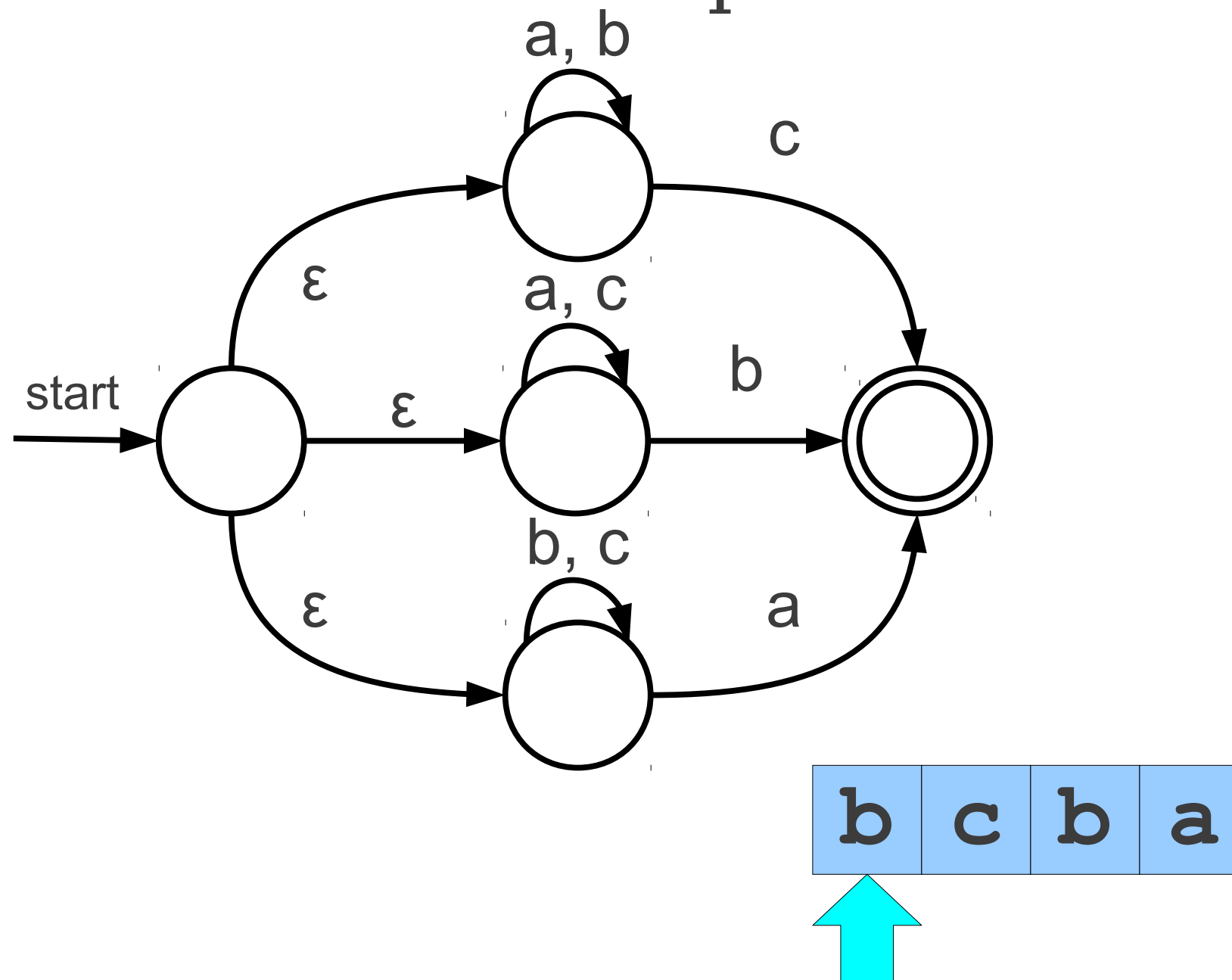


# An Even More Complex Automaton

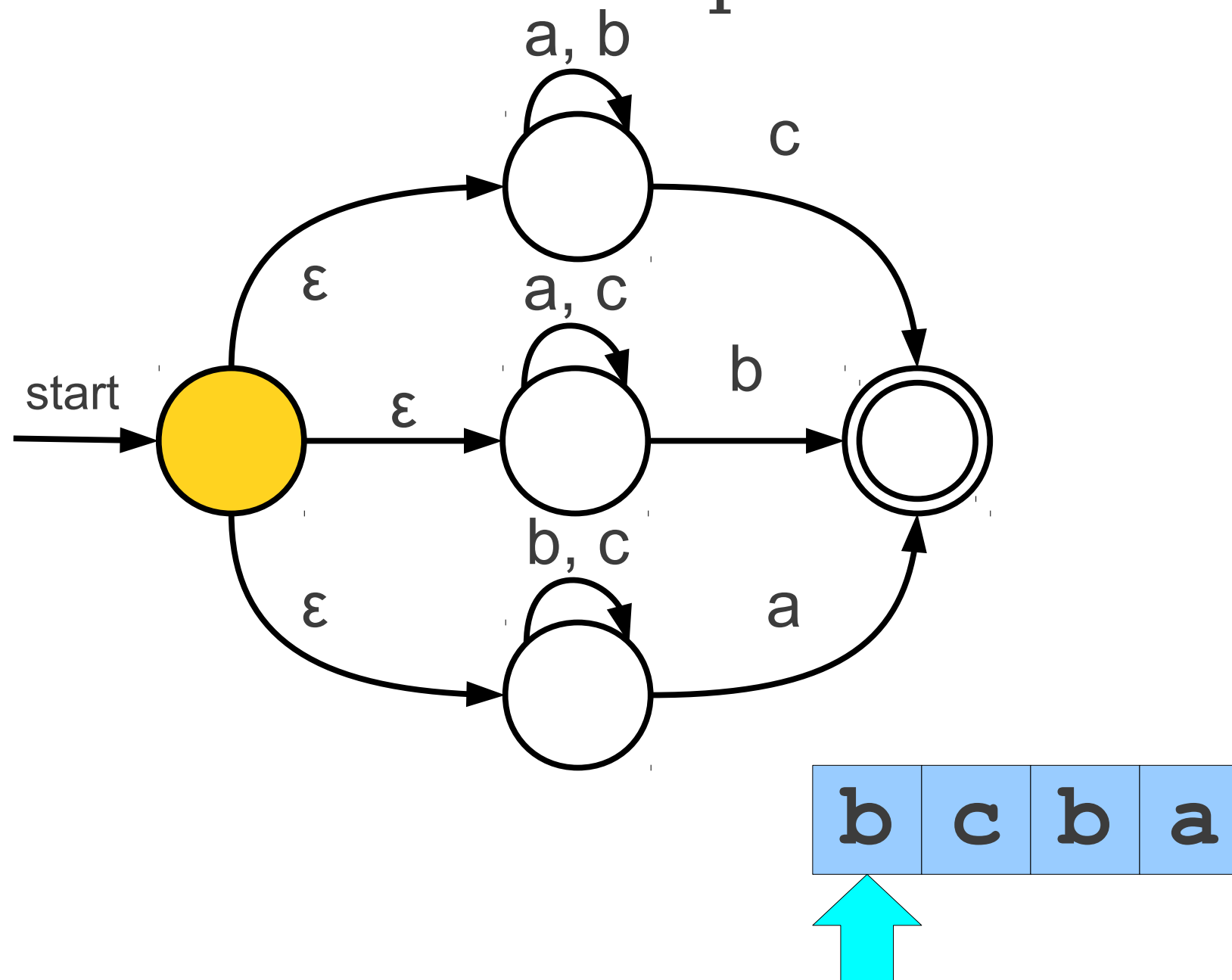


These are called  **$\epsilon$ -transitions**. These transitions are followed automatically and without consuming any input.

# An Even More Complex Automaton

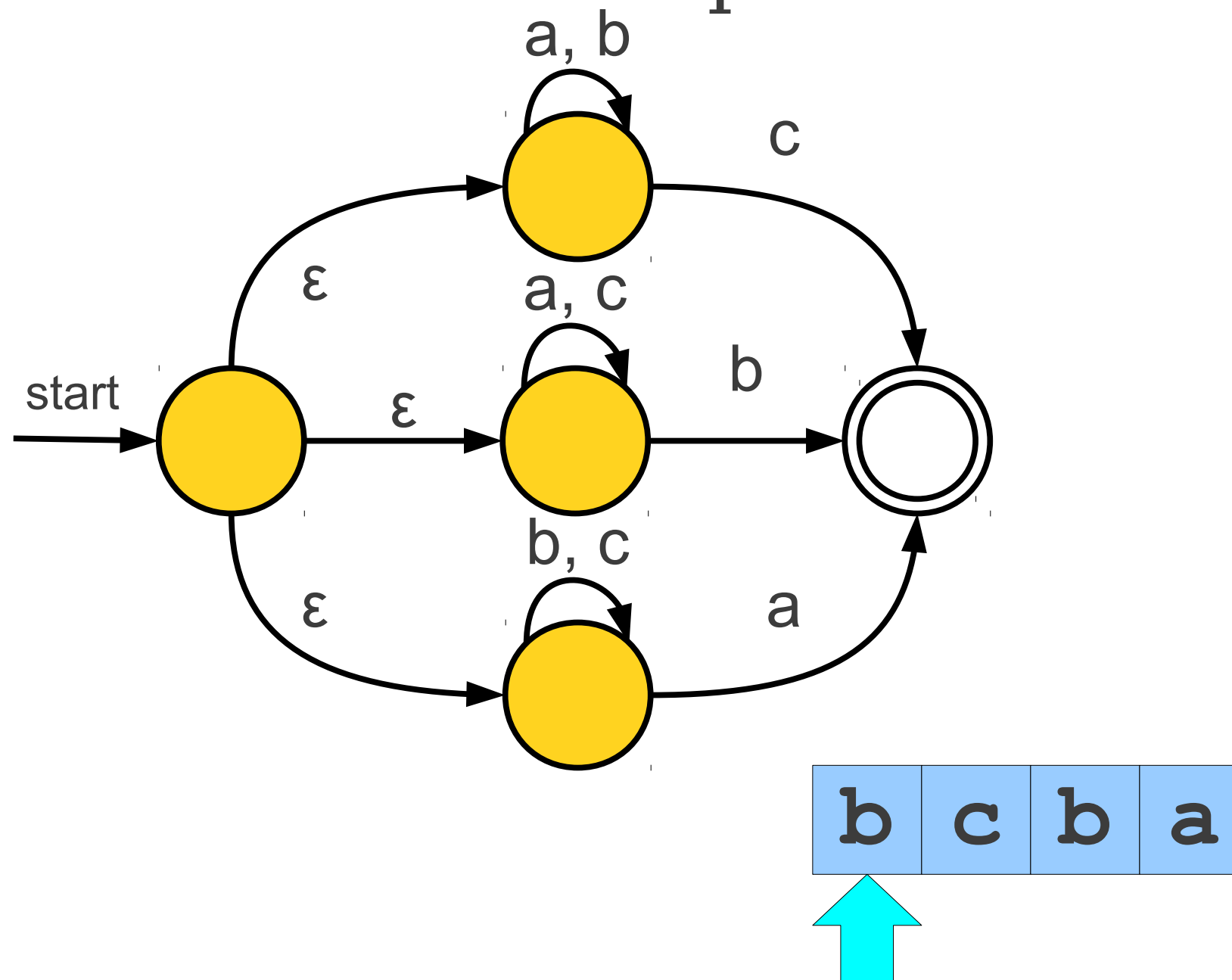


# An Even More Complex Automaton

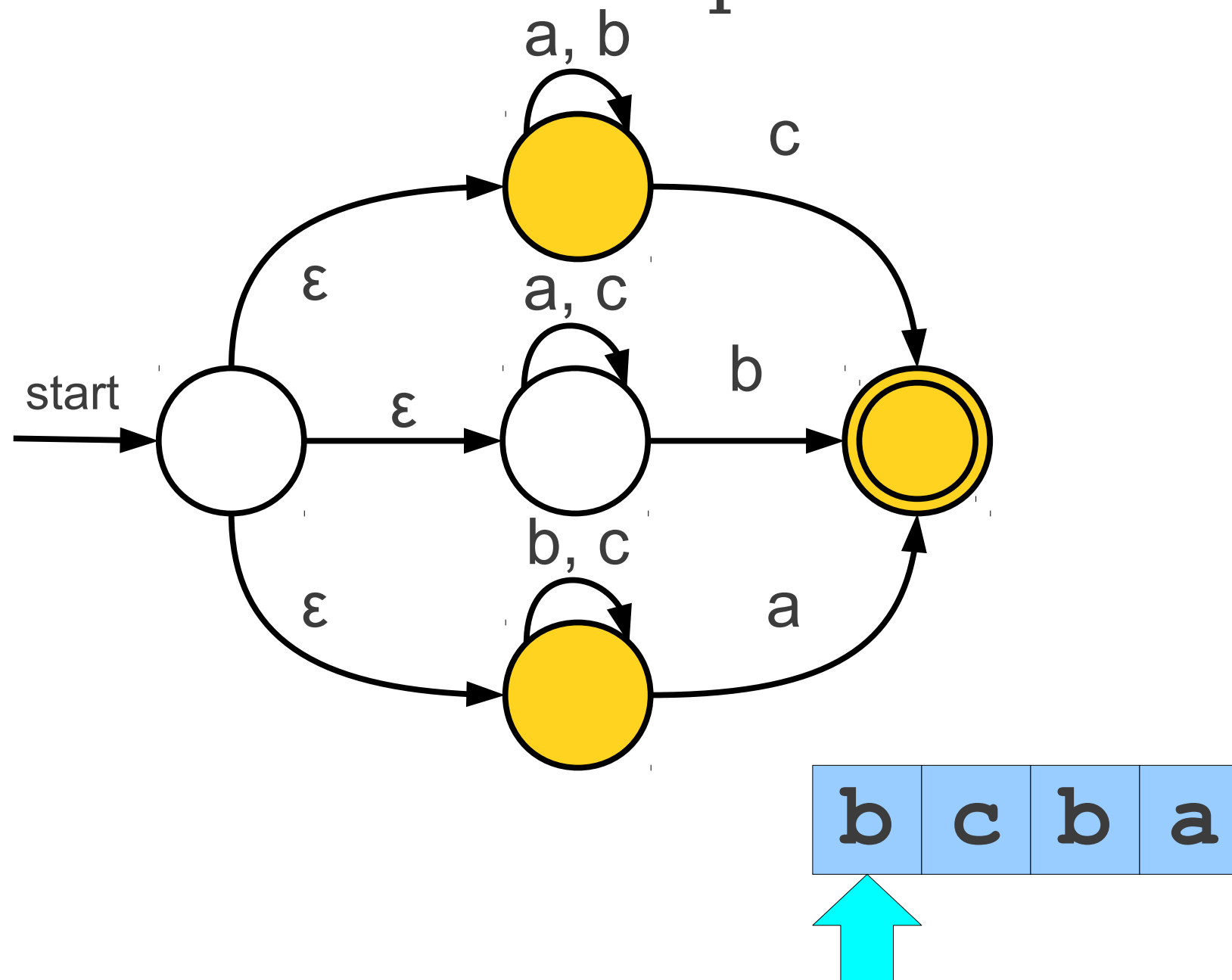




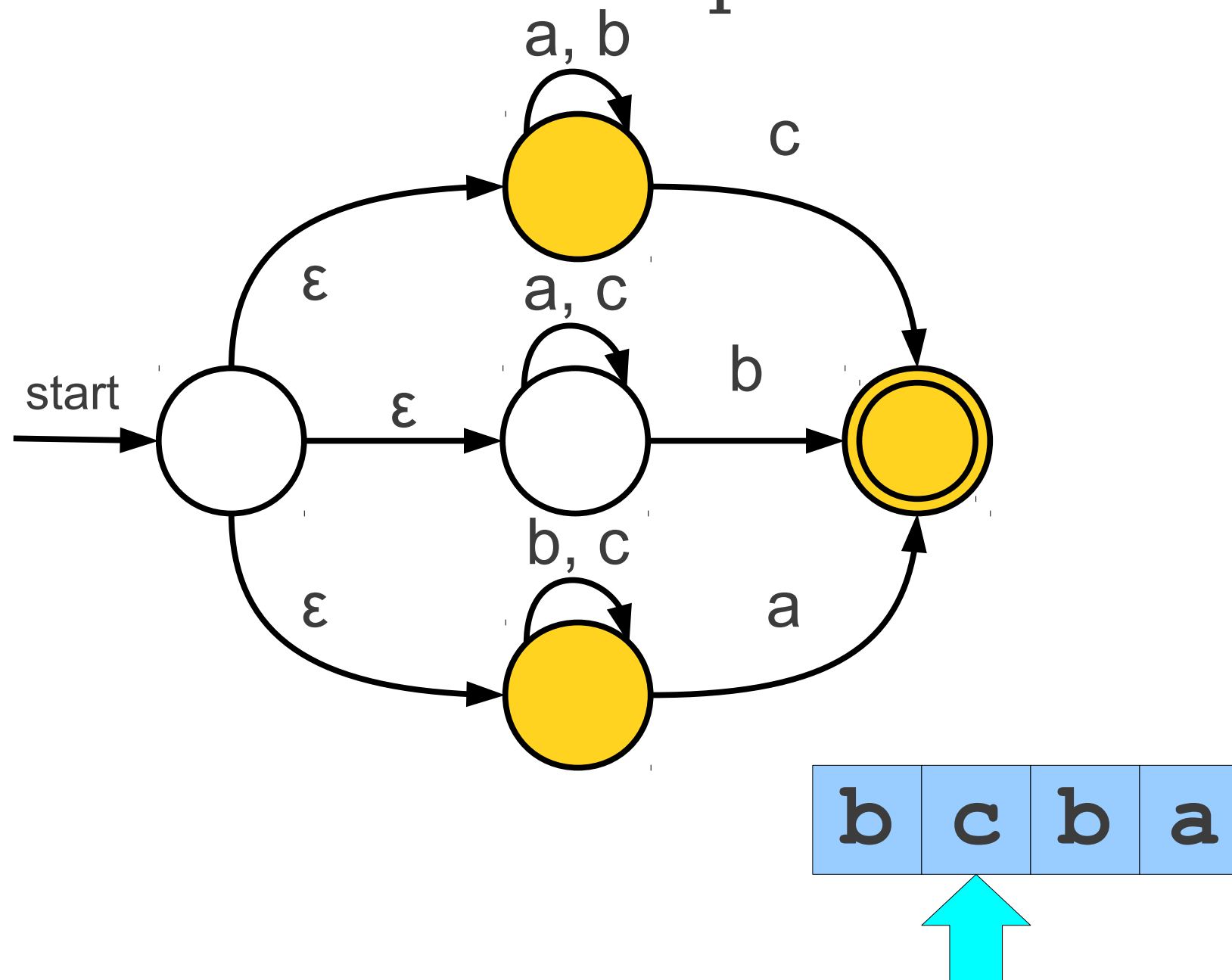
# An Even More Complex Automaton



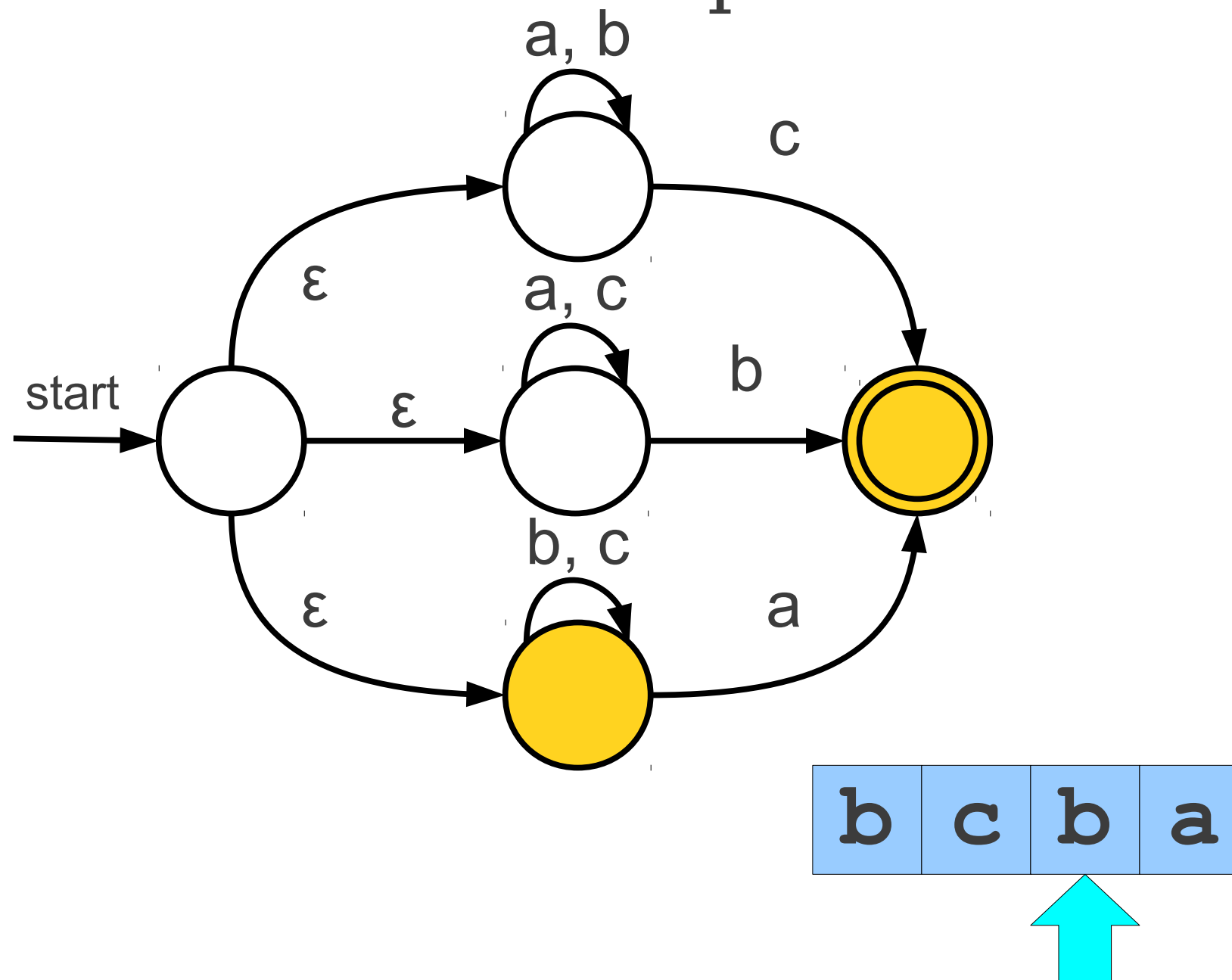
# An Even More Complex Automaton



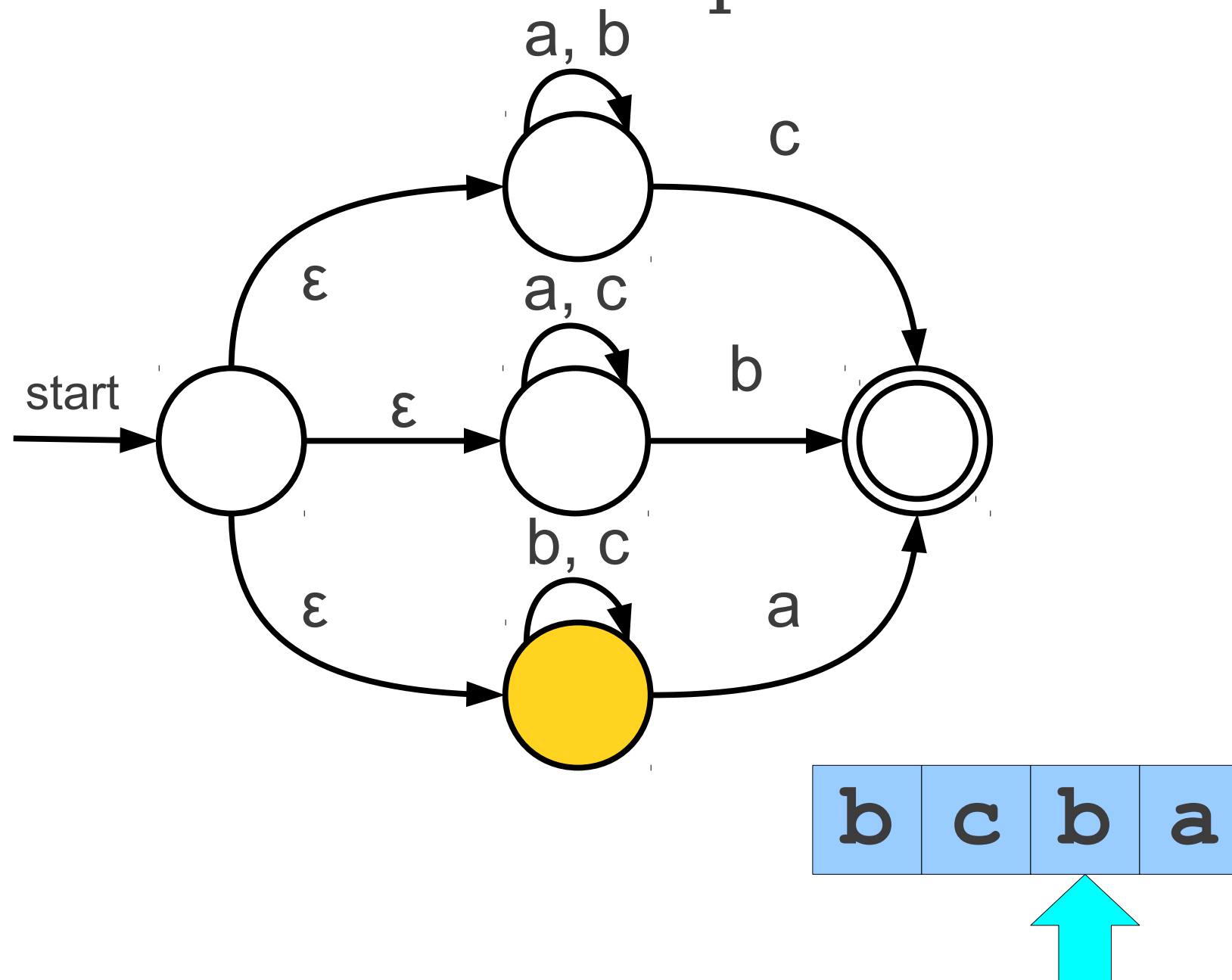
# An Even More Complex Automaton



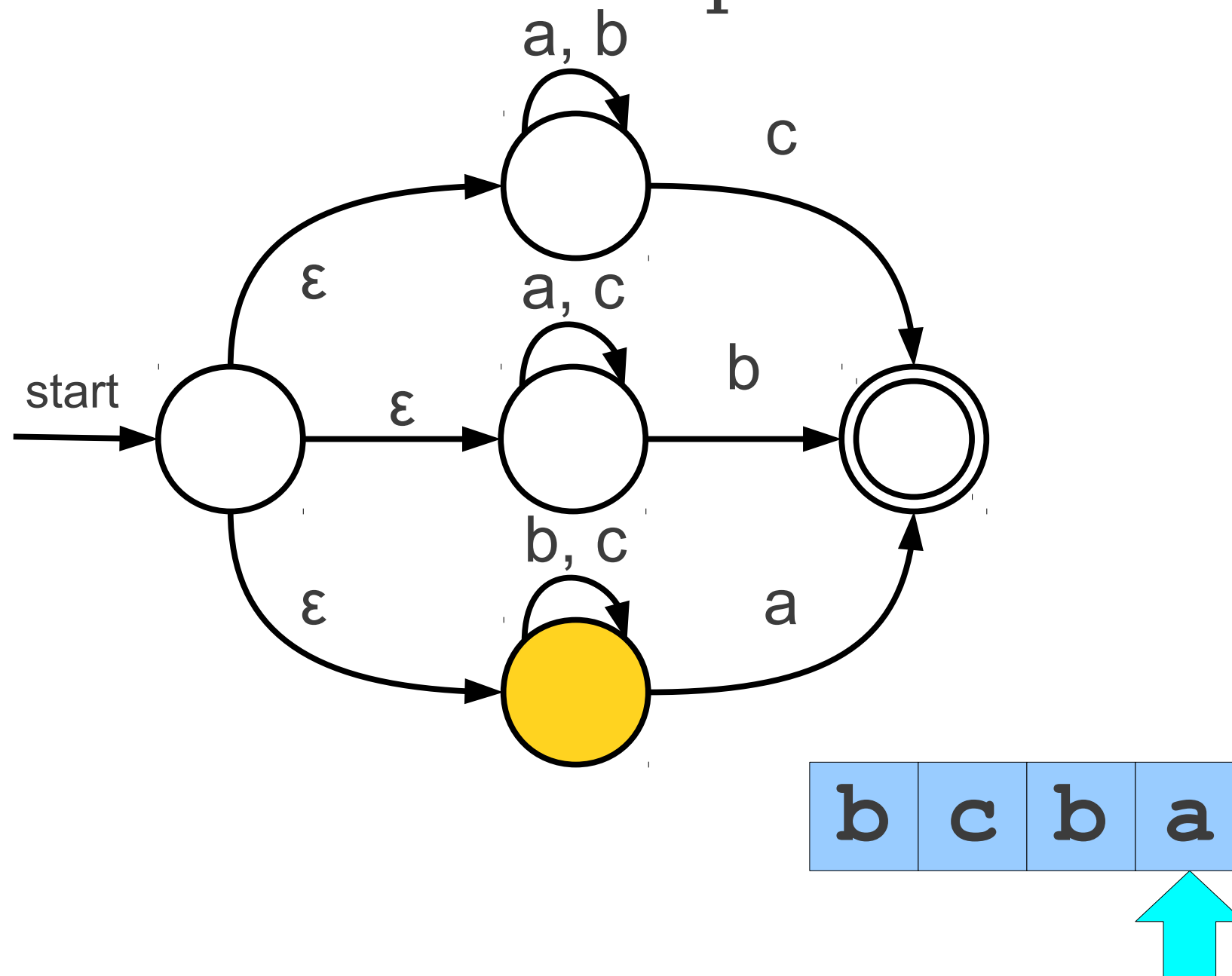
# An Even More Complex Automaton



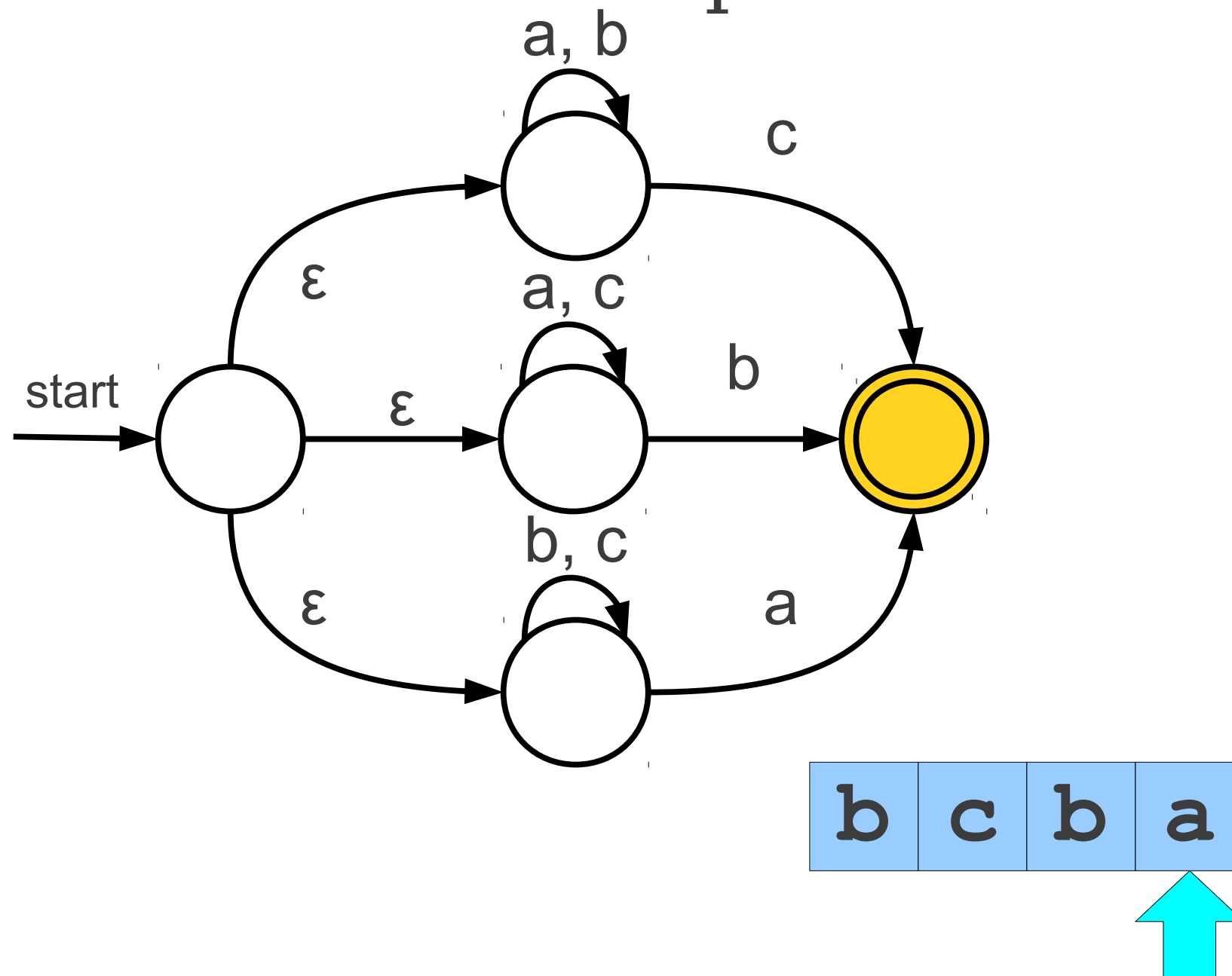
# An Even More Complex Automaton



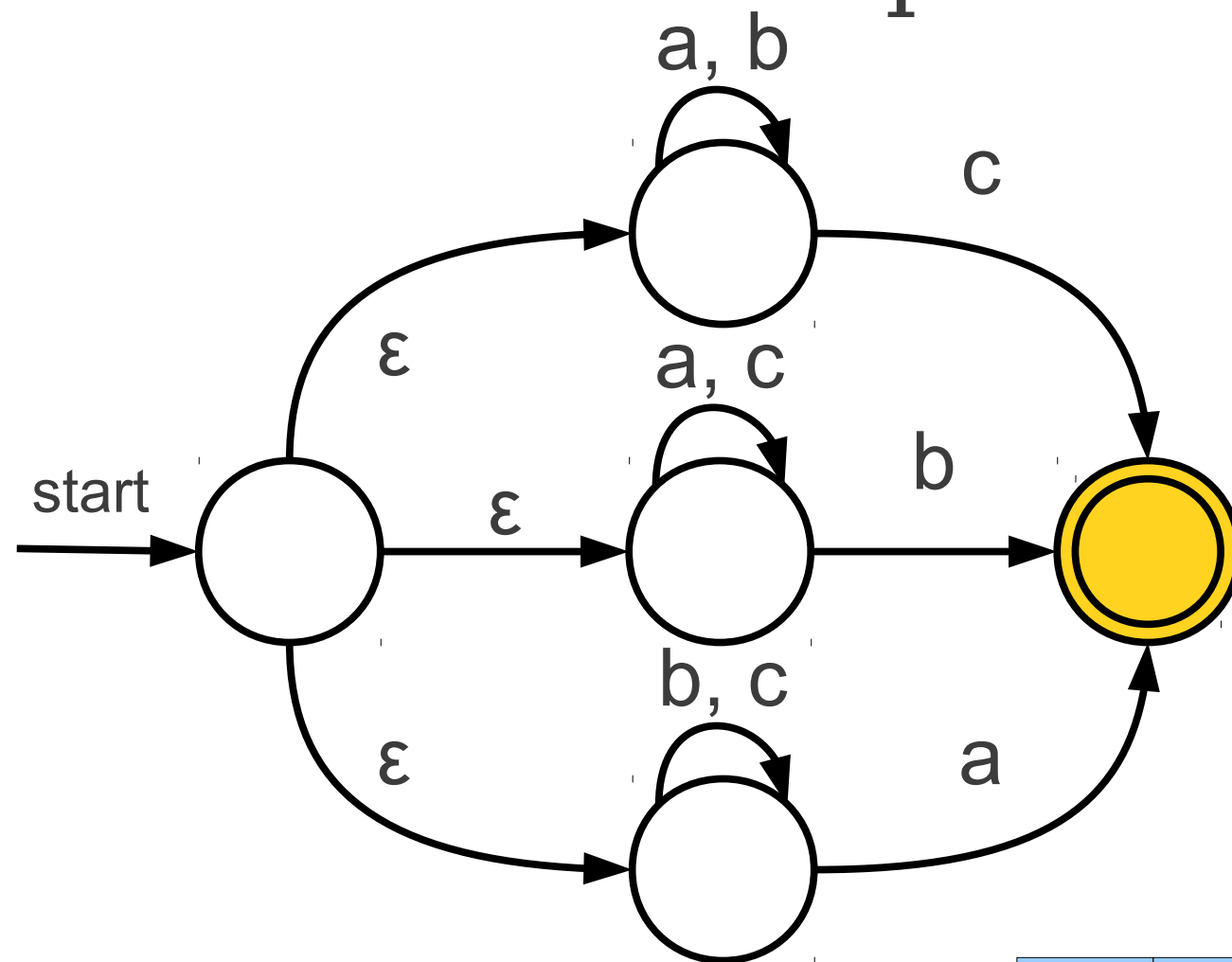
# An Even More Complex Automaton



# An Even More Complex Automaton



# An Even More Complex Automaton



<b>b</b>	<b>c</b>	<b>b</b>	<b>a</b>
----------	----------	----------	----------



# Lexer Generator

- Given regular expressions to describe the language (token types),
  - Step 1: Generates NFA that can recognize the regular language defined
    - existing algorithms
  - Step 2: Transforms NFA to DFA
    - existing algorithms
- Tools: **lex**, **flex**

# Challenges for Lexical Analyzer

- How do we determine which lexemes are associated with each token?
  - Regular expression to describe token type
- When there are multiple ways we could scan the input, how do we know which one to pick?
- How do we address these concerns efficiently?

# Lexing Ambiguities

T_For	for
T_Identifier	[A-Za-z_][A-Za-z0-9_]*

# Lexing Ambiguities

T\_For            for  
T\_Identifier    [A-Za-z\_] [A-Za-z0-9\_]\*

f	o	r	t
---	---	---	---

# Lexing Ambiguities

T\_For

for

T\_Identifier

[A-Za-z\_][A-Za-z0-9\_]\*

f	o	r	t
---	---	---	---

f	o	r	t	
f	o	r		t
f	o	r		t
f	o		r	t
f	o		r	t

f	o	r	t	
f	o	r		t
f	o		r	t
f	o		r	t

# Conflict Resolution

- Assume all tokens are specified as regular expressions.
- Algorithm: **Left-to-right scan**.
- Tiebreaking rule one: **Maximal munch**.
  - Always match the longest possible prefix of the remaining text.

# Lexing Ambiguities

T\_For

for

T\_Identifier

[A-Za-z\_] [A-Za-z0-9\_]\*

f	o	r	t
---	---	---	---

f	o	r	t
---	---	---	---

# Implementing Maximal Munch

- Given a set of regular expressions, how can we use them to implement maximum munch?



- Example

# Implementing Maximal Munch

T_Do	do
T_Double	double
T_Mystery	[A-Za-z]

# Implementing Maximal Munch

T\_Do

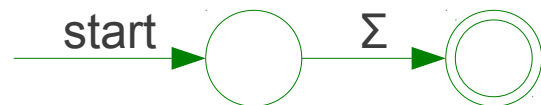
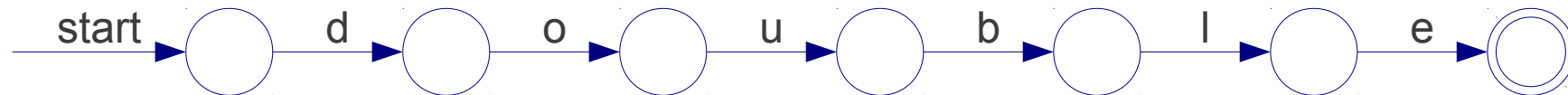
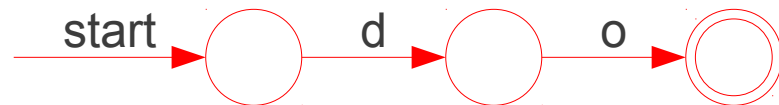
do

T\_Double

double

T\_Mystery

[A-Za-z]



# Implementing Maximal Munch

T\_Do

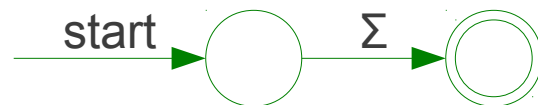
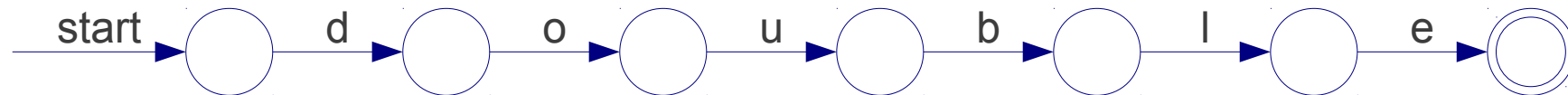
do

T\_Double

double

T\_Mystery

[A-Za-z]



D	O	U	B	D	O	U	B	L	E
---	---	---	---	---	---	---	---	---	---

# Implementing Maximal Munch

T\_Do

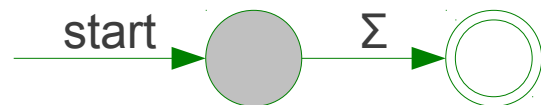
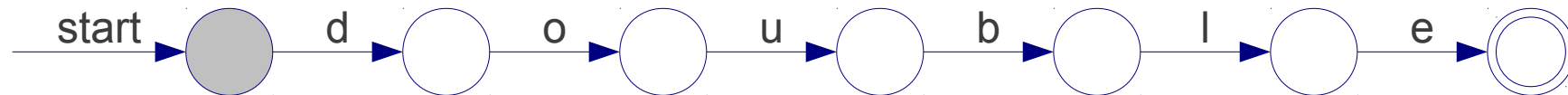
do

T\_Double

double

T\_Mystery

[A-Za-z]



D	O	U	B	D	O	U	B	L	E
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# Implementing Maximal Munch

T\_Do

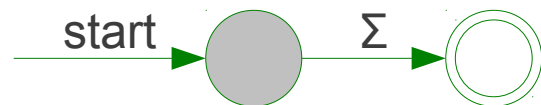
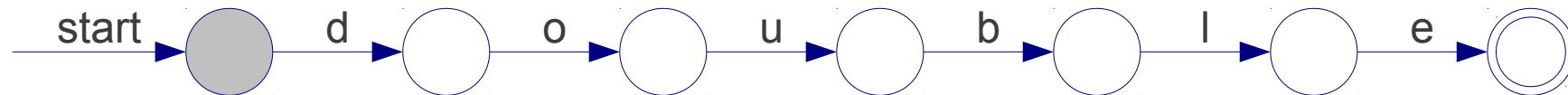
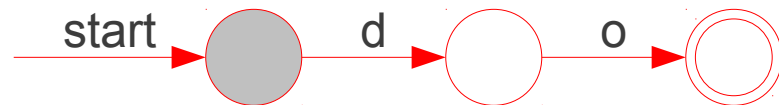
do

T\_Double

double

T\_Mystery

[A-Za-z]



D	O	U	B	D	O	U	B	L	E
---	---	---	---	---	---	---	---	---	---



# Implementing Maximal Munch

T\_Do

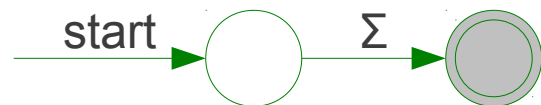
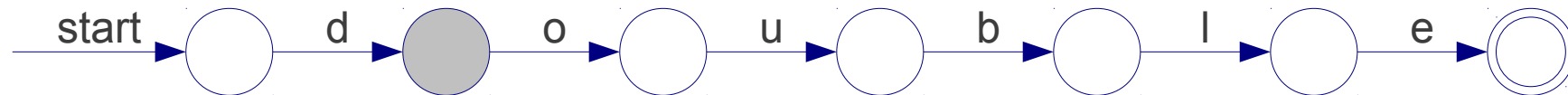
do

T\_Double

double

T\_Mystery

[A-Za-z]



D	O	U	B	D	O	U	B	L	E
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# Implementing Maximal Munch

T\_Do

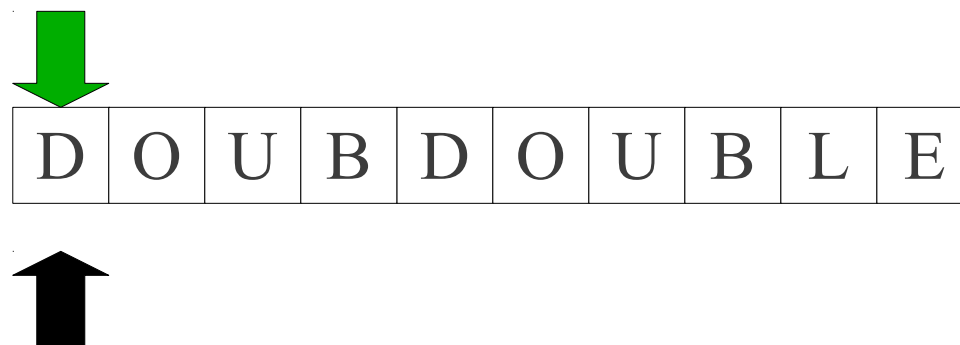
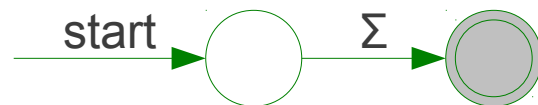
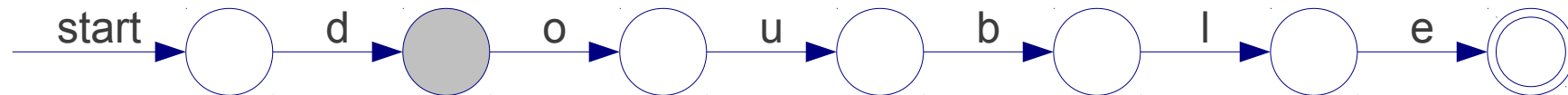
do

T\_Double

double

T\_Mystery

[A-Za-z]





# Implementing Maximal Munch

T\_Do

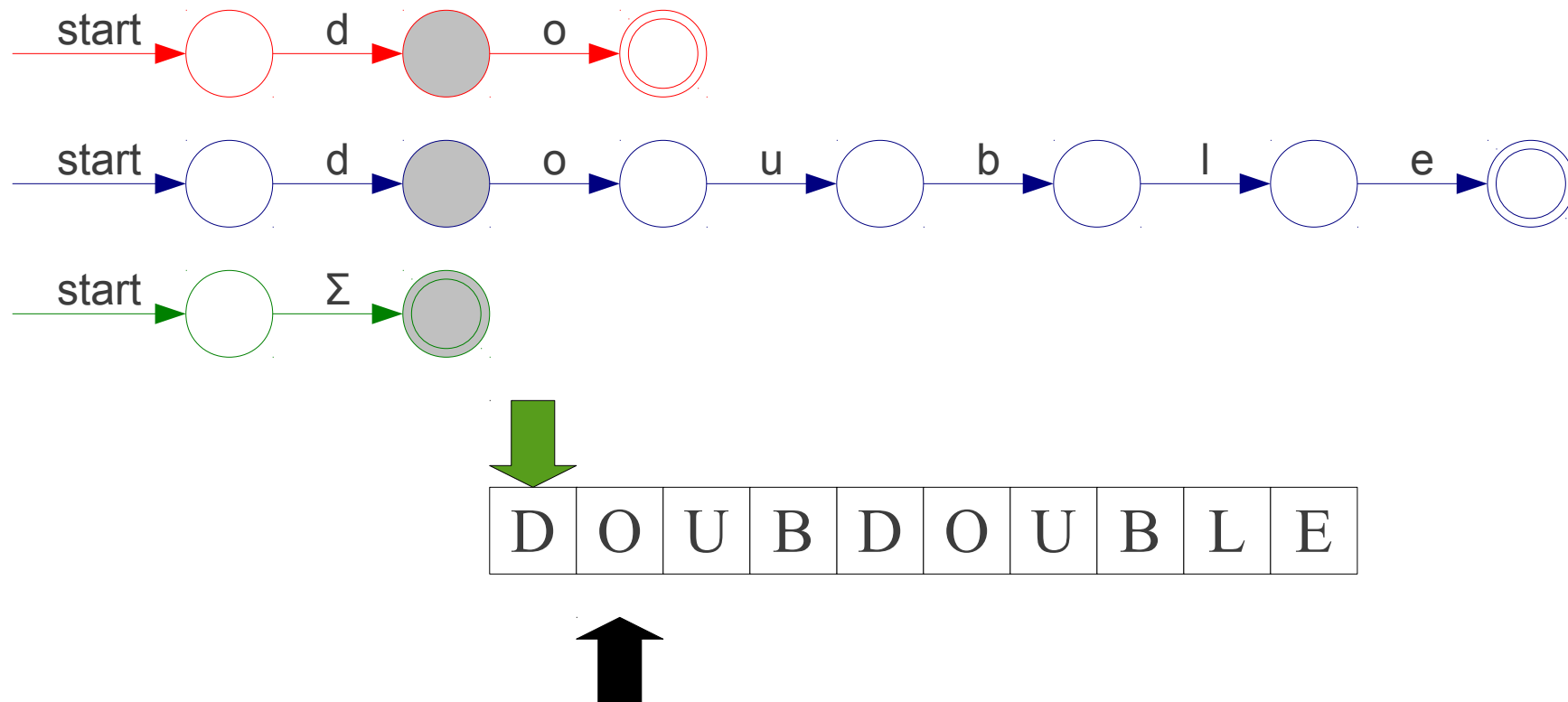
do

T\_Double

double

T\_Mystery

[A-Za-z]



# Implementing Maximal Munch

T\_Do

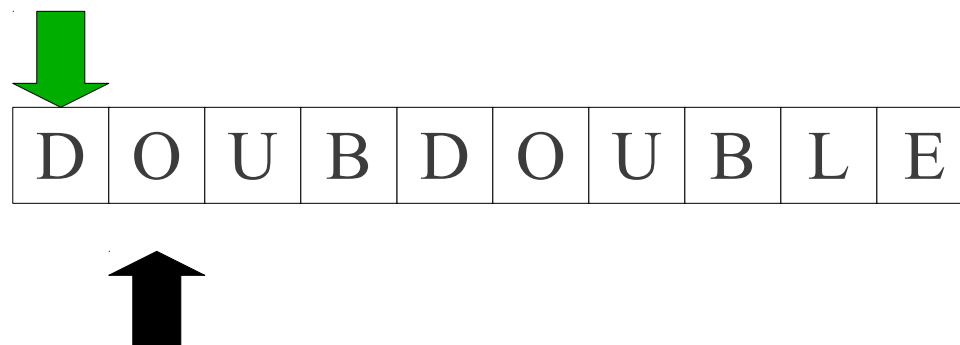
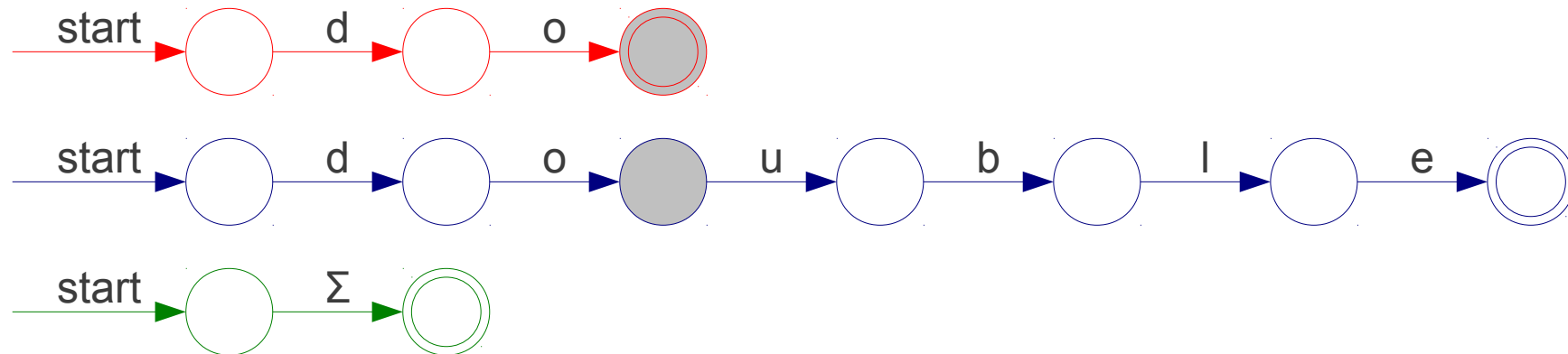
do

T\_Double

double

T\_Mystery

[A-Za-z]



# Implementing Maximal Munch

T\_Do

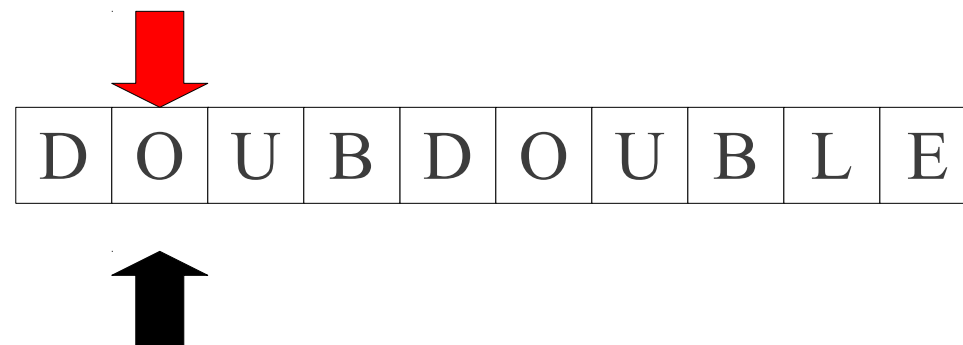
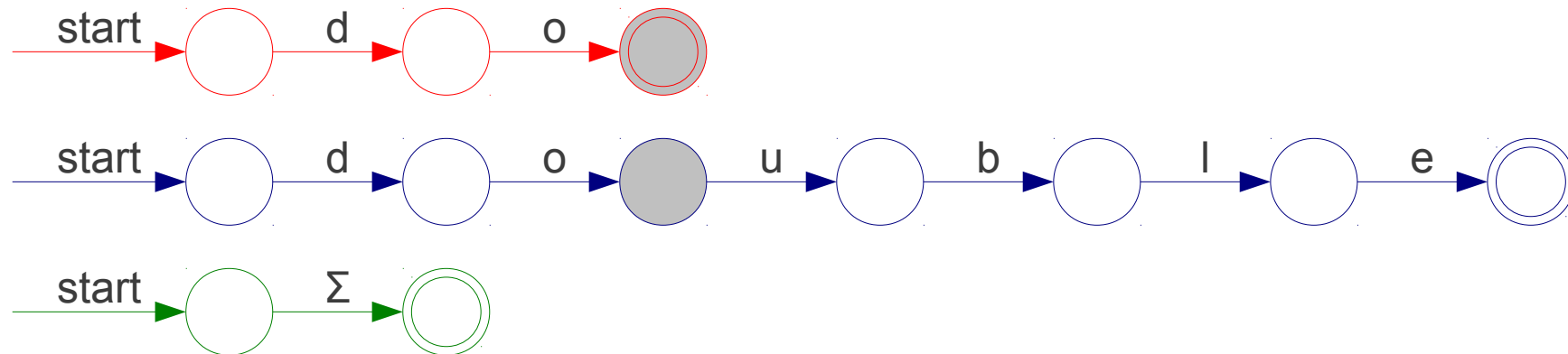
do

T\_Double

double

T\_Mystery

[A-Za-z]



# Implementing Maximal Munch

T\_Do

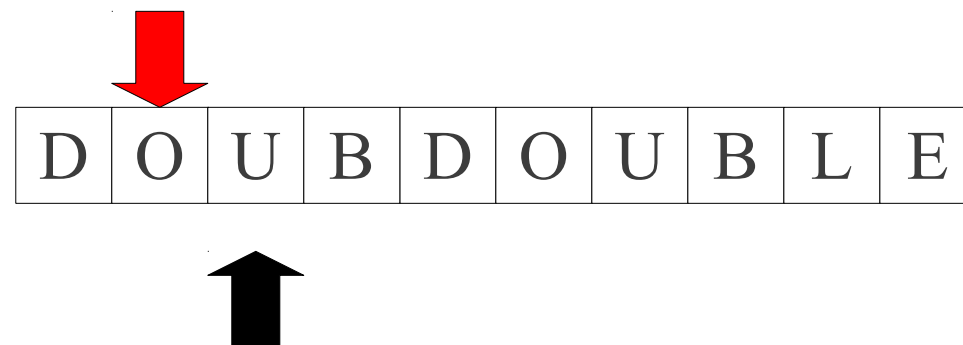
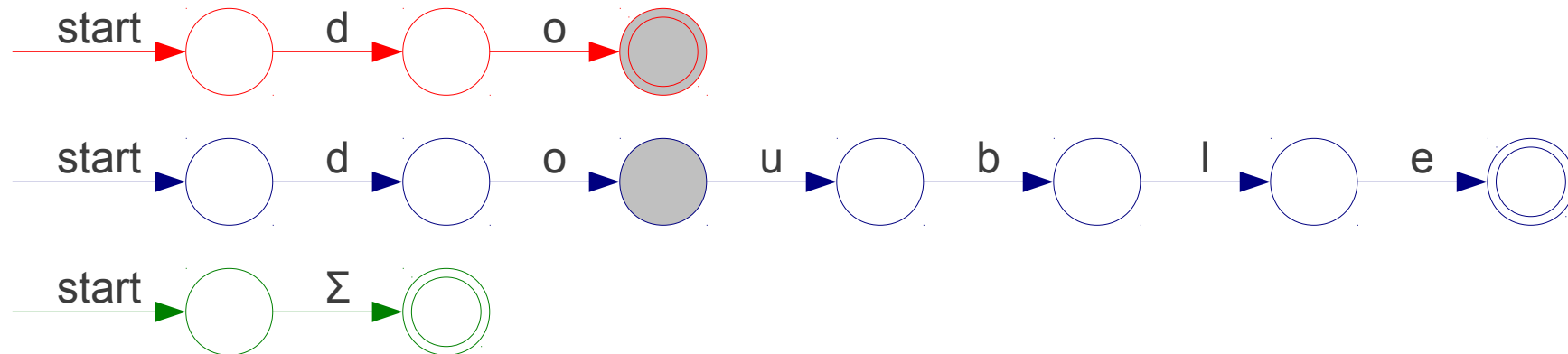
do

T\_Double

double

T\_Mystery

[A-Za-z]



# Implementing Maximal Munch

T\_Do

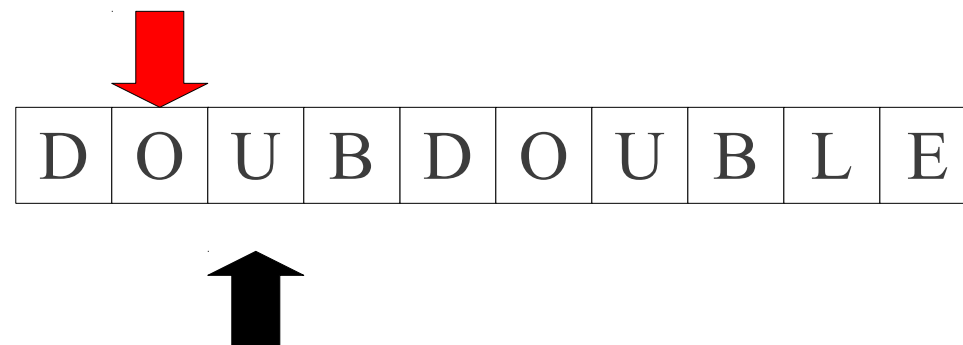
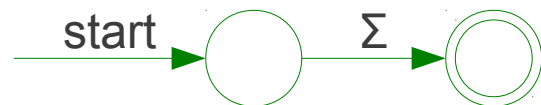
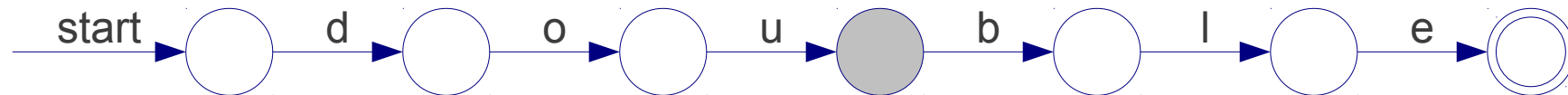
do

T\_Double

double

T\_Mystery

[A-Za-z]



# Implementing Maximal Munch

T\_Do

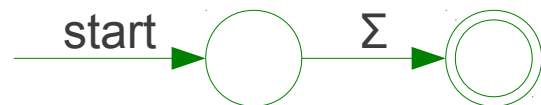
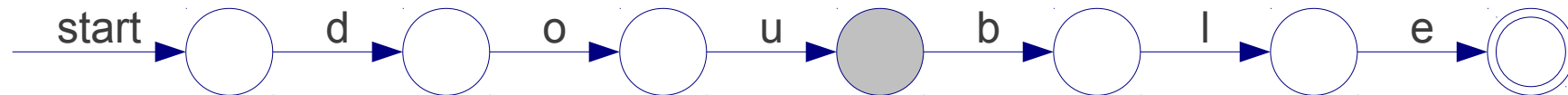
do

T\_Double

double

T\_Mystery

[A-Za-z]



# Implementing Maximal Munch

T\_Do

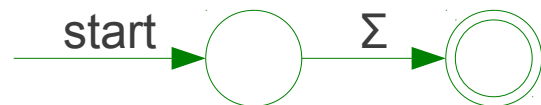
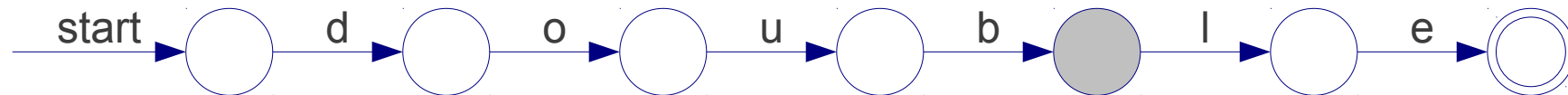
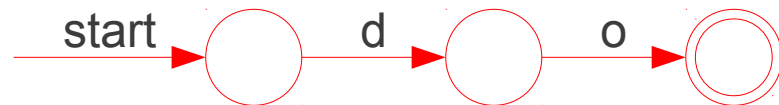
do

T\_Double

double

T\_Mystery

[A-Za-z]



# Implementing Maximal Munch

T\_Do

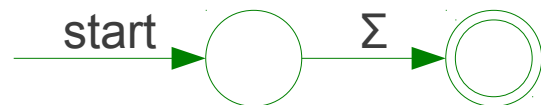
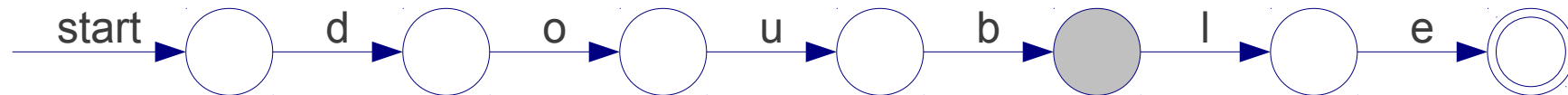
do

T\_Double

double

T\_Mystery

[A-Za-z]





# Implementing Maximal Munch

T\_Do

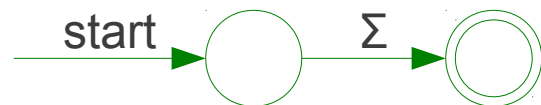
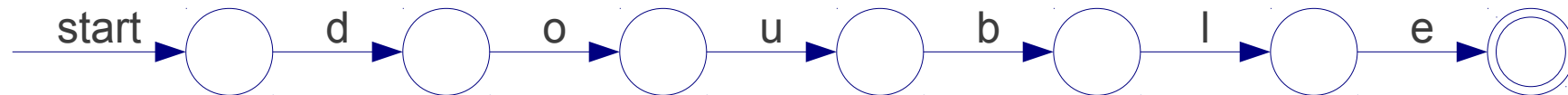
do

T\_Double

double

T\_Mystery

[A-Za-z]



# Implementing Maximal Munch

T\_Do

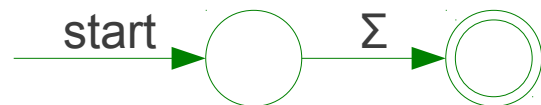
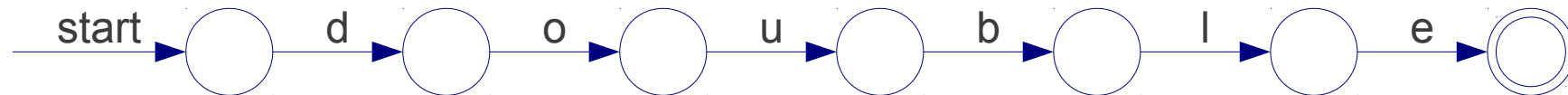
do

T\_Double

double

T\_Mystery

[A-Za-z]



D O

U B D O U B L E



# Implementing Maximal Munch

T\_Do

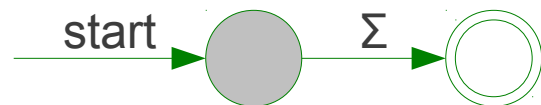
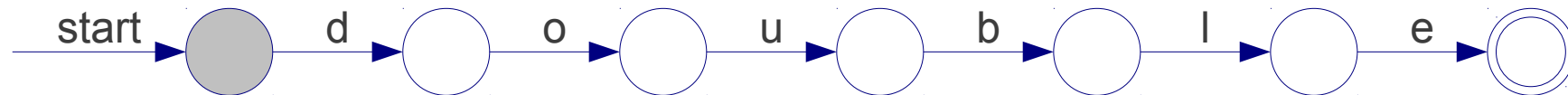
do

T\_Double

double

T\_Mystery

[A-Za-z]



D O

U B D O U B L E



# Implementing Maximal Munch

T\_Do

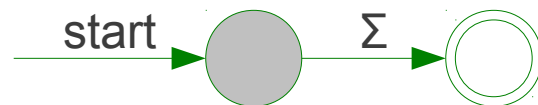
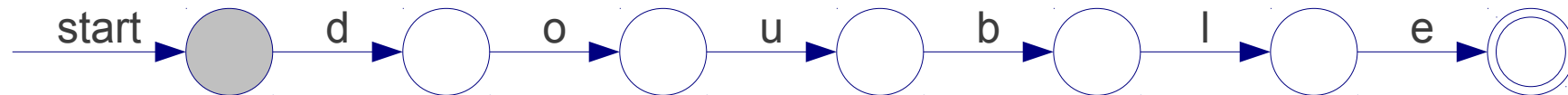
do

T\_Double

double

T\_Mystery

[A-Za-z]



**D O**

U	B	D	O	U	B	L	E
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# Implementing Maximal Munch

T\_Do

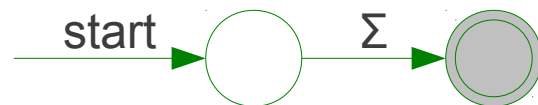
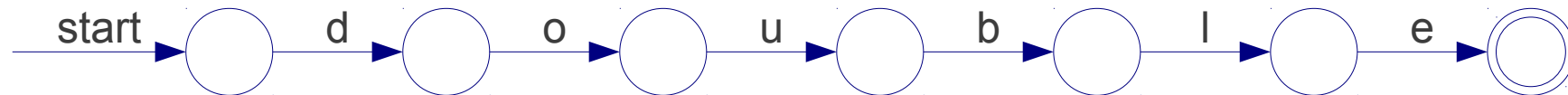
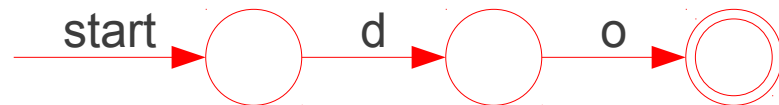
do

T\_Double

double

T\_Mystery

[A-Za-z]



D O

U B D O U B L E



# Implementing Maximal Munch

T\_Do

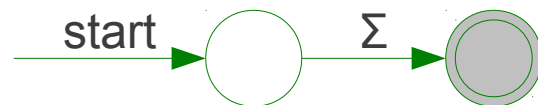
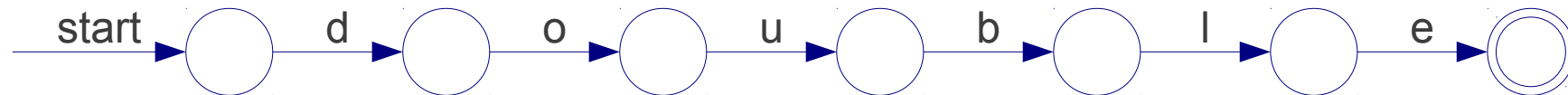
do

T\_Double

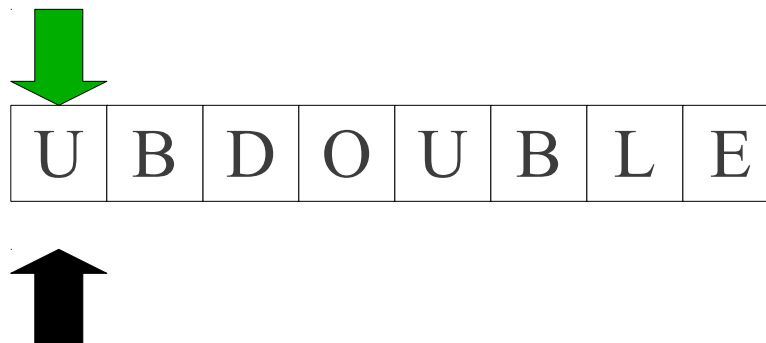
double

T\_Mystery

[A-Za-z]



D O



# Implementing Maximal Munch

T\_Do

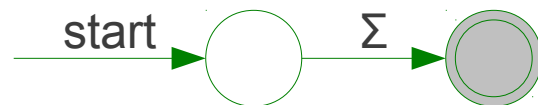
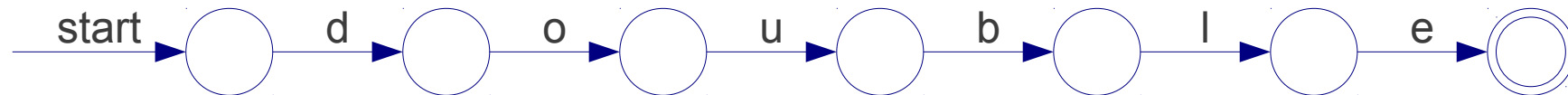
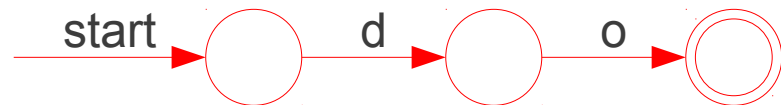
do

T\_Double

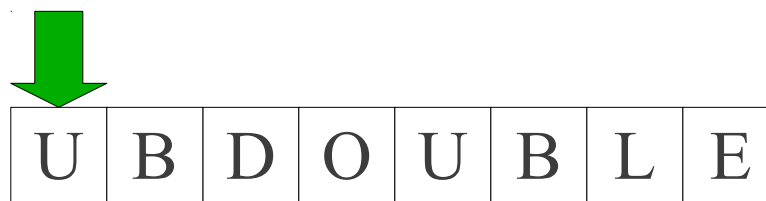
double

T\_Mystery

[A-Za-z]



D O



# Implementing Maximal Munch

T\_Do

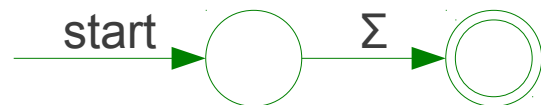
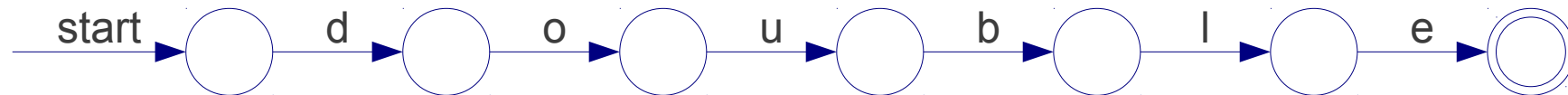
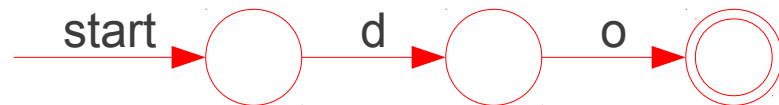
do

T\_Double

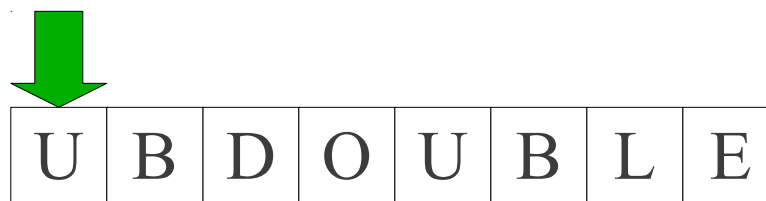
double

T\_Mystery

[A-Za-z]



D O





# Implementing Maximal Munch

T\_Do

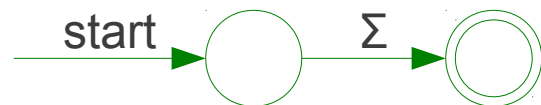
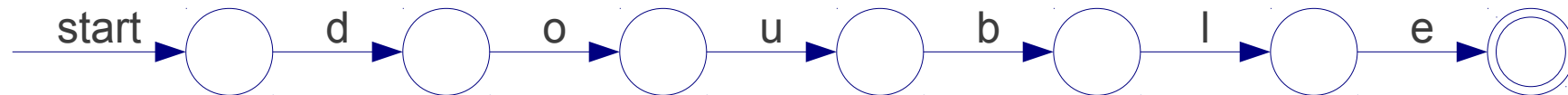
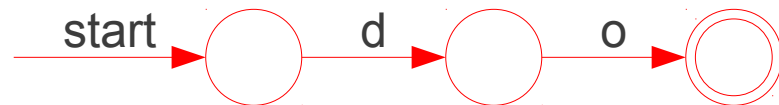
do

T\_Double

double

T\_Mystery

[A-Za-z]



D O

U

B D O U B L E



# Implementing Maximal Munch

T\_Do

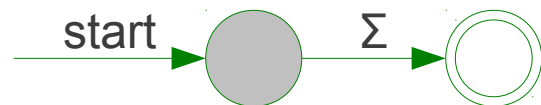
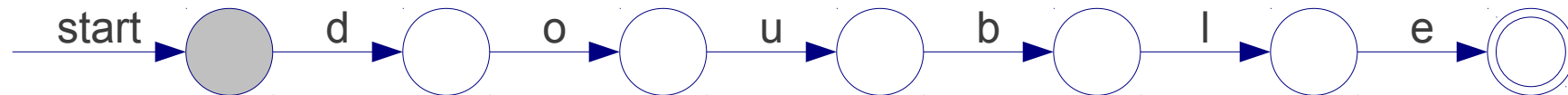
do

T\_Double

double

T\_Mystery

[A-Za-z]



D O

U

B D O U B L E



# Implementing Maximal Munch

T\_Do

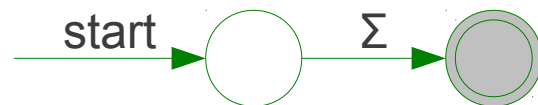
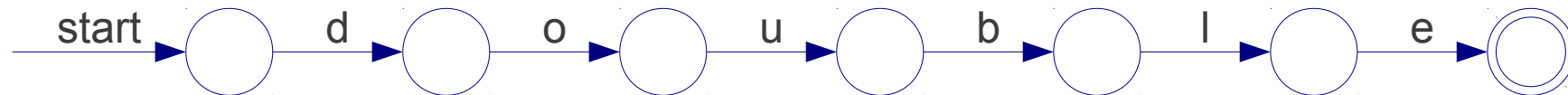
do

T\_Double

double

T\_Mystery

[A-Za-z]



D O

U

B D O U B L E



# Implementing Maximal Munch

T\_Do

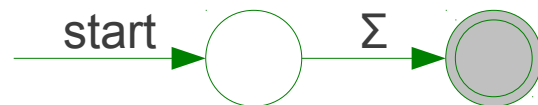
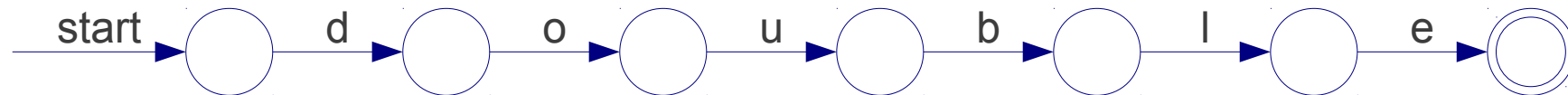
do

T\_Double

double

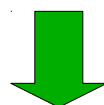
T\_Mystery

[A-Za-z]



D O

U



B D O U B L E



# Implementing Maximal Munch

T\_Do

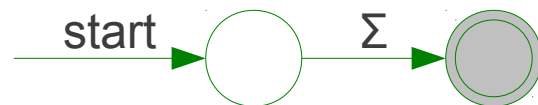
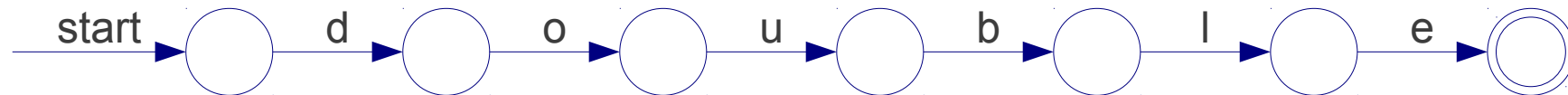
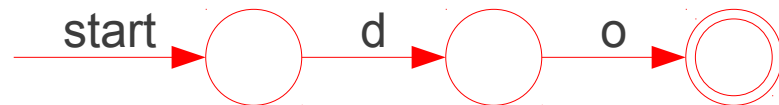
do

T\_Double

double

T\_Mystery

[A-Za-z]



D O

U



# Implementing Maximal Munch

T\_Do

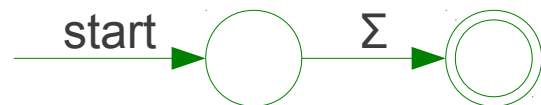
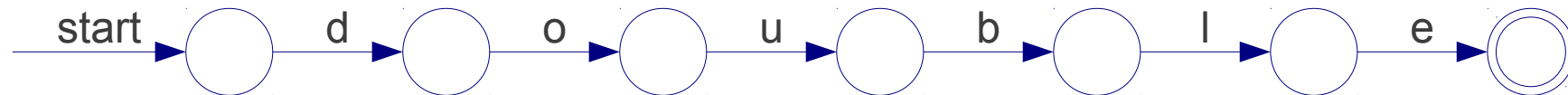
do

T\_Double

double

T\_Mystery

[A-Za-z]



**D O**

**U**



# Implementing Maximal Munch

T\_Do

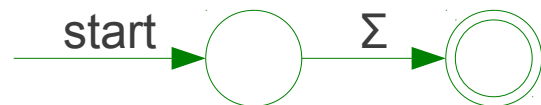
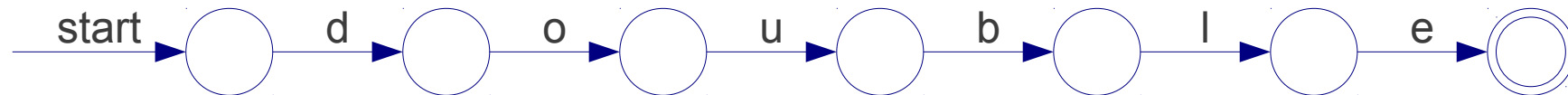
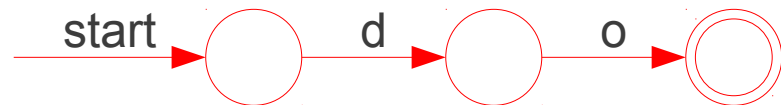
do

T\_Double

double

T\_Mystery

[A-Za-z]



D O

U

B

D O U B L E



# Implementing Maximal Munch

T\_Do

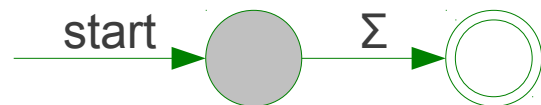
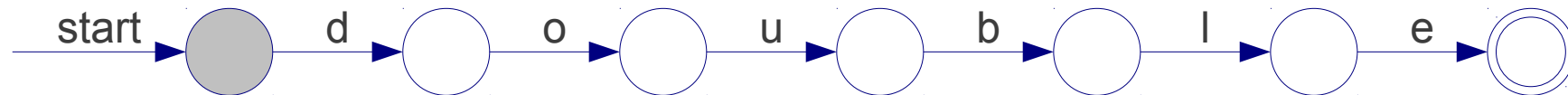
do

T\_Double

double

T\_Mystery

[A-Za-z]



D O

U

B

D O U B L E





# Implementing Maximal Munch

T\_Do

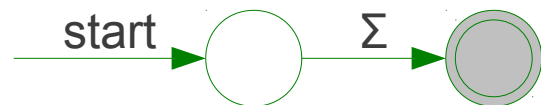
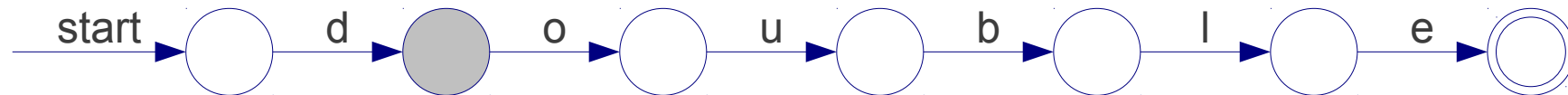
do

T\_Double

double

T\_Mystery

[A-Za-z]



D O

U

B

D O U B L E



# Implementing Maximal Munch

T\_Do

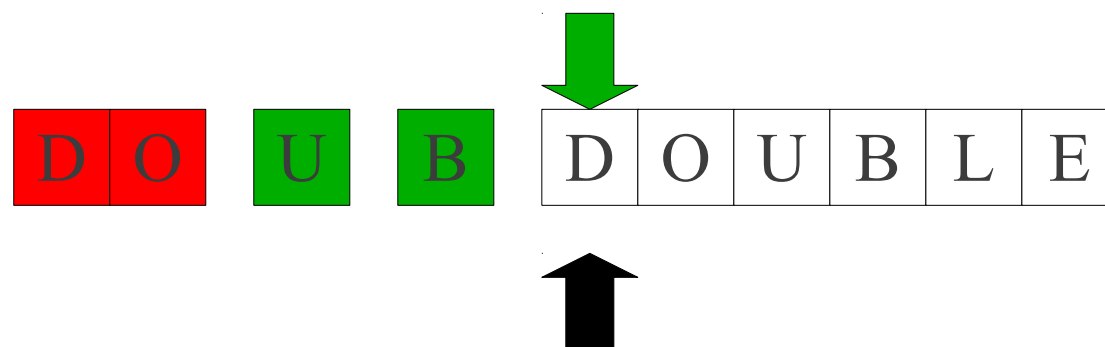
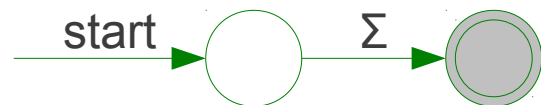
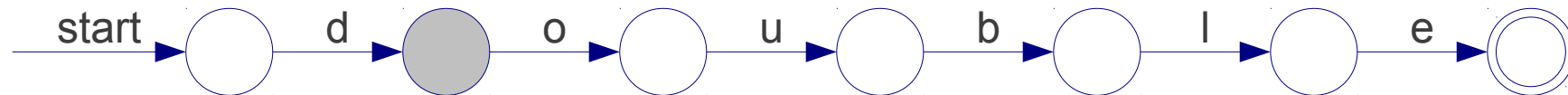
do

T\_Double

double

T\_Mystery

[A-Za-z]



# Implementing Maximal Munch

T\_Do

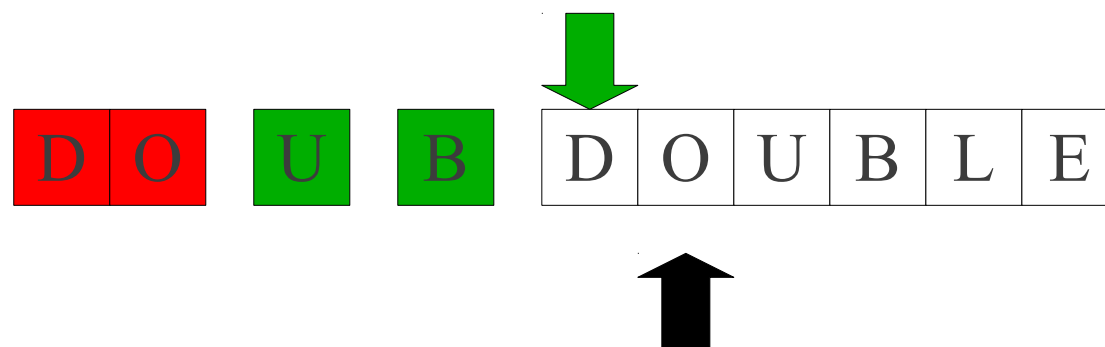
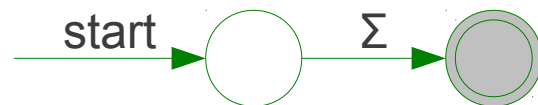
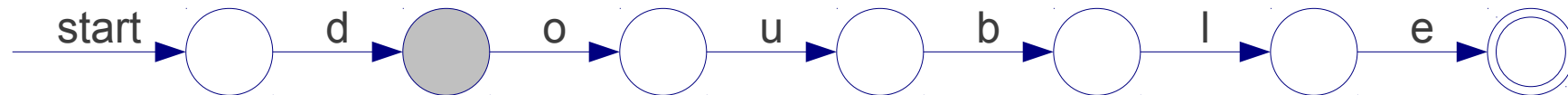
do

T\_Double

double

T\_Mystery

[A-Za-z]



# Implementing Maximal Munch

T\_Do

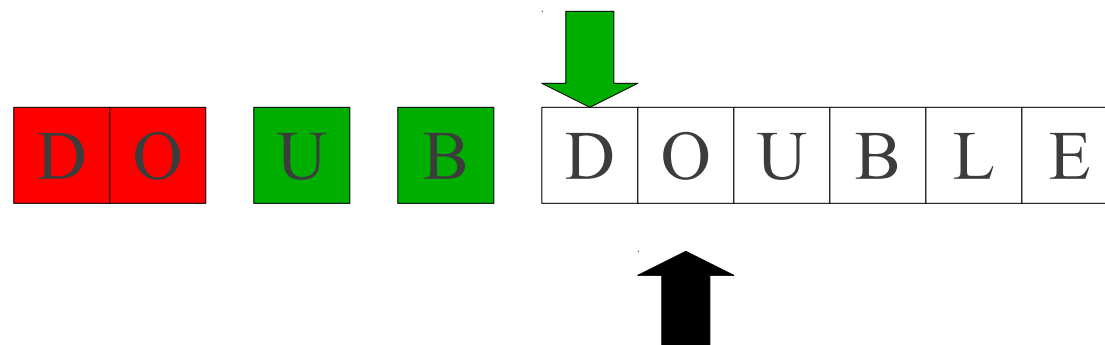
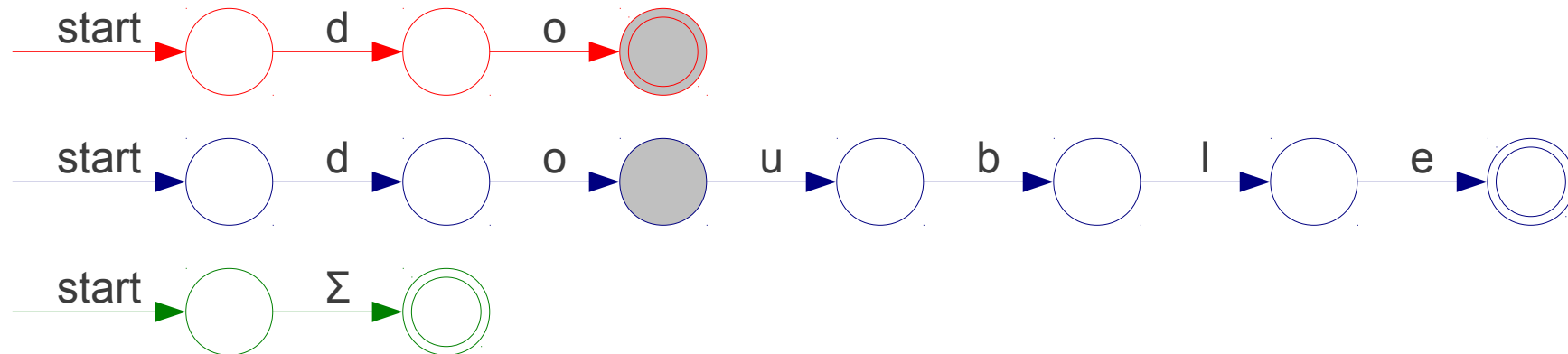
do

T\_Double

double

T\_Mystery

[A-Za-z]



# Implementing Maximal Munch

T\_Do

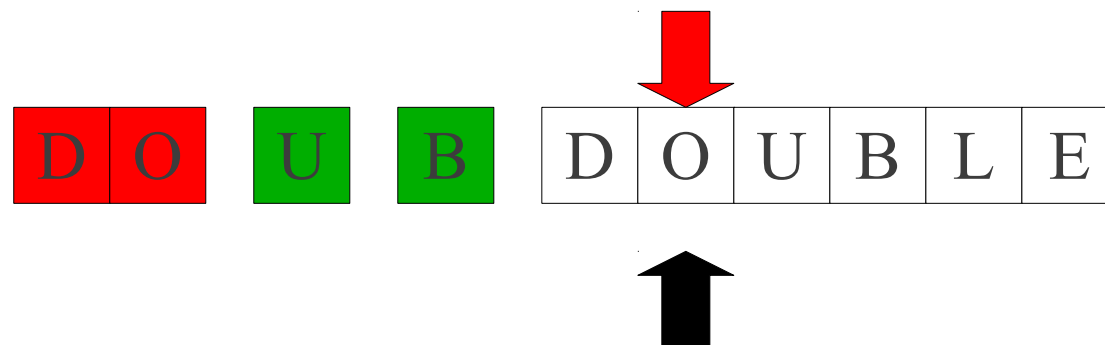
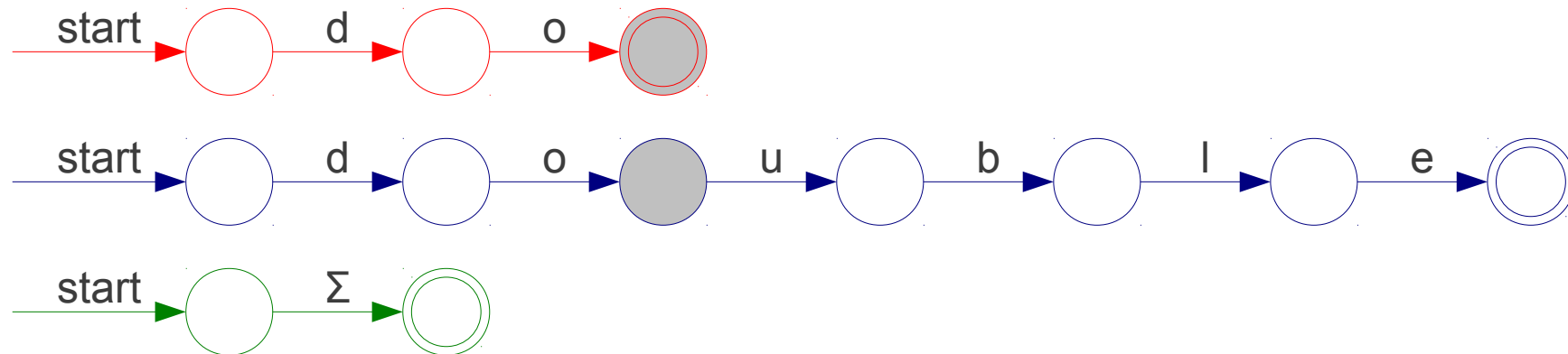
do

T\_Double

double

T\_Mystery

[A-Za-z]



# Implementing Maximal Munch

T\_Do

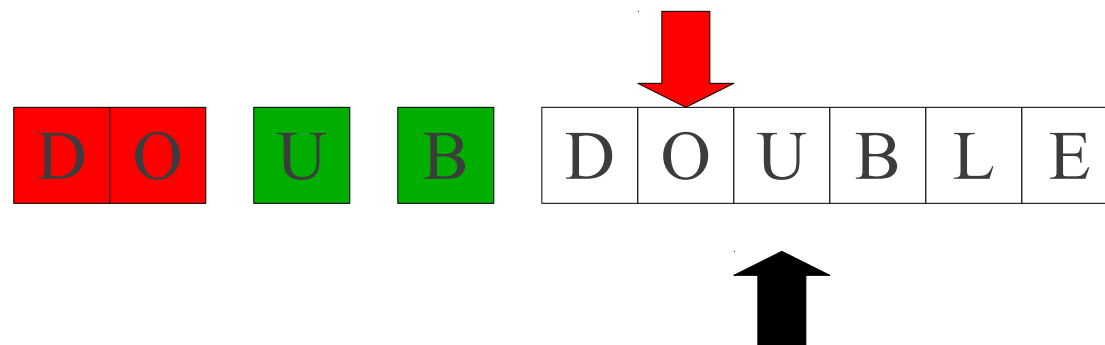
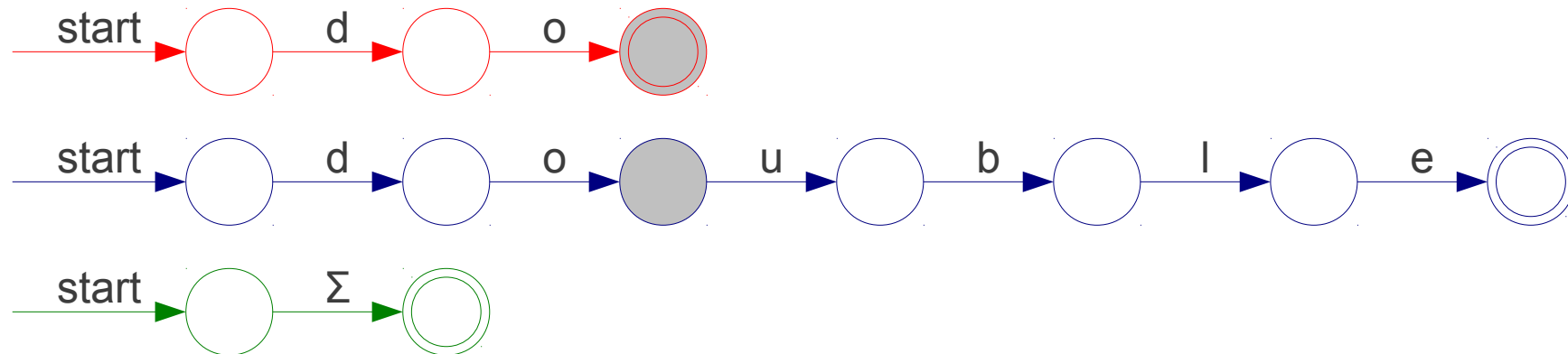
do

T\_Double

double

T\_Mystery

[A-Za-z]



# Implementing Maximal Munch

T\_Do

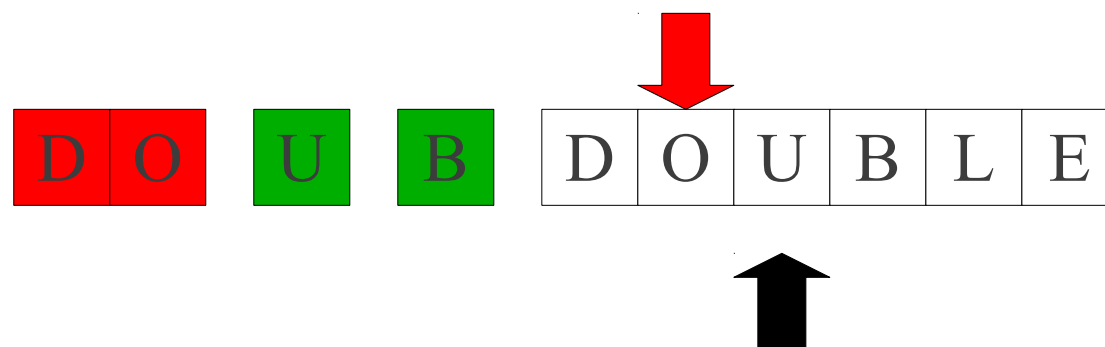
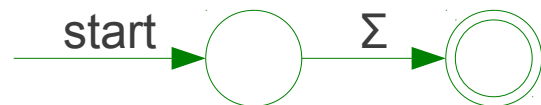
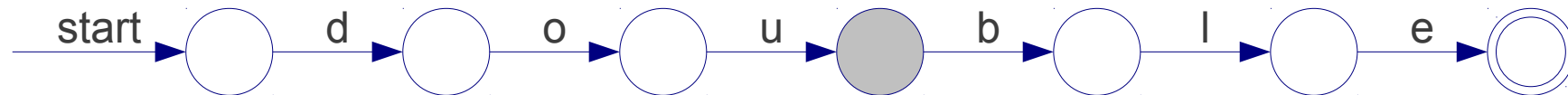
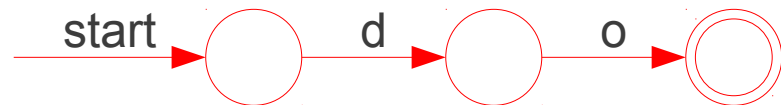
do

T\_Double

double

T\_Mystery

[A-Za-z]



# Implementing Maximal Munch

T\_Do

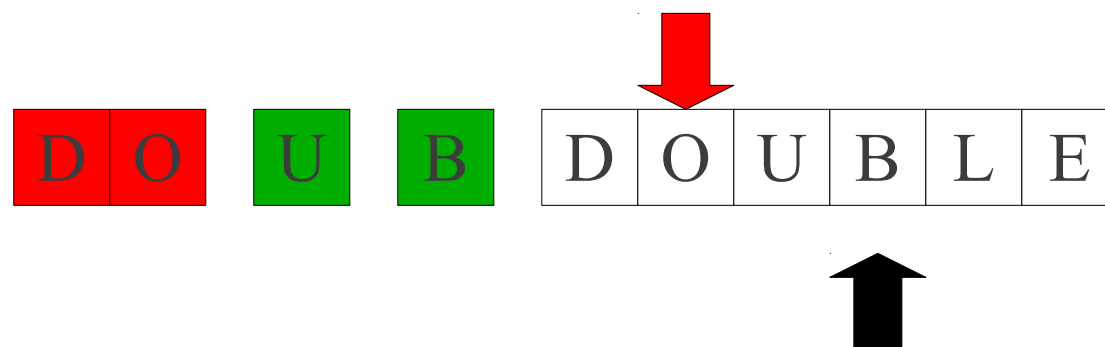
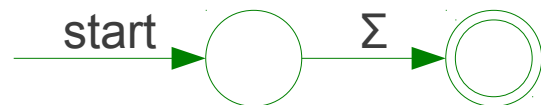
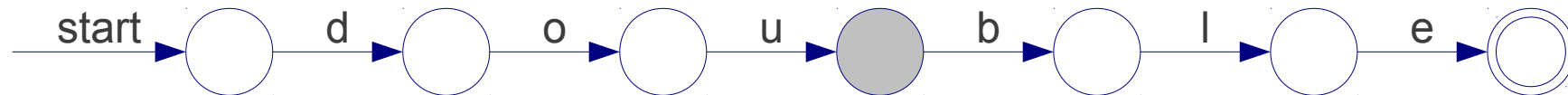
do

T\_Double

double

T\_Mystery

[A-Za-z]





# Implementing Maximal Munch

T\_Do

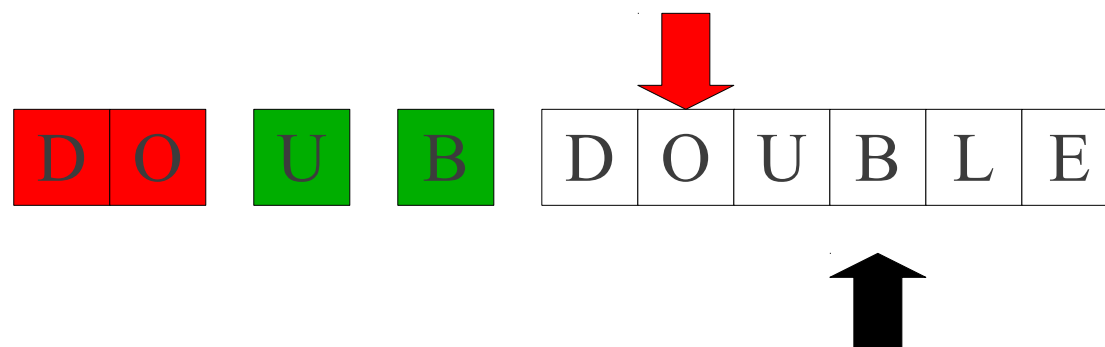
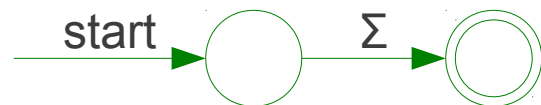
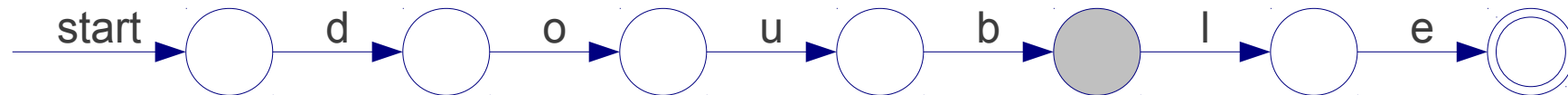
do

T\_Double

double

T\_Mystery

[A-Za-z]



# Implementing Maximal Munch

T\_Do

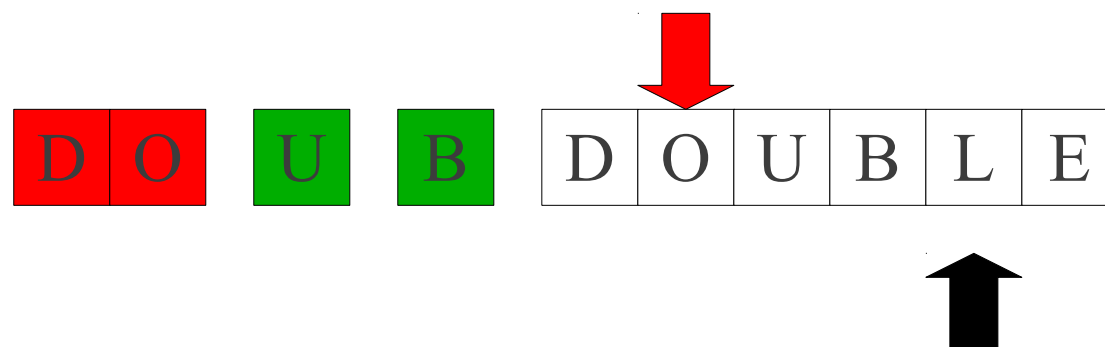
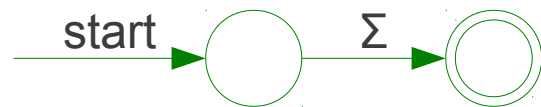
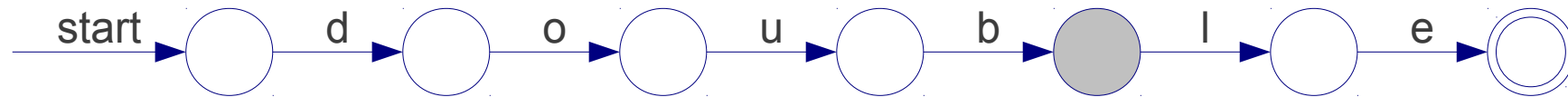
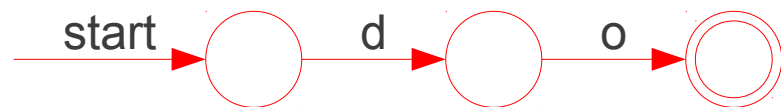
do

T\_Double

double

T\_Mystery

[A-Za-z]



# Implementing Maximal Munch

T\_Do

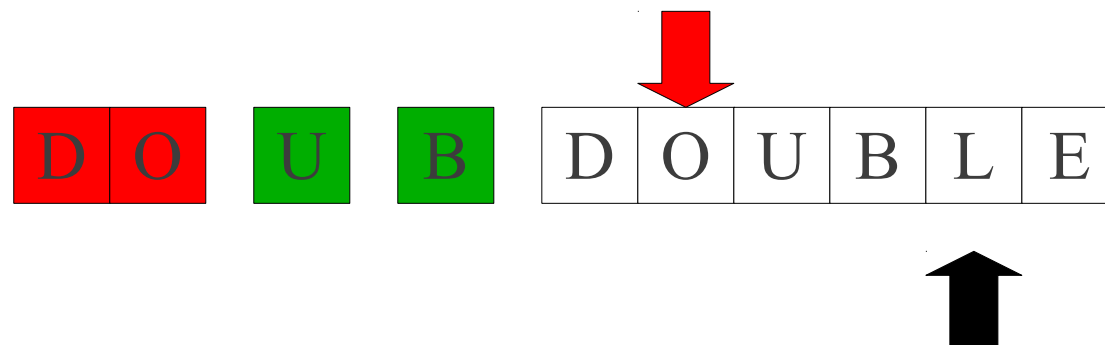
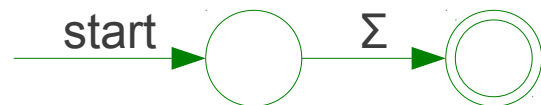
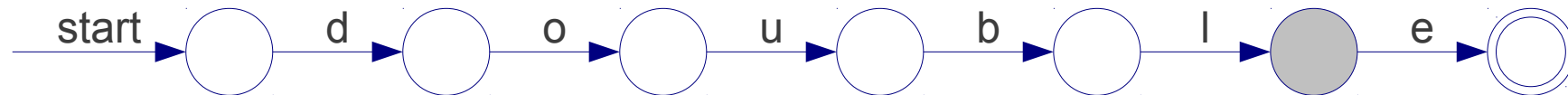
do

T\_Double

double

T\_Mystery

[A-Za-z]



# Implementing Maximal Munch

T\_Do

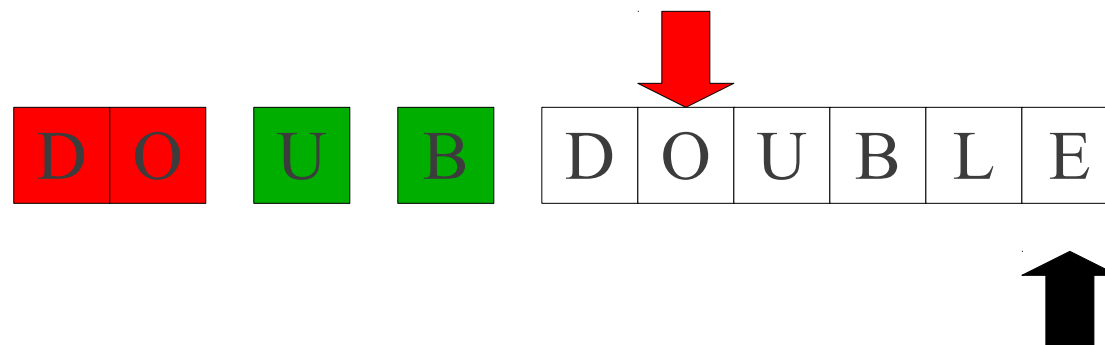
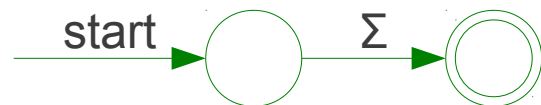
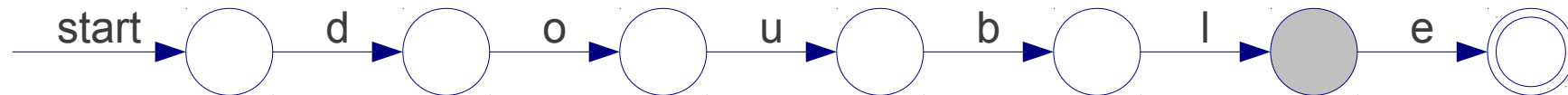
do

T\_Double

double

T\_Mystery

[A-Za-z]



# Implementing Maximal Munch

T\_Do

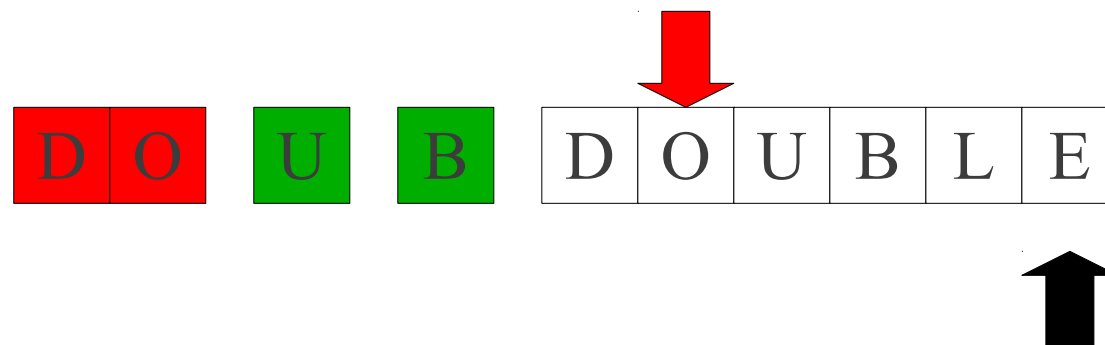
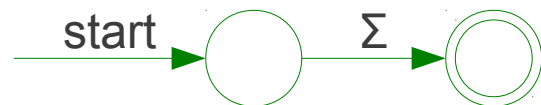
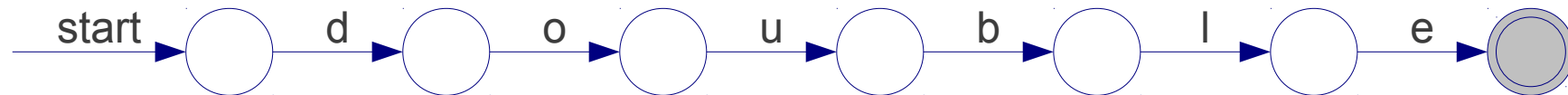
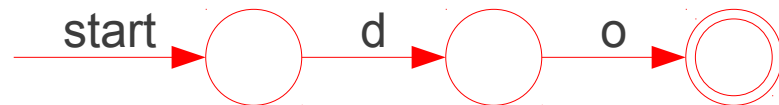
do

T\_Double

double

T\_Mystery

[A-Za-z]



# Implementing Maximal Munch

T\_Do

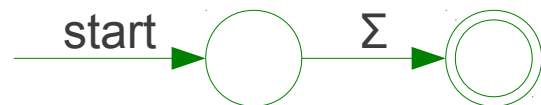
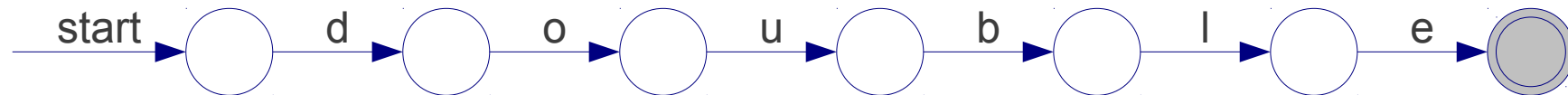
do

T\_Double

double

T\_Mystery

[A-Za-z]

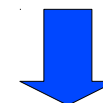


D O

U

B

D O U B L E



# Implementing Maximal Munch

T\_Do

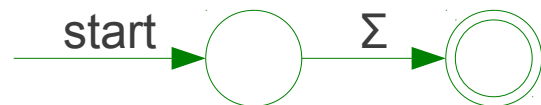
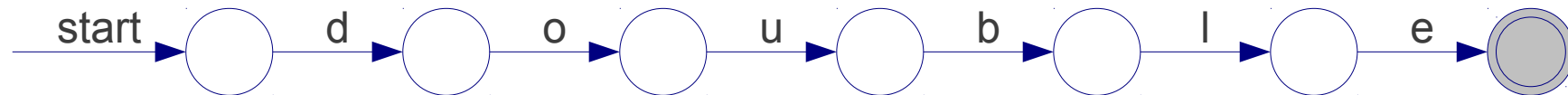
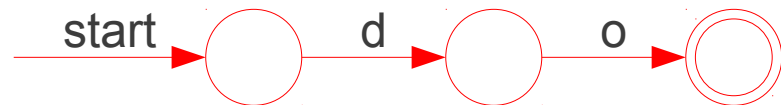
do

T\_Double

double

T\_Mystery

[A-Za-z]



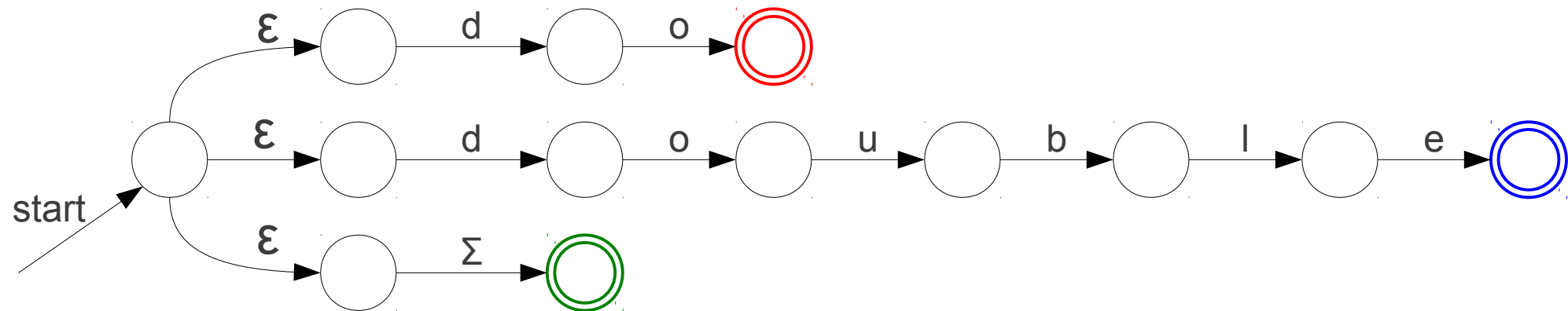
D O

U

B

D O U B L E

# A Minor Simplification





# Other Conflicts

T\_Do do  
T\_Double double  
T\_Identifier [A-Za-z\_] [A-Za-z0-9\_]\*

d	o	u	b	l	e
---	---	---	---	---	---

# More Tiebreaking

- When two regular expressions apply, choose the one with the greater “priority.”
- Simple priority system: **pick the rule that was defined first.**

# Other Conflicts

T\_Do

do

T\_Double

double

T\_Identifier [A-Za-z\_] [A-Za-z0-9\_]\*

d	o	u	b	l	e
---	---	---	---	---	---

d	o	u	b	l	e
d	o	u	b	l	e

# Other Conflicts

T\_Do

do

T\_Double

double

T\_Identifier [A-Za-z\_] [A-Za-z0-9\_]\*

d	o	u	b	l	e
---	---	---	---	---	---

d	o	u	b	l	e
---	---	---	---	---	---

# Implement a lexical analyzer

- Step 1: Use regular expressions to describe token types (keyword, identifier, integer constant..)

Number = digit + ...

Keyword = 'if' + 'else' + ...

Identifier = letter (letter + digit)\*

OpenPar = '('

...

Then construct Regular language R, matching all lexemes for all tokens

$R = \text{Keyword} + \text{Identifier} + \text{Number} + \dots$

$= R_1 + R_2 + \dots$

- Step 2: Use DFA/NFA to recognize the regular language
- But...good news. you don't need to implement the algorithms to transform your regular expressions to DFA/NFA to recognize it
- **flex**: given regular expressions -> output c code that does lexical analysis (it internally generates DFA)

# Lexical analyzer

REs + priorities + longest matching token rule  
= definition of a lexical analyzer

# DFA vs. NFA

- NFAs and DFAs recognize the same set of languages (regular languages)
  - For a given NFA, there exists a DFA, and vice versa
- DFAs are faster to execute
  - There are no choices to consider
  - Tradeoff: simplicity
    - For a given language DFA can be exponentially larger than NFA.

# Automating Lexical Analyzer (scanner) Construction

To convert a specification into code:

- 1 Write down the RE for the input language
- 2 Build a big NFA
- 3 Build the DFA that simulates the NFA
- 4 Systematically shrink the DFA
- 5 Turn it into code

Scanner generators

- Lex and Flex work along these lines
- Algorithms are well-known and well-understood



# Automating Lexical Analyzer (scanner) Construction

**RE  $\rightarrow$  NFA** (*Thompson's construction*)

- Build an NFA for each term
- Combine them with  $\epsilon$ -moves

**NFA  $\rightarrow$  DFA** (*subset construction*)

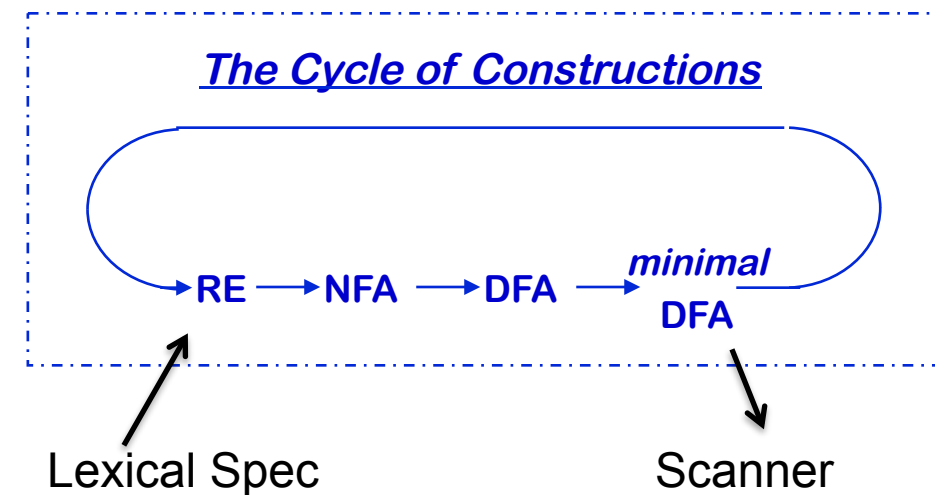
- Build the simulation

**DFA  $\rightarrow$  Minimal DFA**

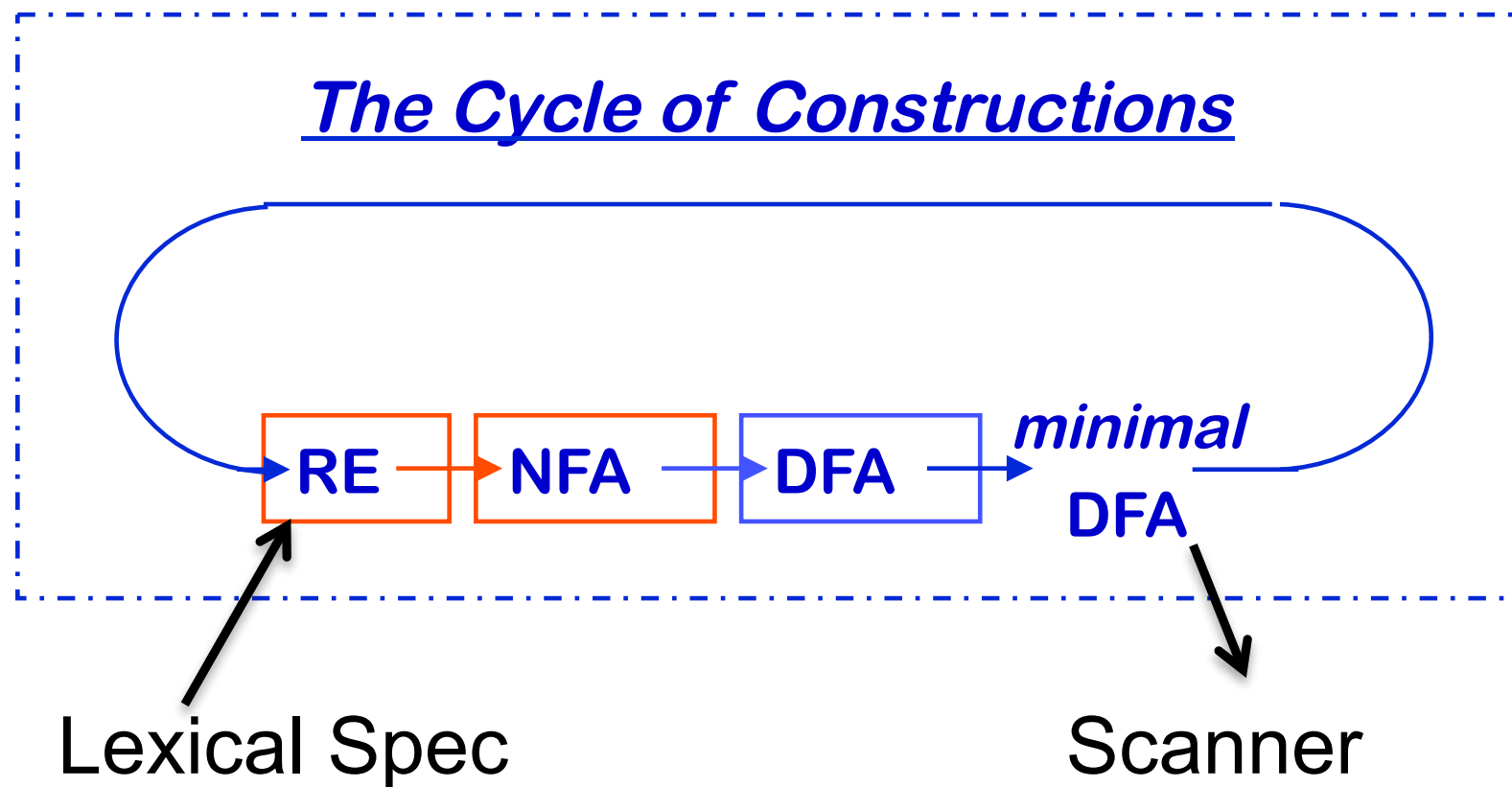
- Hopcroft's algorithm

**DFA  $\rightarrow$  RE** (*Not part of the scanner construction*)

- All pairs, all paths problem
- Take the union of all paths from  $s_0$  to an accepting state



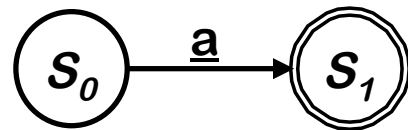
## *The Cycle of Constructions*



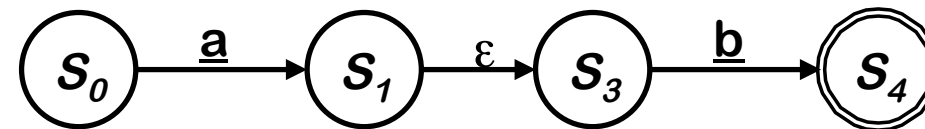
# RE $\rightarrow$ NFA using Thompson's Construction

Key idea

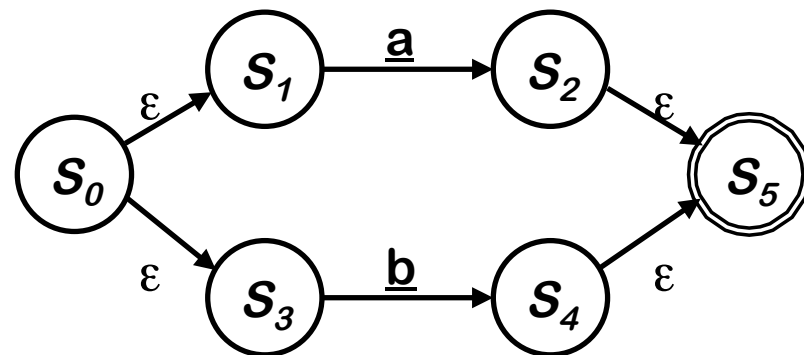
- NFA pattern for each symbol & each operator
- Join them with  $\epsilon$  moves in precedence order



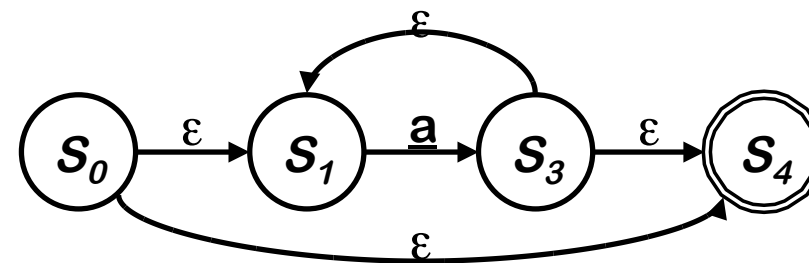
NFA for a



NFA for ab



NFA for a | b



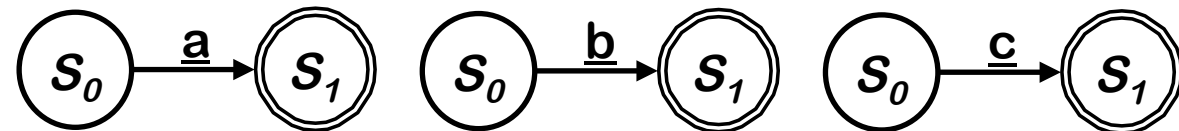
NFA for a\*

Ken Thompson, CACM, 1968

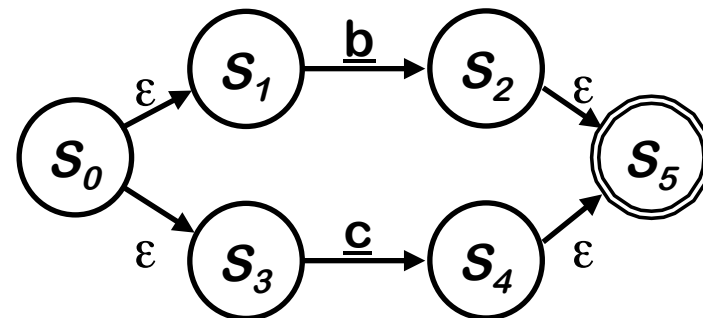
# Example of Thompson's Construction

Let's try  $\underline{a} (\underline{b} | \underline{c})^*$

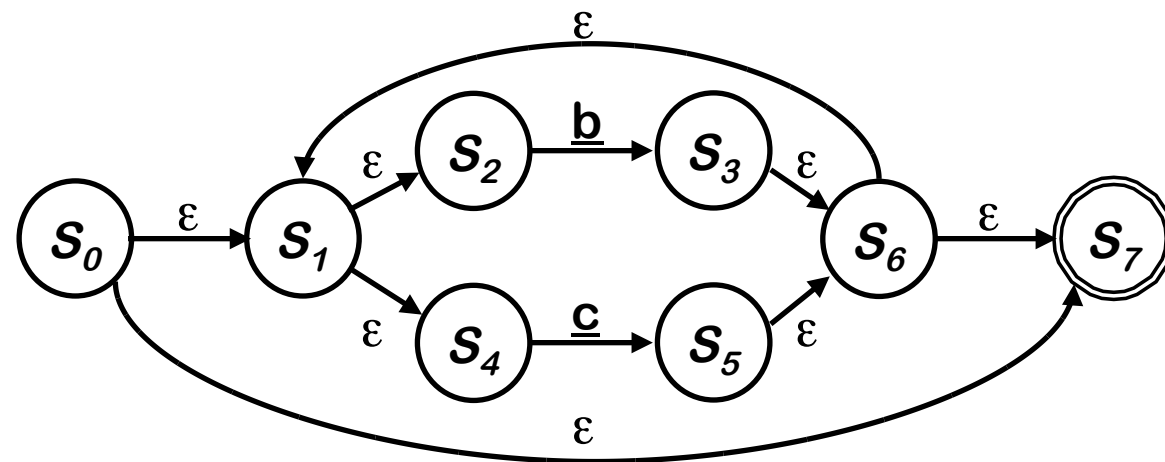
1.  $\underline{a}$ ,  $\underline{b}$ , &  $\underline{c}$



2.  $\underline{b} | \underline{c}$

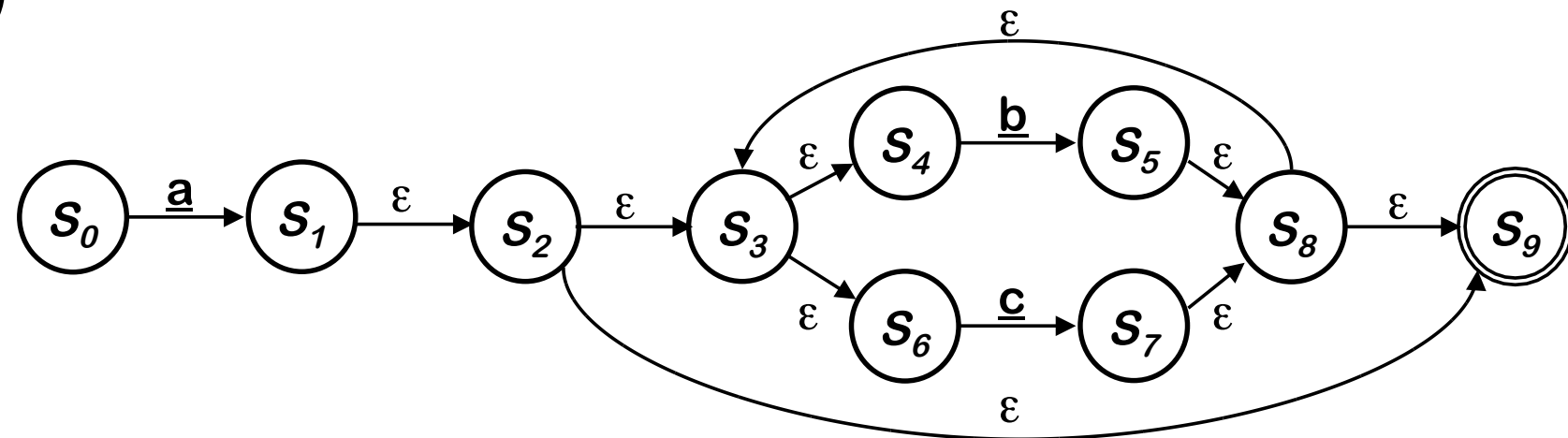


3.  $(\underline{b} | \underline{c})^*$

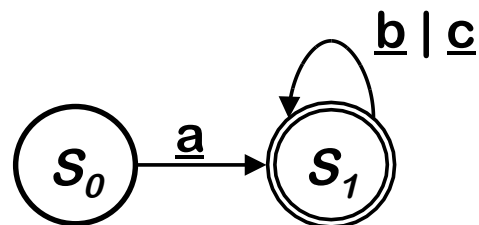


# Example of Thompson's Construction (con't)

4.  $\underline{a} (\underline{b} | \underline{c})^*$

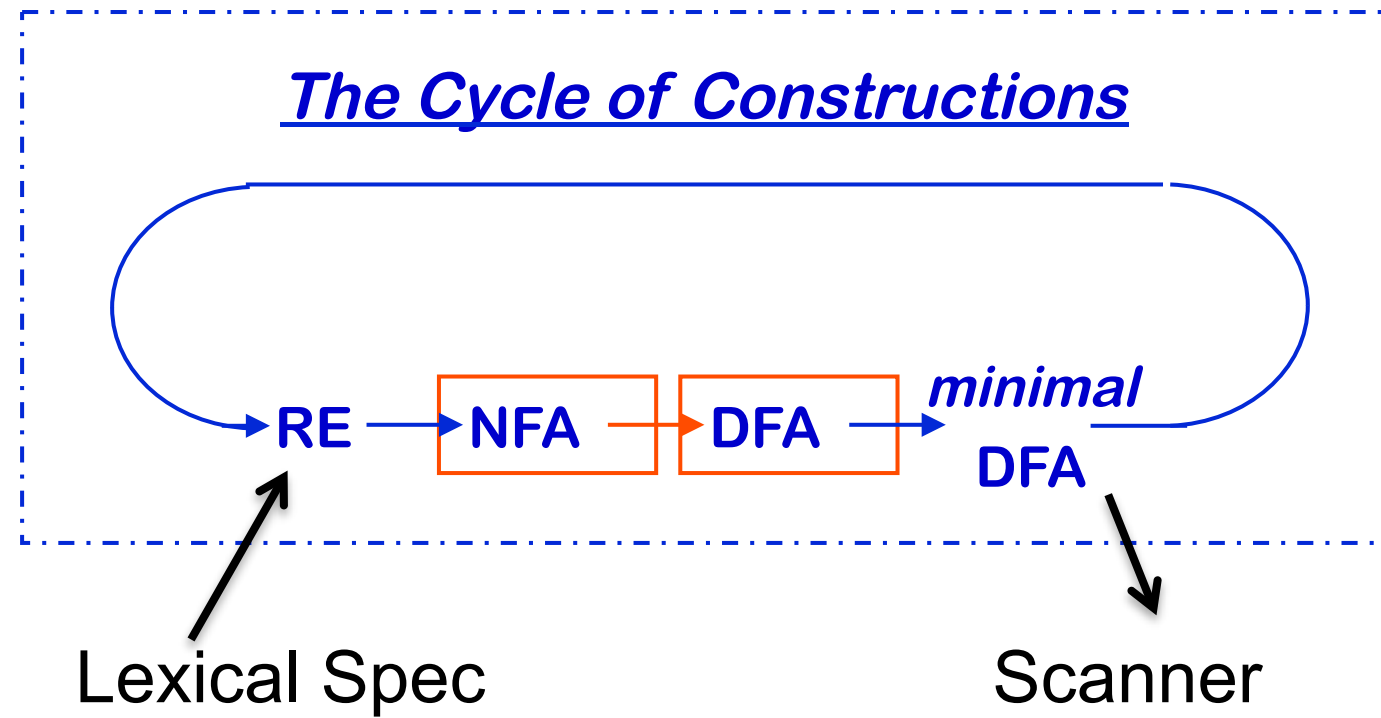


Of course, a human would design something simpler ...



But, we can automate production of the more complex one ...

### *The Cycle of Constructions*



# NFA to DFA : Trick

---

- Simulate the NFA
- Each state of DFA
  - = a non-empty subset of states of the NFA
- Start state
  - = the set of NFA states reachable through  $\epsilon$ -moves from NFA start state
- Add a transition  $S \xrightarrow{a} S'$  to DFA iff
  - $S'$  is the set of NFA states reachable from any state in  $S$  after seeing the input  $a$ , considering  $\epsilon$ -moves as well

## NFA to DFA : cont..

---

- An NFA may be in many states at any time
- How many different states ?
- If there are N states, the NFA must be in some subset of those N states
- How many subsets are there?

$$2^N - 1 = \text{finitely many}$$

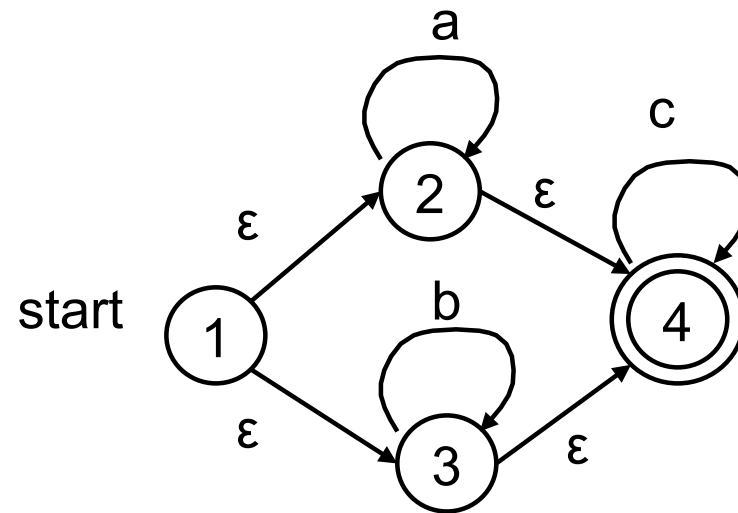


# NFA to DFA

---

- Remove the non-determinism
  - States with multiple outgoing edges due to same input
  - $\epsilon$  transitions

$(a^* | b^*) c^*$

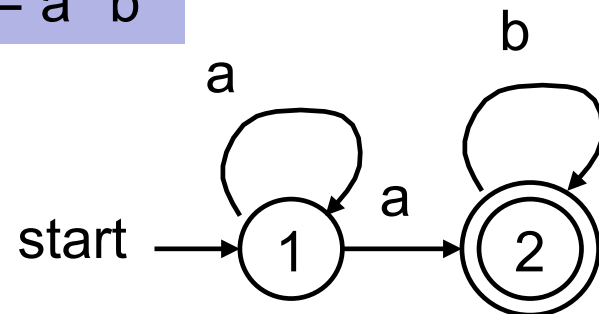


# NFA to DFA (2)

---

- Multiple transitions
  - Solve by subset construction
  - Build new DFA based upon the set of states each representing a unique subset of states in NFA

$R = a^+ b^*$



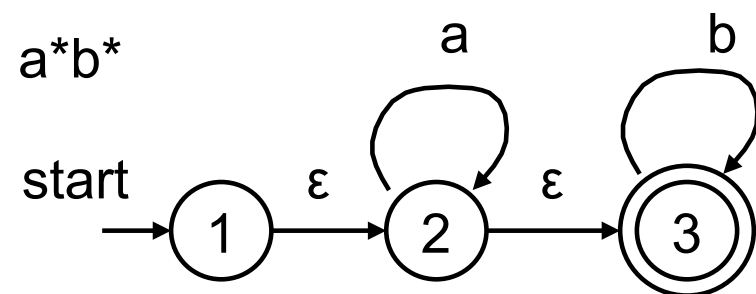
$\epsilon$ -closure(1) = {1} include state "1"

(1,a)  $\rightarrow$  {1,2} include state "1/2"

(1,b)  $\rightarrow$  ERROR

# NFA to DFA (3)

- $\epsilon$  transitions
  - Any state reachable by an  $\epsilon$  transition is “part of the state”
  - $\epsilon$ -closure - Any state reachable from S by  $\epsilon$  transitions is in the  $\epsilon$ -closure; treat  $\epsilon$ -closure as 1 big state, always include  $\epsilon$ -closure as part of the state

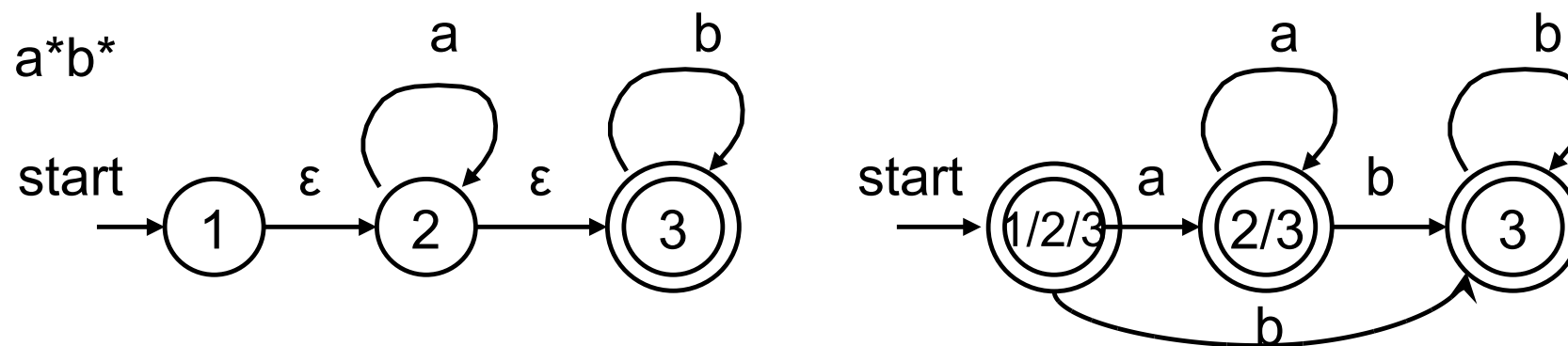


1.  $\epsilon$ -closure(1) = {1,2,3}; include 1/2/3
2. Move(1/2/3, a) = {2, 3} +  $\epsilon$ -closure(2,3) = {2,3} ; include 2/3
3. Move(1/2/3, b) = {3} +  $\epsilon$ -closure(3) = {3} ; include state 3
4. Move(2/3, a) = {2} +  $\epsilon$ -closure(2) = {2,3}
5. Move(2/3, b) = {3} +  $\epsilon$ -closure(3) = {3}
6. Move(3, b) = {3} +  $\epsilon$ -closure(3) = {3}

# NFA to DFA (3)

- $\epsilon$  transitions

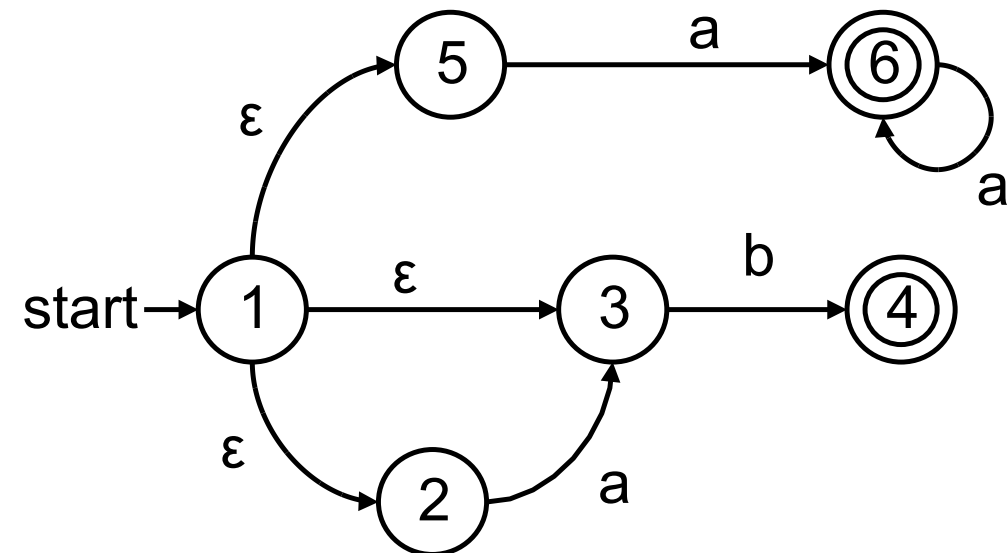
- Any state reachable by an  $\epsilon$  transition is “part of the state”
- $\epsilon$ -closure - Any state reachable from S by  $\epsilon$  transitions is in the  $\epsilon$ -closure; treat  $\epsilon$ -closure as 1 big state, always include  $\epsilon$ -closure as part of the state



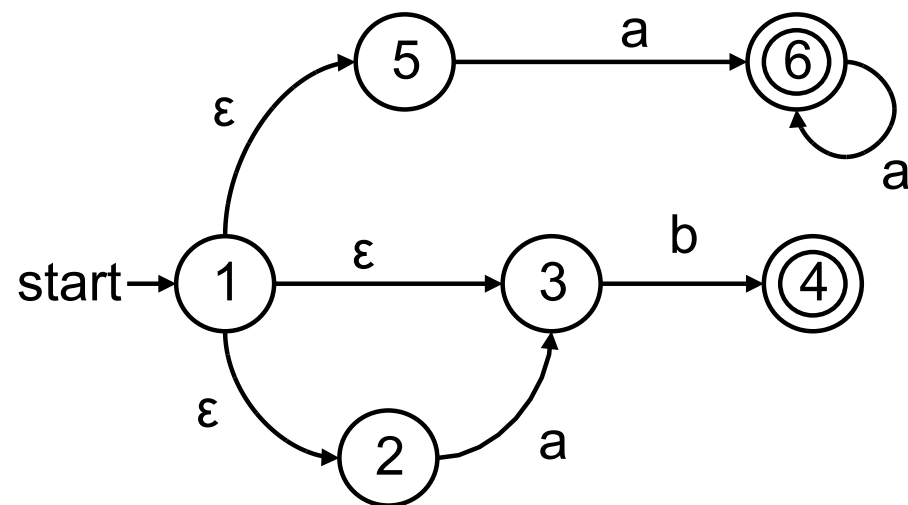
1.  $\epsilon$ -closure(1) = {1,2,3};
2. Move(1/2/3, a) = {2, 3} +  $\epsilon$ -closure(2,3) = {2,3} ; include 2/3
3. Move(1/2/3, b) = {3} +  $\epsilon$ -closure(3) = {3} ; include state 3
4. Move(2/3, a) = {2} +  $\epsilon$ -closure(2) = {2,3}
5. Move(2/3, b) = {3} +  $\epsilon$ -closure(3) = {3}
6. Move(3, b) = {3} +  $\epsilon$ -closure(3) = {3}

# NFA to DFA - Example

---



# NFA to DFA - Example



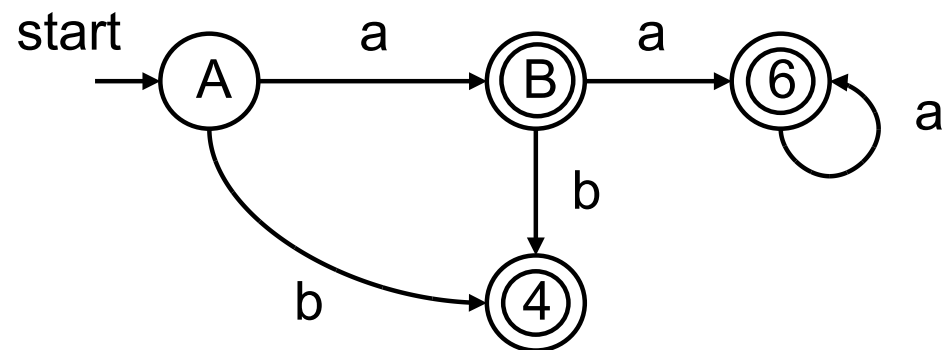
$\epsilon\text{-closure}(1) = \{1, 2, 3, 5\}$

Create a new state  $A = \{1, 2, 3, 5\}$

$\text{move}(A, a) = \{3, 6\} + \epsilon\text{-closure}(3, 6) = \{3, 6\}$

Create  $B = \{3, 6\}$

$\text{move}(A, b) = \{4\} + \epsilon\text{-closure}(4) = \{4\}$



$\text{move}(B, a) = \{6\} + \epsilon\text{-closure}(6) = \{6\}$

$\text{move}(B, b) = \{4\} + \epsilon\text{-closure}(4) = \{4\}$

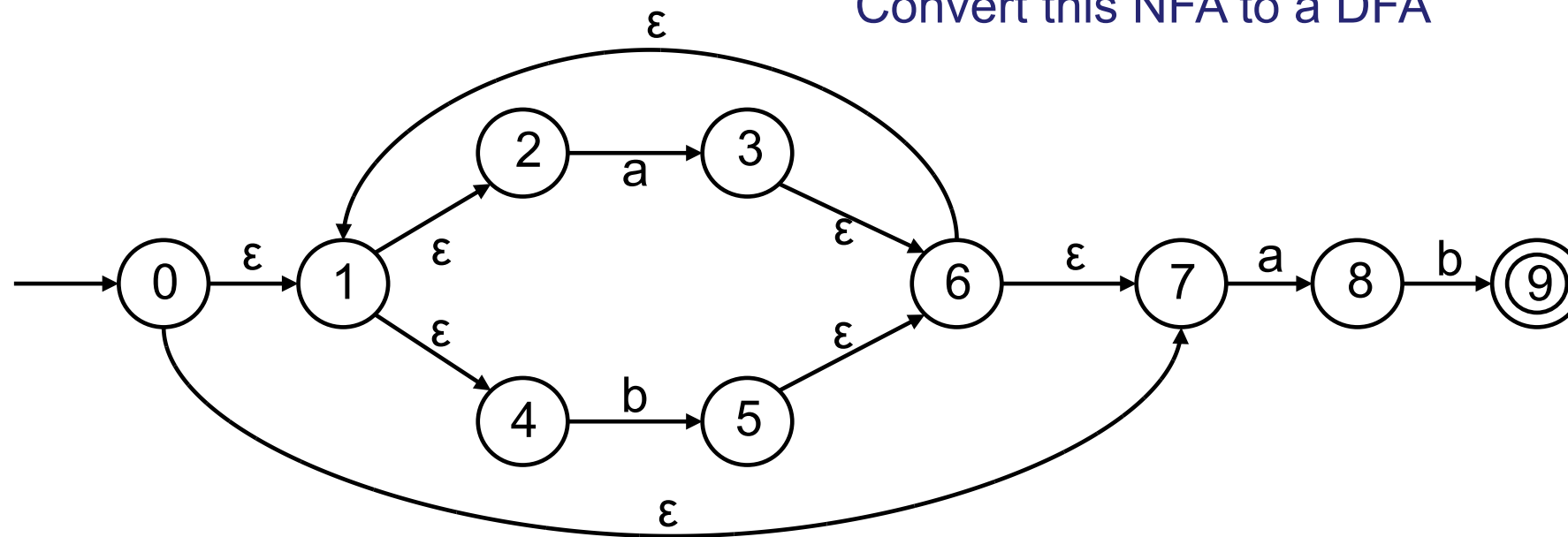
$\text{move}(6, a) = \{6\} + \epsilon\text{-closure}(6) = \{6\}$

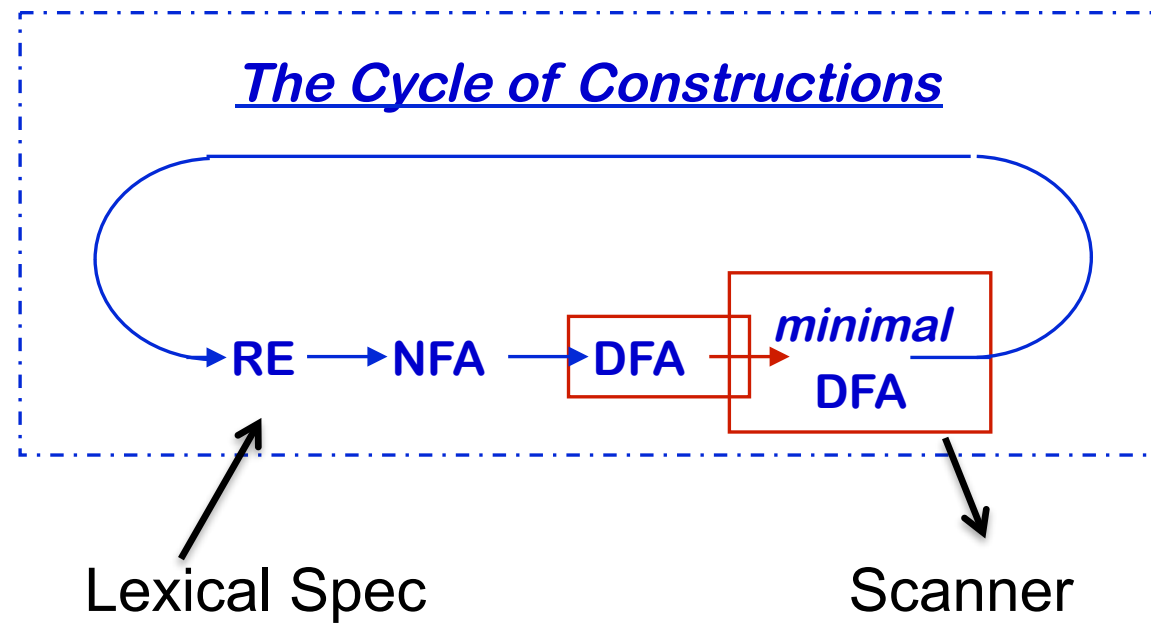
$\text{move}(6, b) \rightarrow \text{ERROR}$

$\text{move}(4, a|b) \rightarrow \text{ERROR}$

# Class Problem

Convert this NFA to a DFA

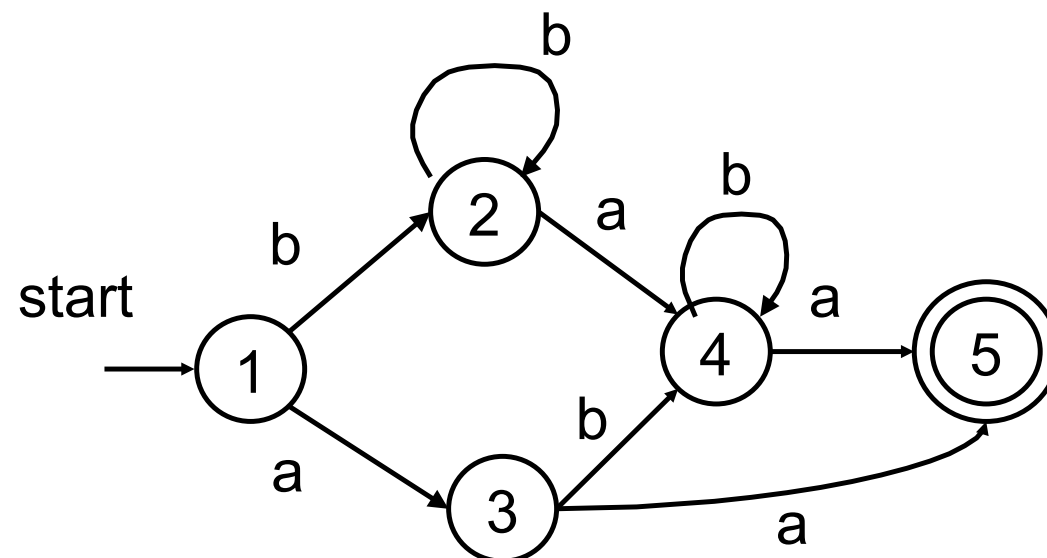




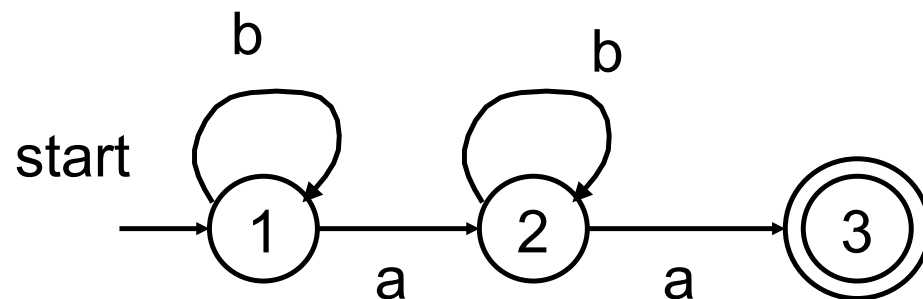


# State Minimization

- Resulting DFA can be quite large
  - Contains redundant or equivalent states

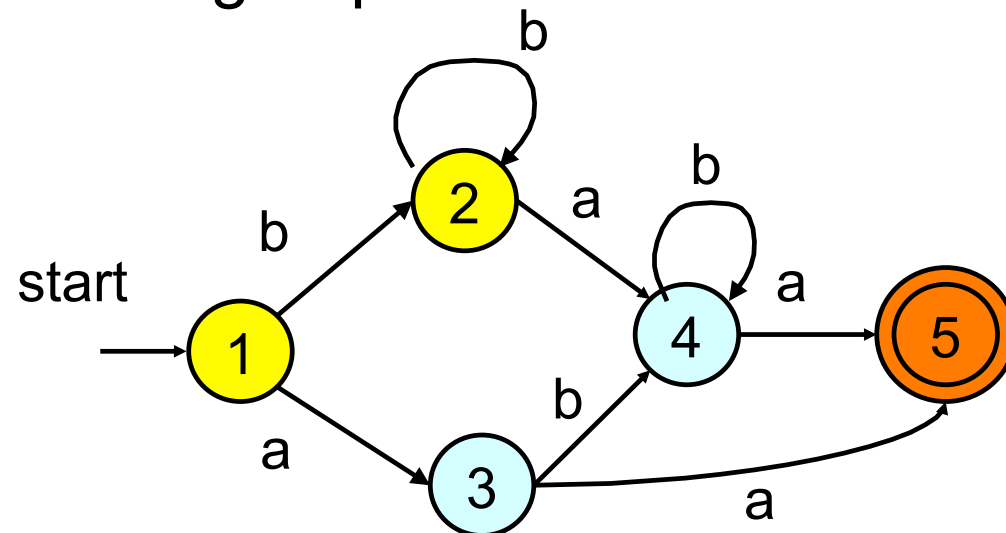


Both DFAs accept  
 $b^*ab^*a$

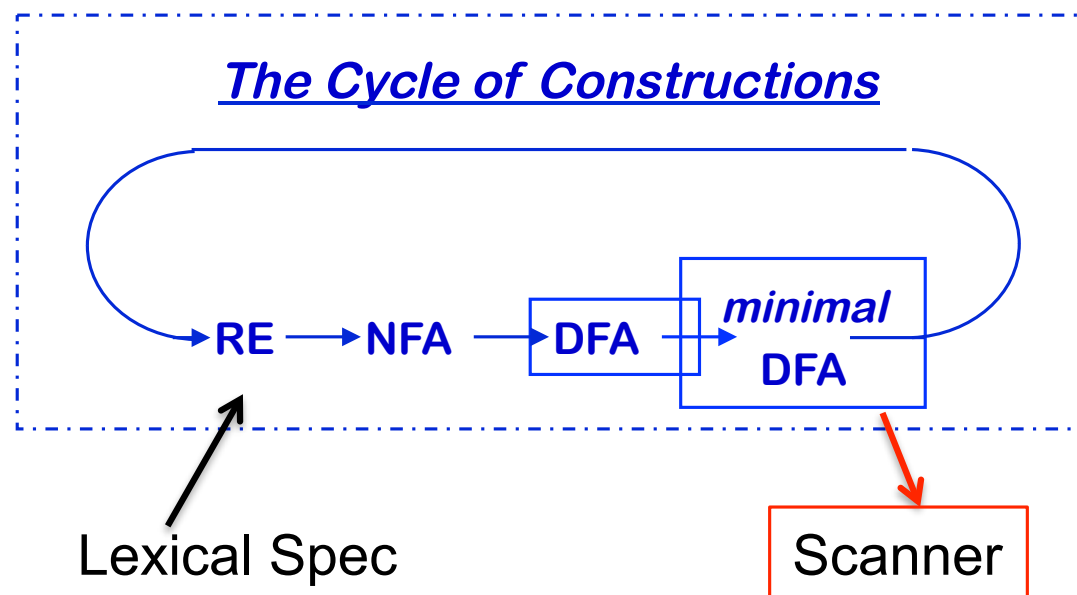


# State Minimization (2)

- Idea – find groups of equivalent states and merge them
  - All transitions from states in group G1 go to states in another group G2
  - Construct minimized DFA such that there is 1 state for each group of states



Basic strategy: identify distinguishing transitions

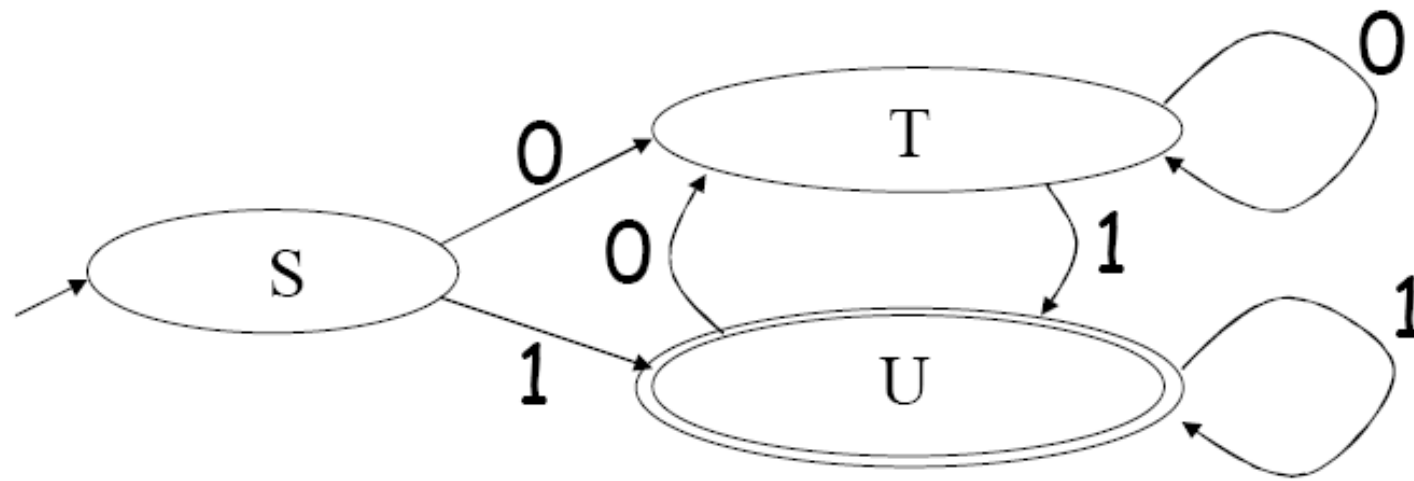


# DFA Implementation

---

- A DFA can be implemented by a 2D table T
  - One dimension is “states”
  - Other dimension is “input symbol”
  - For every transition  $S_i \xrightarrow{a} S_k$  define  $T[i,a] = k$
- DFA “execution”
  - If in state  $S_i$  and input  $a$ , read  $T[i,a] = k$  and skip to state  $S_k$
  - Very efficient

## DFA Table Implementation : Example



	0	1
S	T	U
T	T	U
U	T	U

# Implementation Cont ..

---

- NFA -> DFA conversion is at the heart of tools such as flex
- But, DFAs can be huge
- In practice, flex-like tools trade off speed for space in the choice of NFA and DFA representations