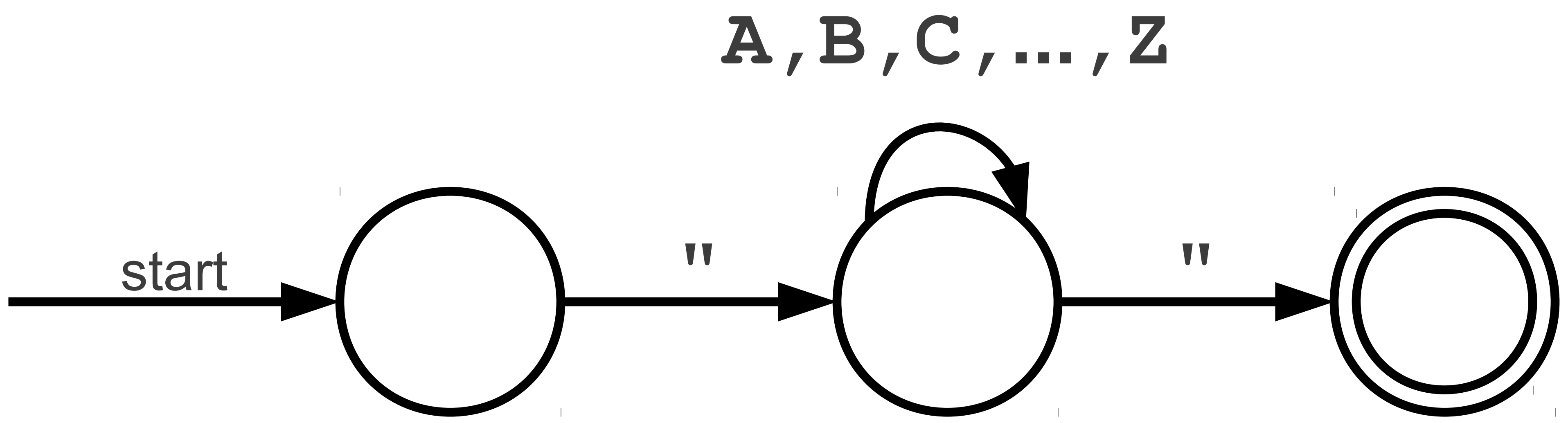
Lexical Analysis 2:
Automata and Lexer Generators

Recognizing Regular Languages

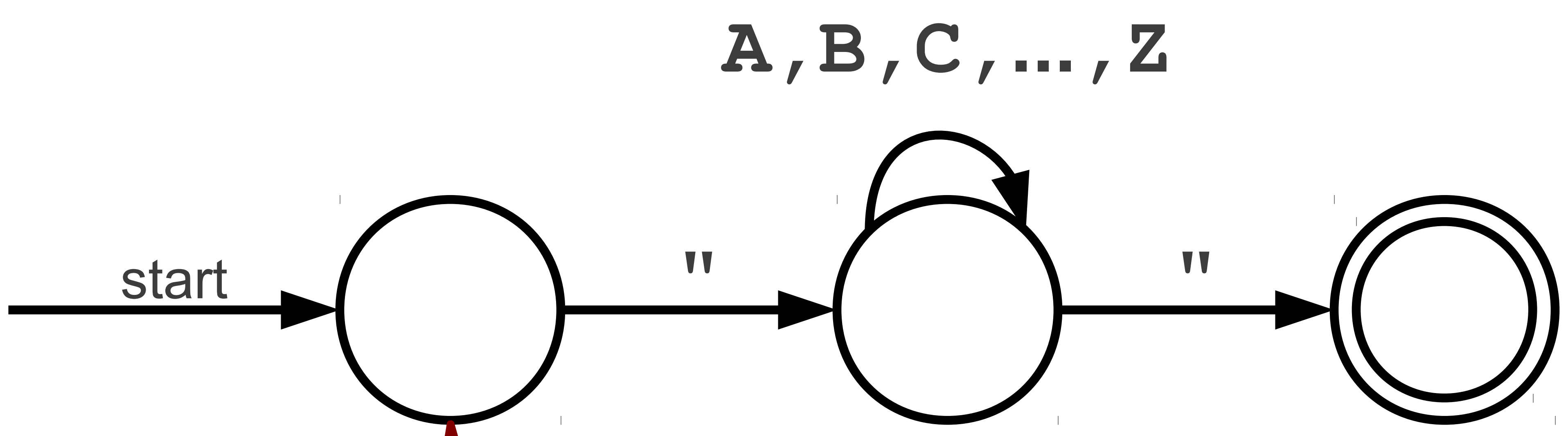
How can we efficiently implement a recognizer for a regular language?

- Finite Automata
- DFA (Deterministic Finite Automata)
- NFA (Non-deterministic Finite Automata)

A Simple Automaton

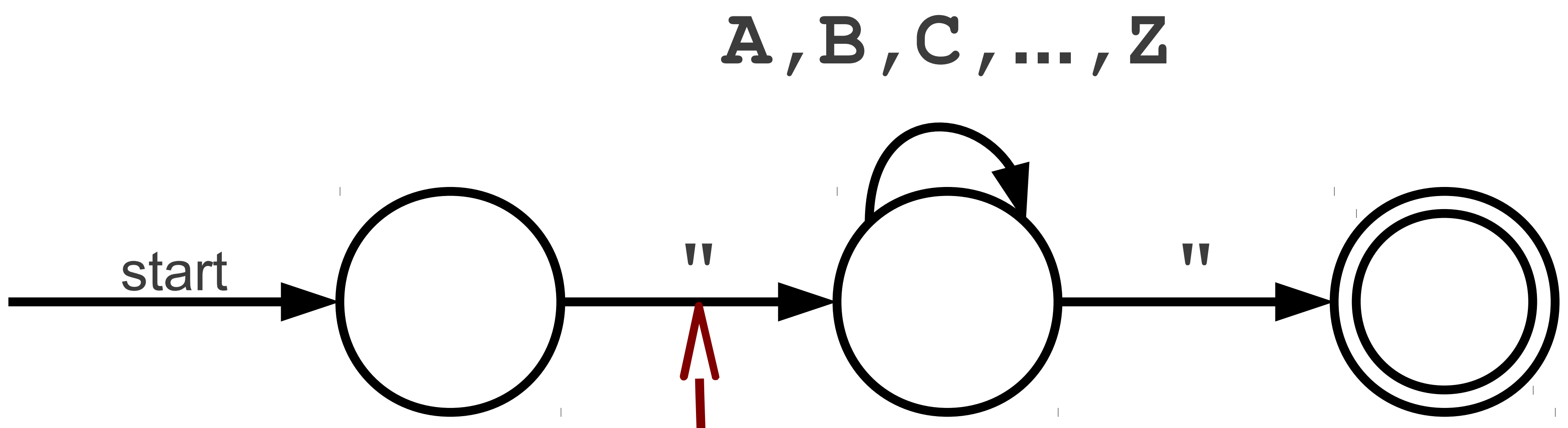


A Simple Automaton



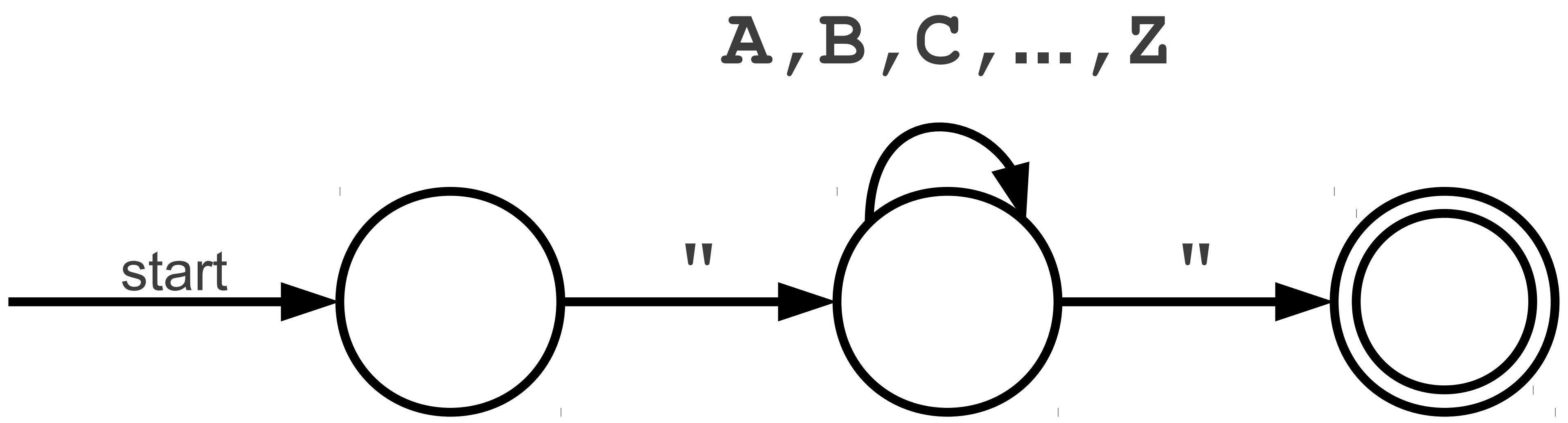
Each circle is a state of the automaton. The automaton's configuration is determined by what state(s) it is in.

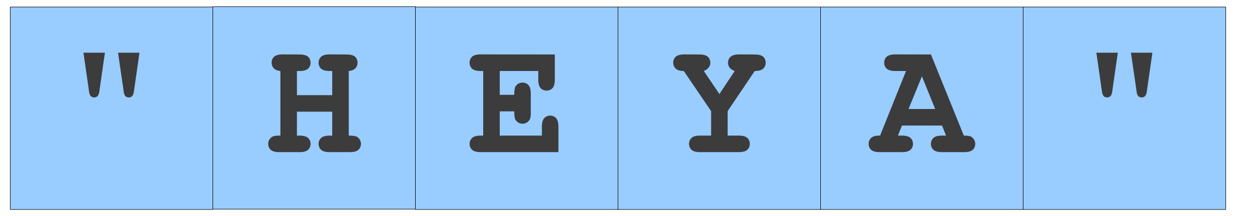
A Simple Automaton



These arrows are called **transitions**. The automaton changes which state(s) it is in by following transitions.

A Simple Automaton

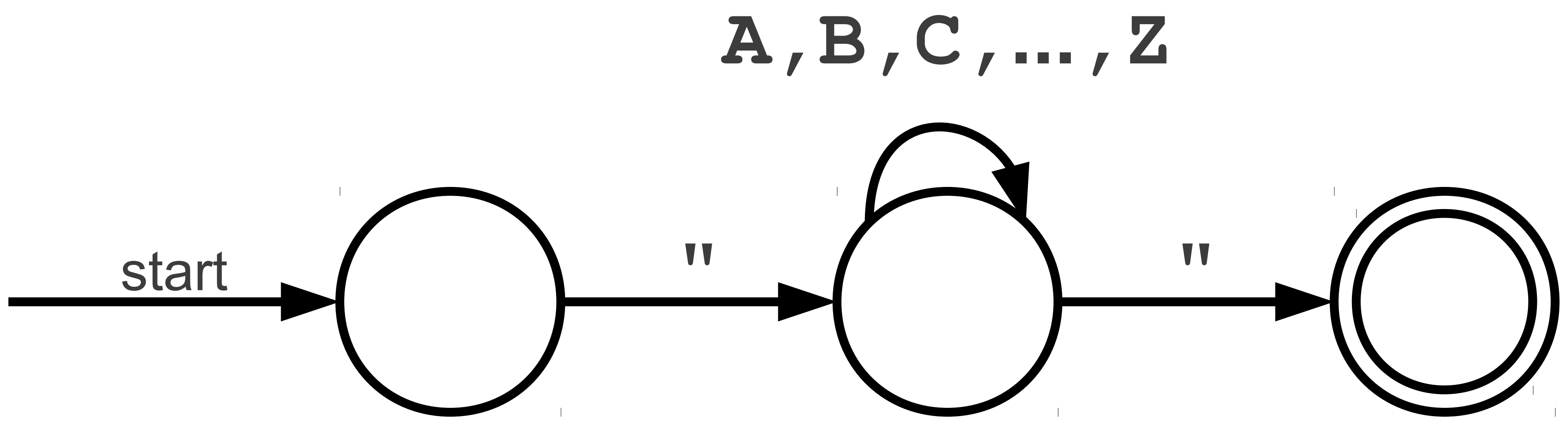




Finite Automata: Takes an input string and determines whether it's a valid sentence of a language

accept or reject

A Simple Automaton



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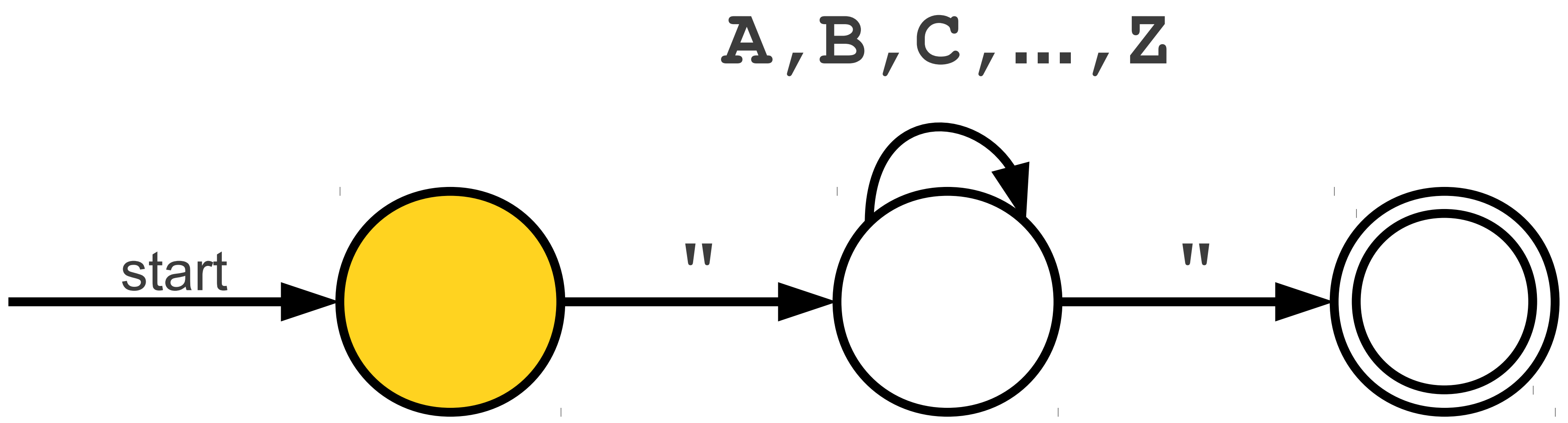

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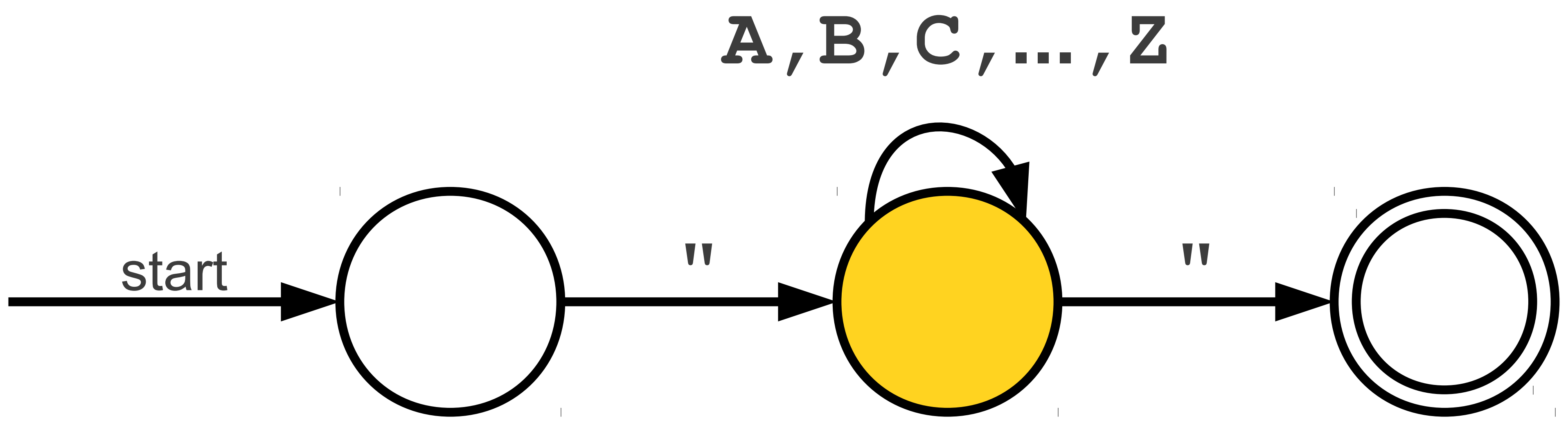
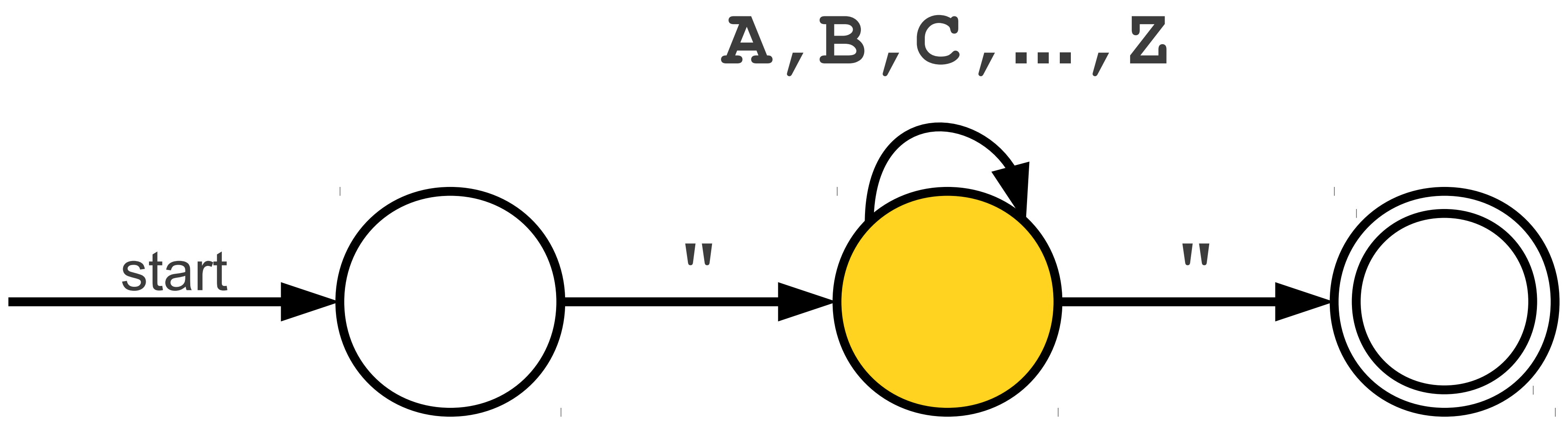
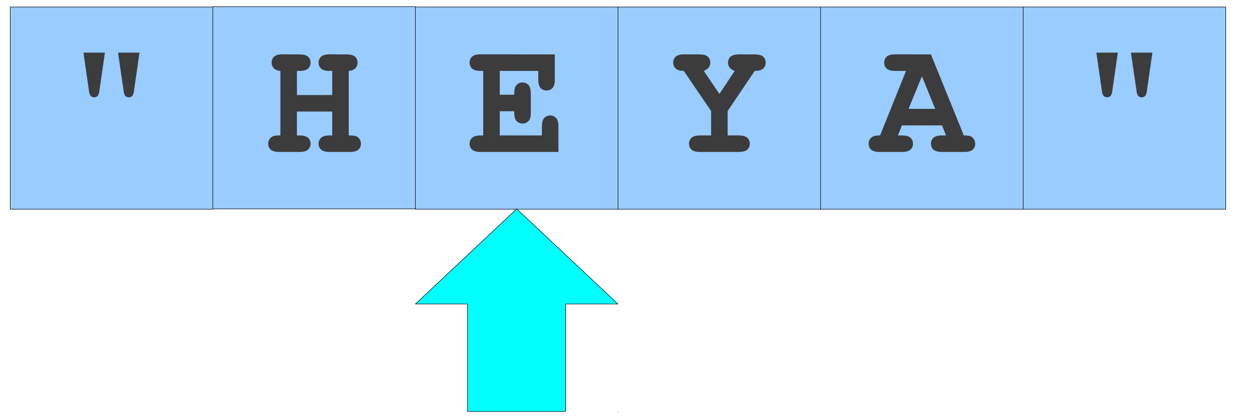
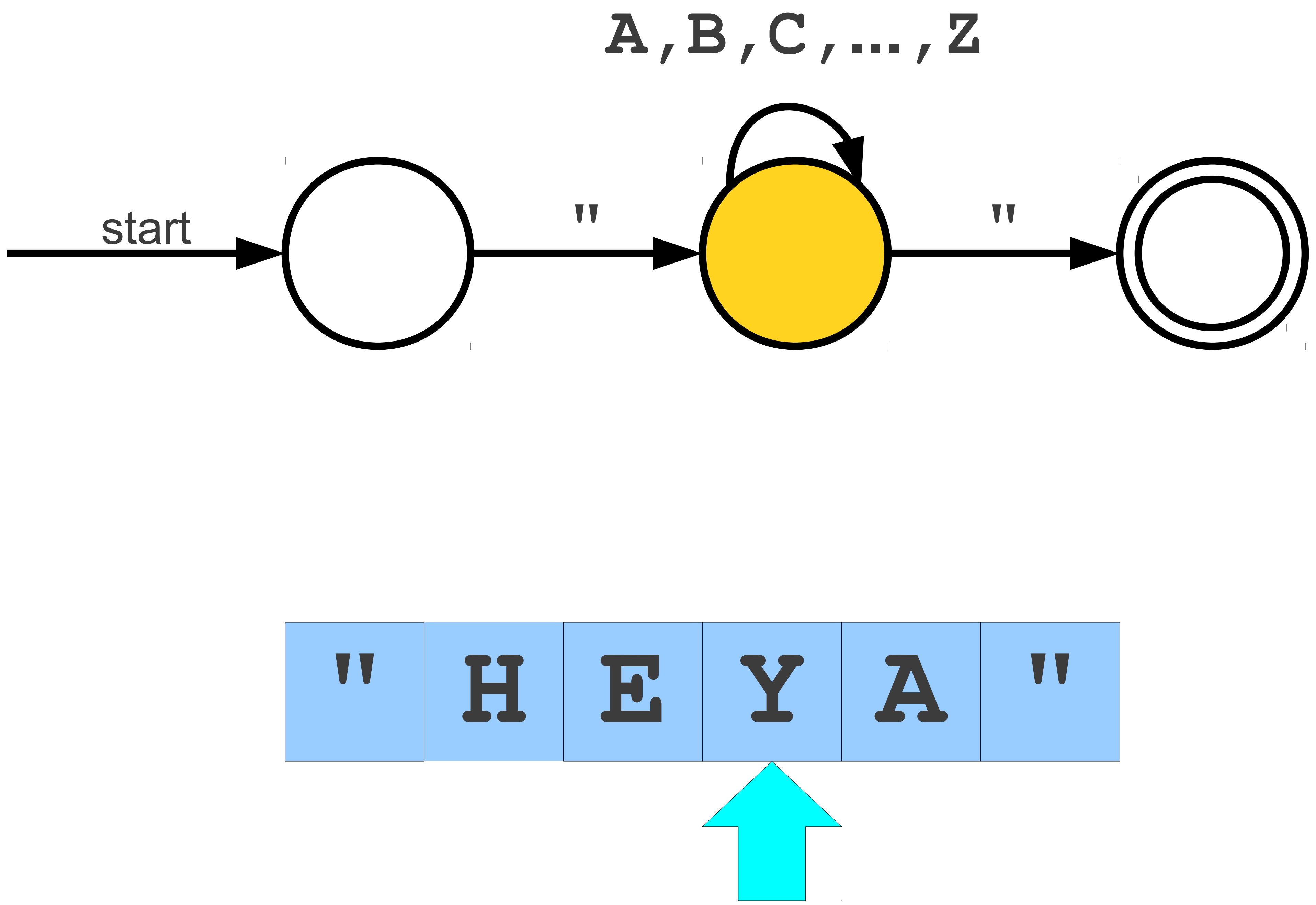


A Simple Automaton

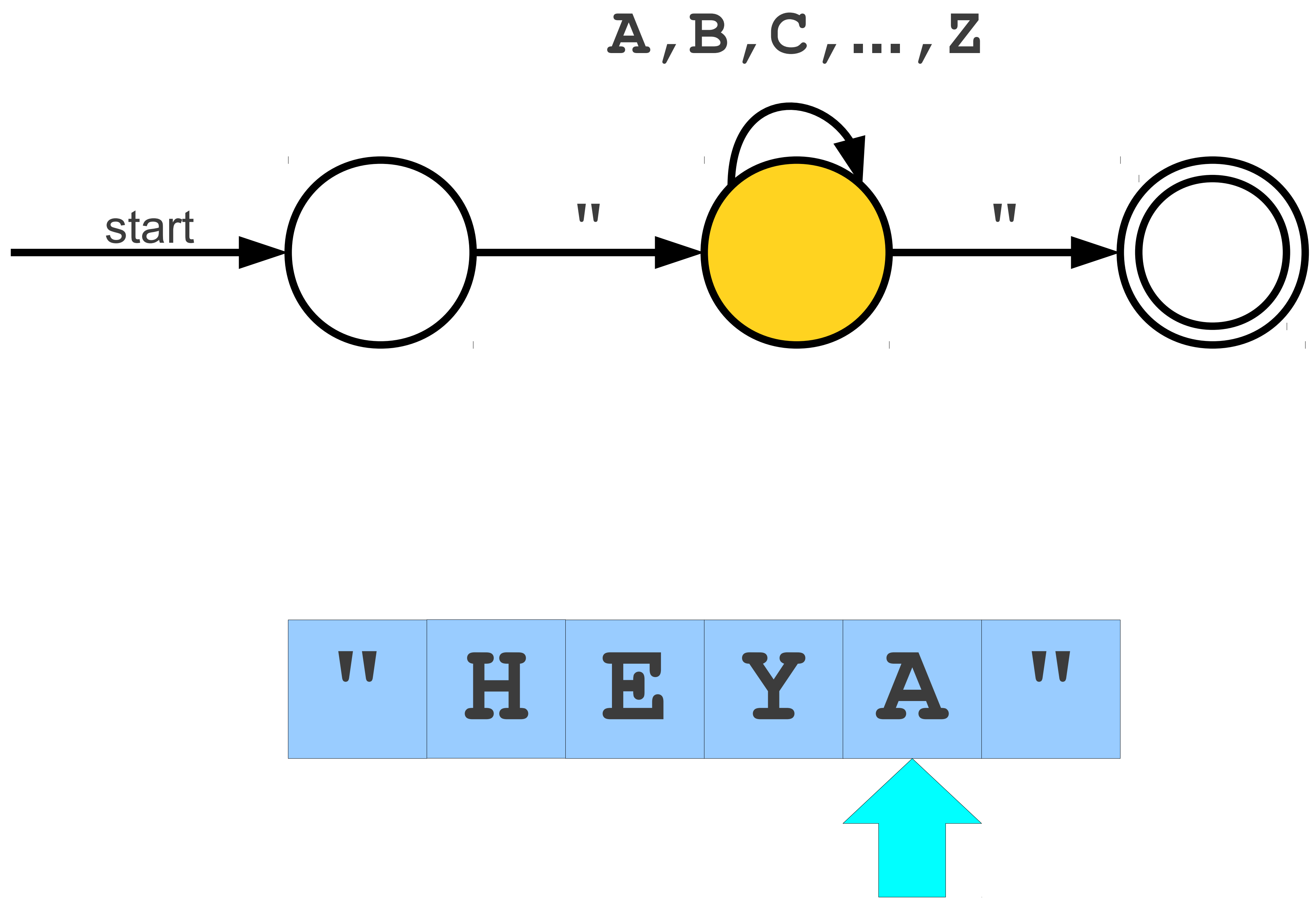




A Simple Automaton



A Simple Automaton



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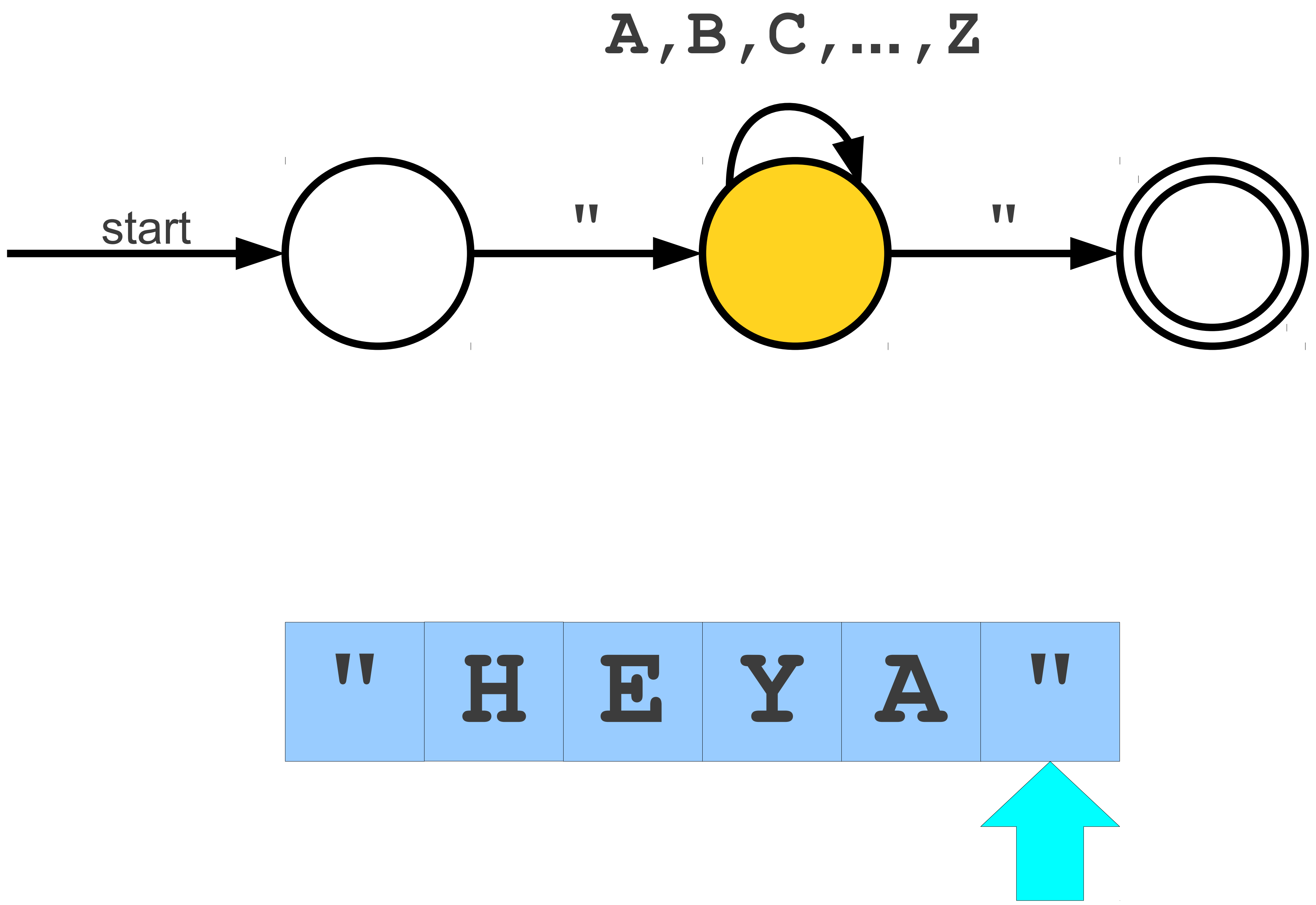
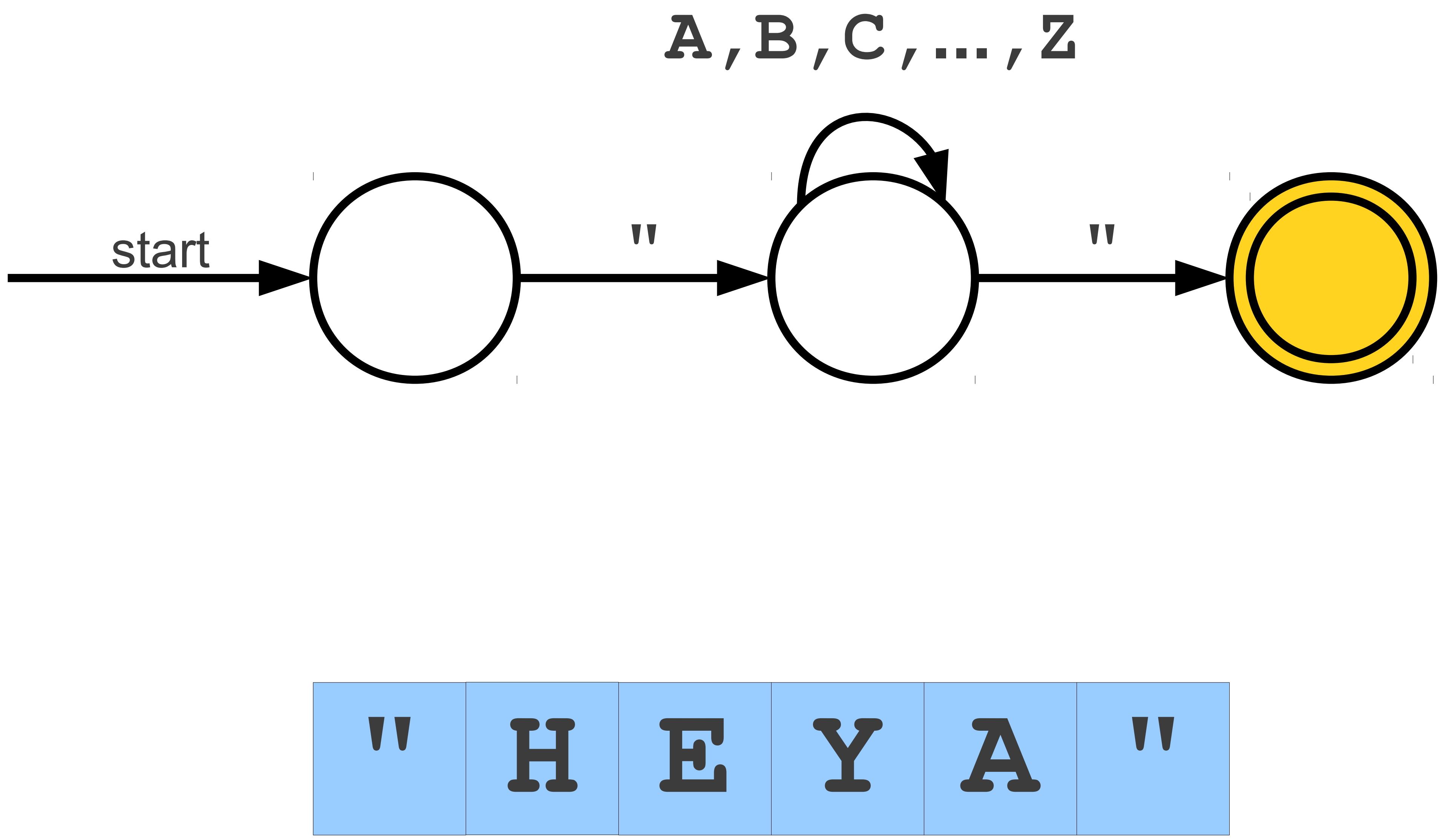
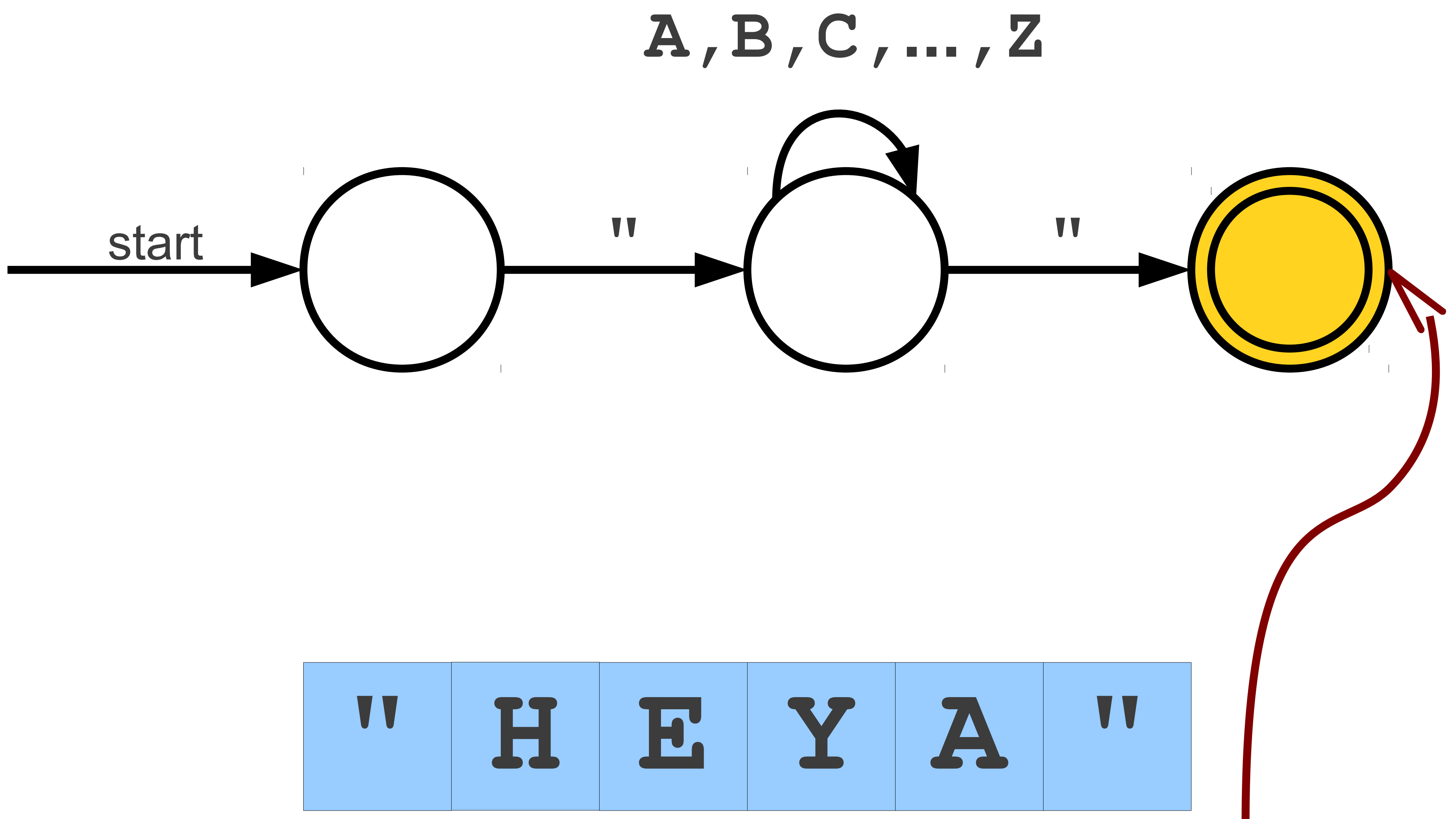


A Simple Automaton

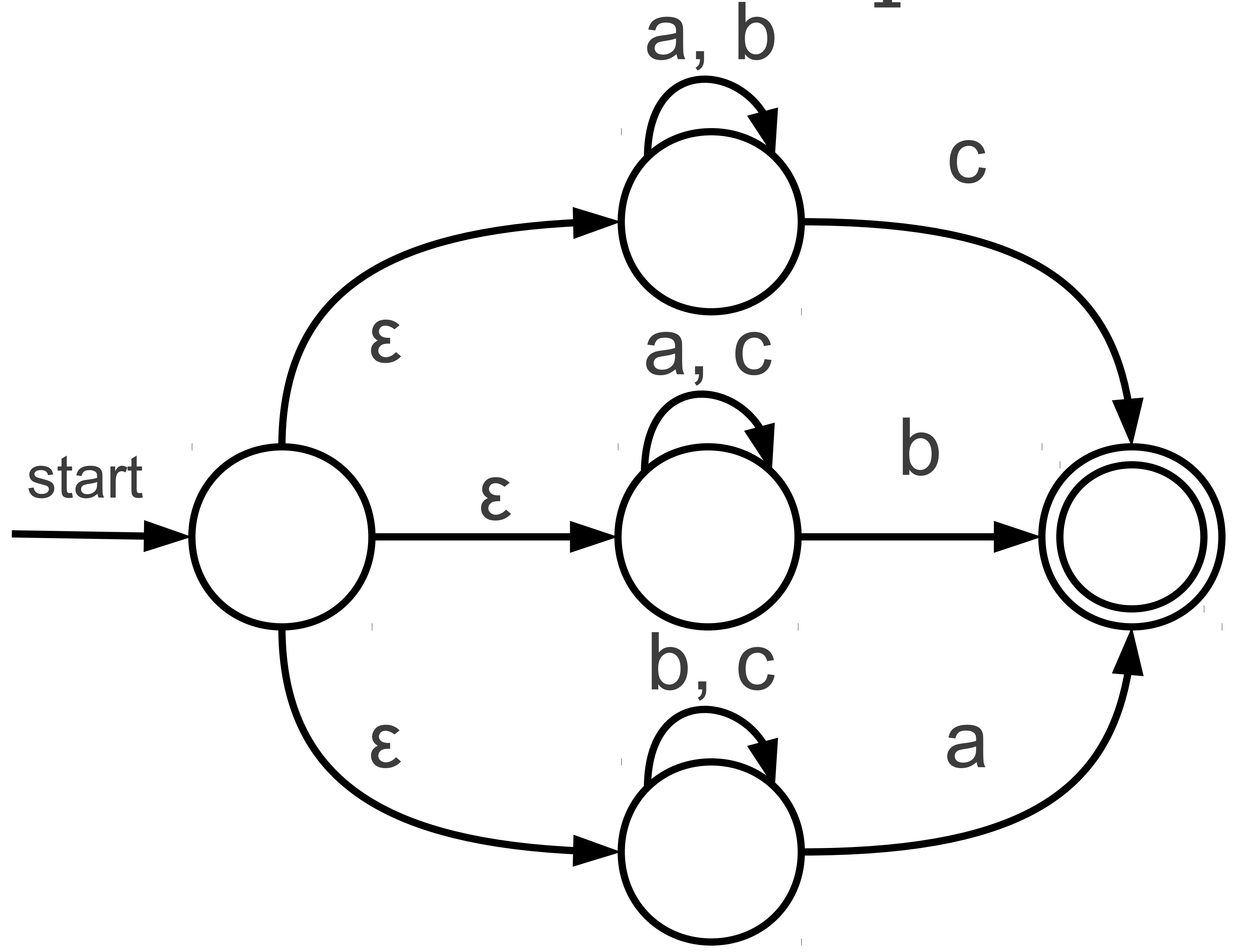


A Simple Automaton



The double circle indicates that this state is an accepting state. The automaton accepts the string if it ends in an accepting state.

An Even More Complex Automaton



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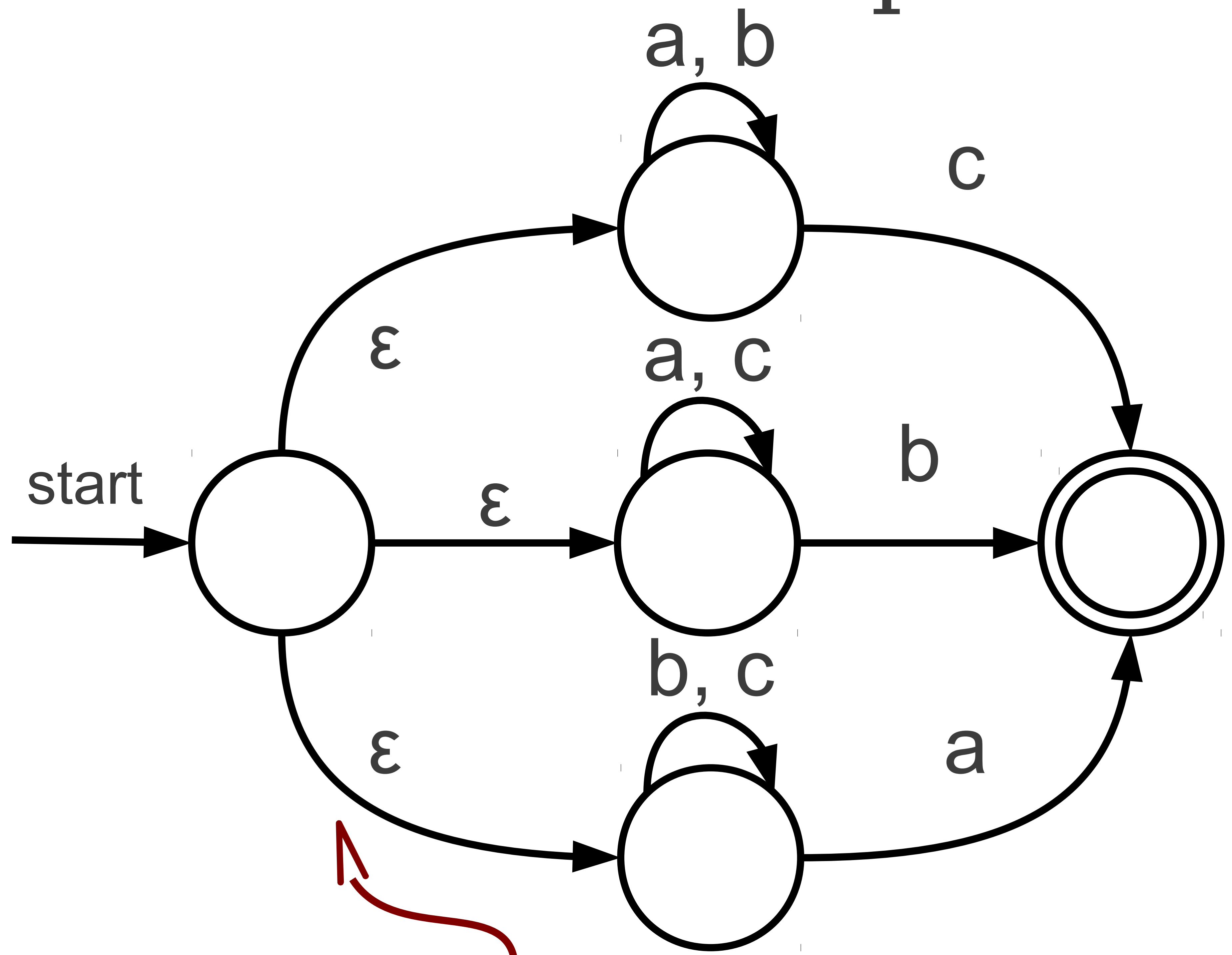
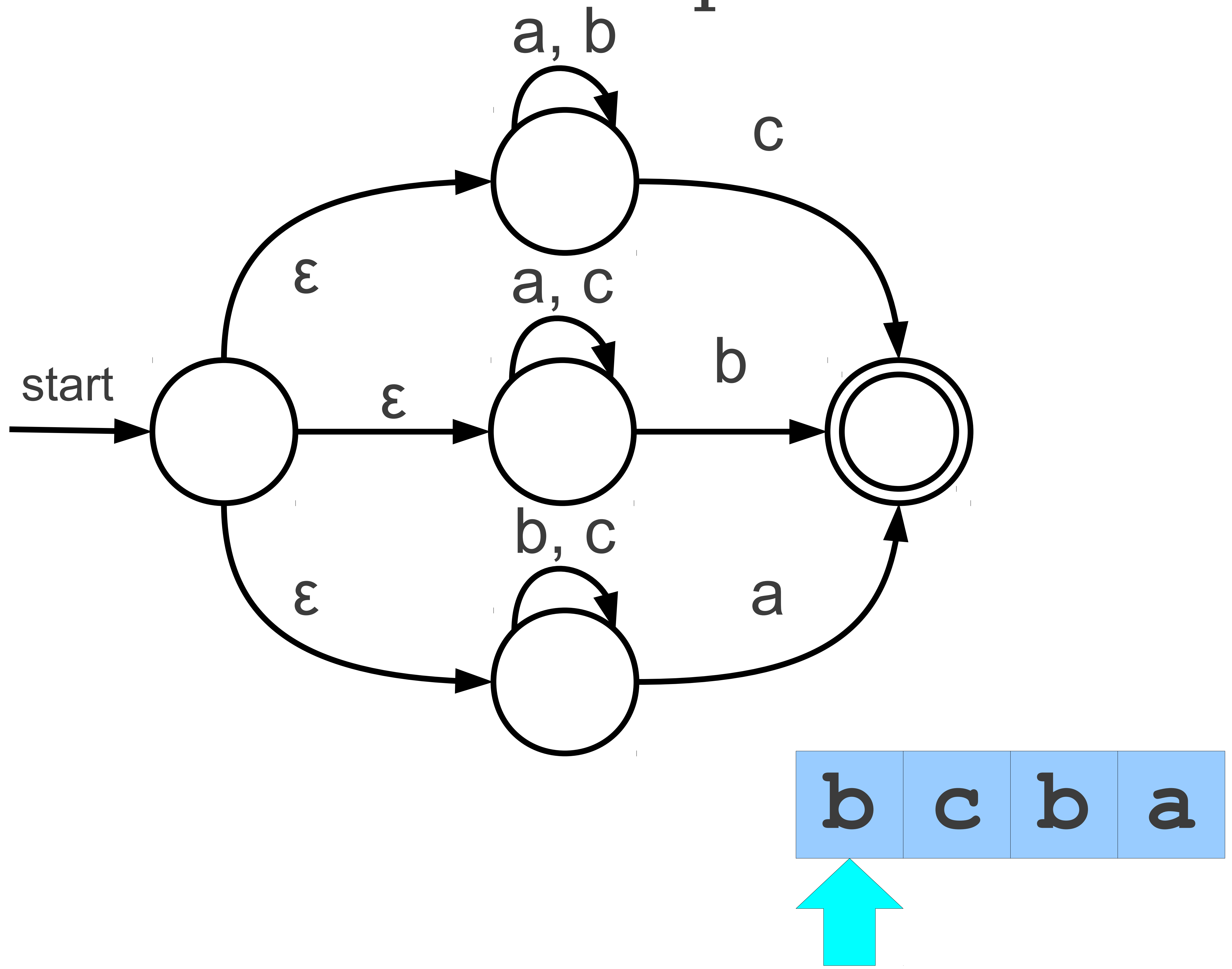


These are called **ε-transitions**. These transitions are followed automatically and without consuming any input.

An Even More Complex Automaton



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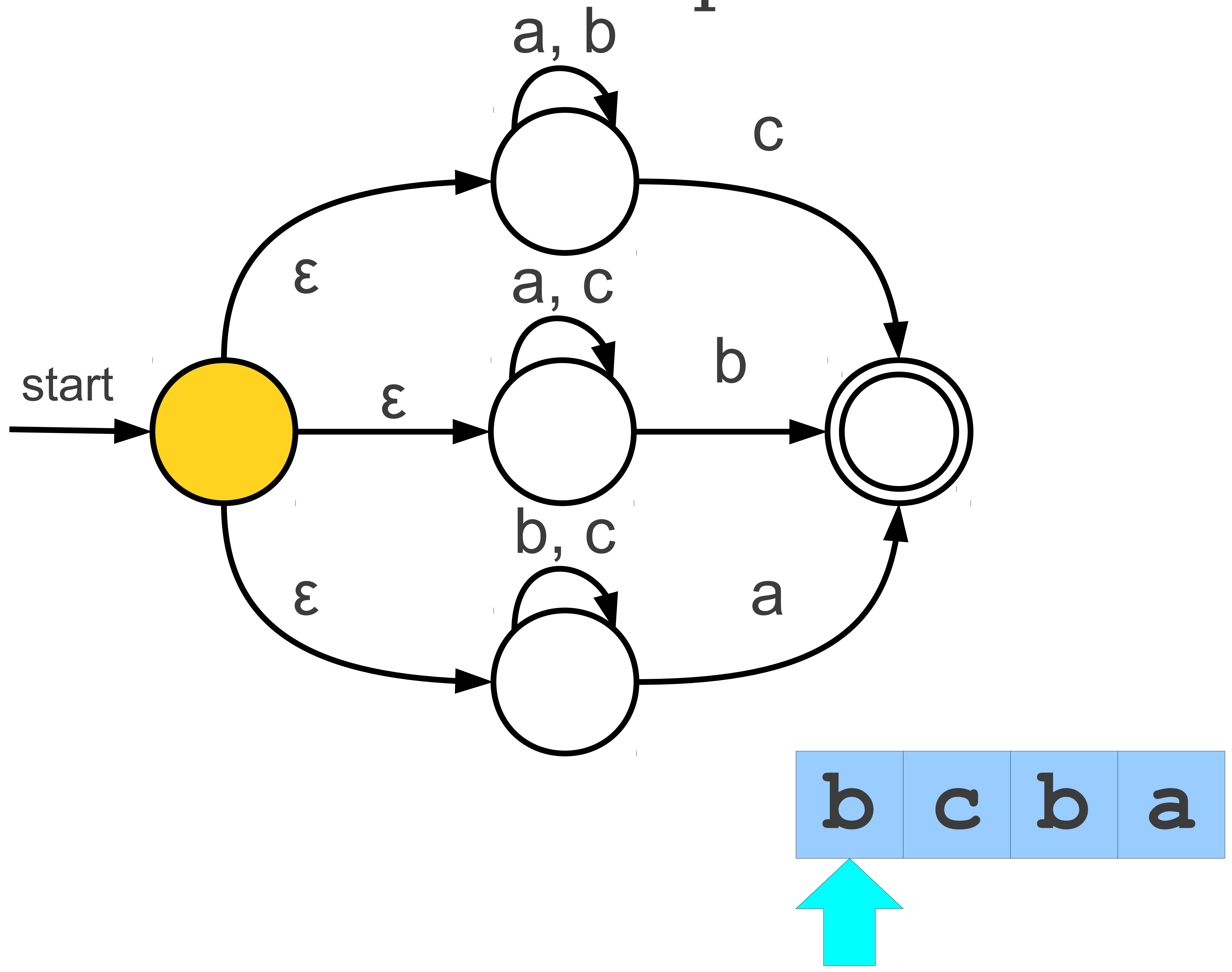

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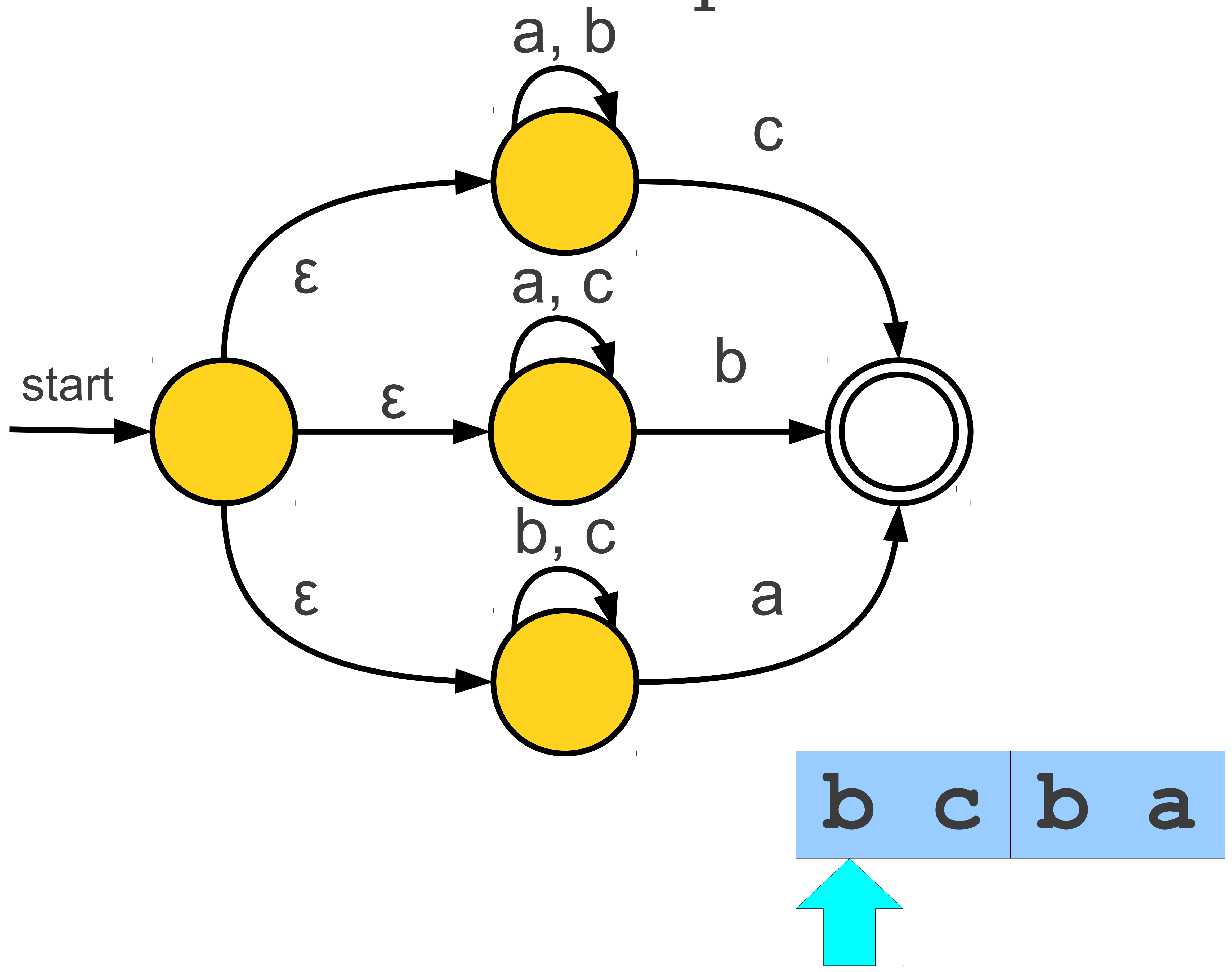

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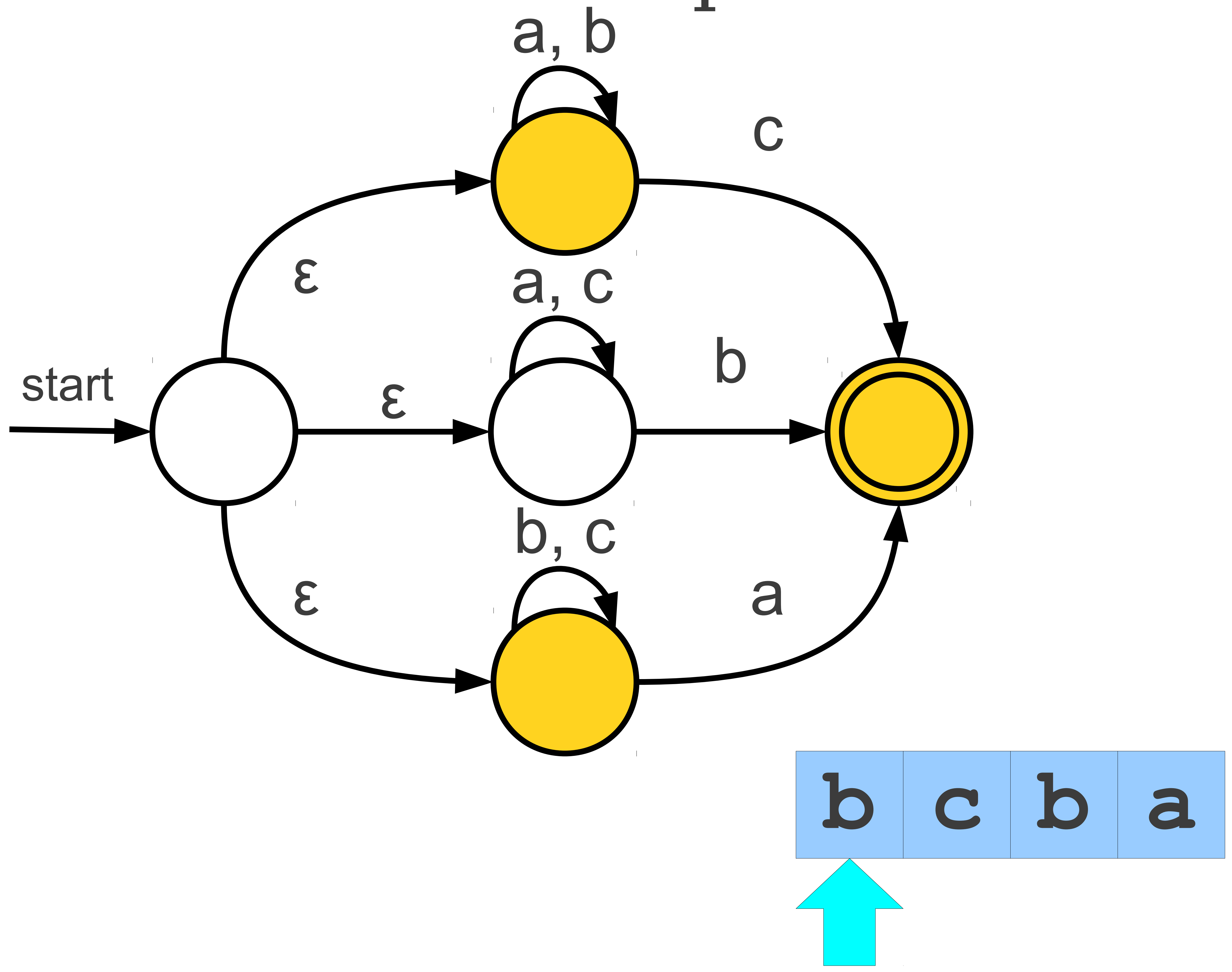

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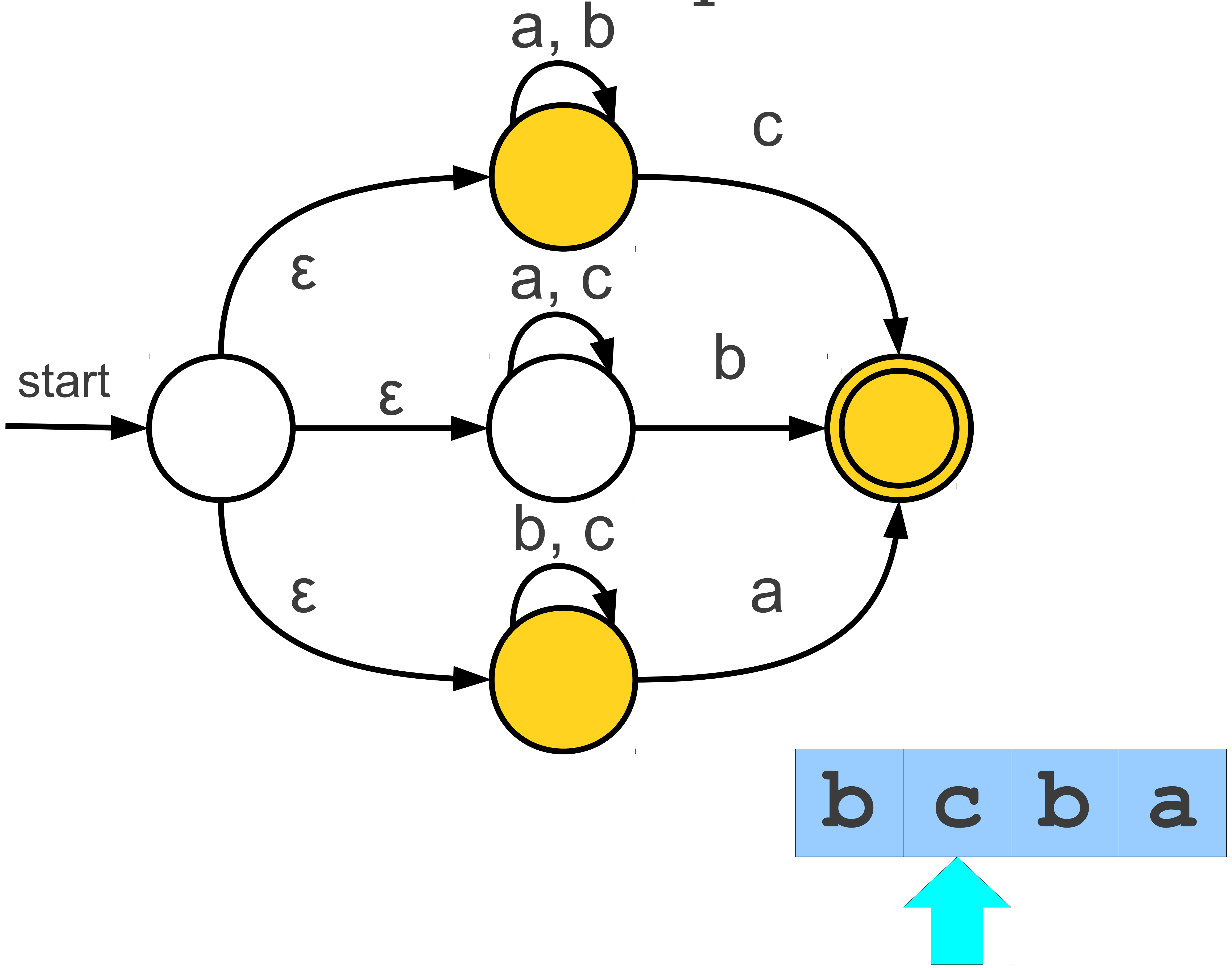

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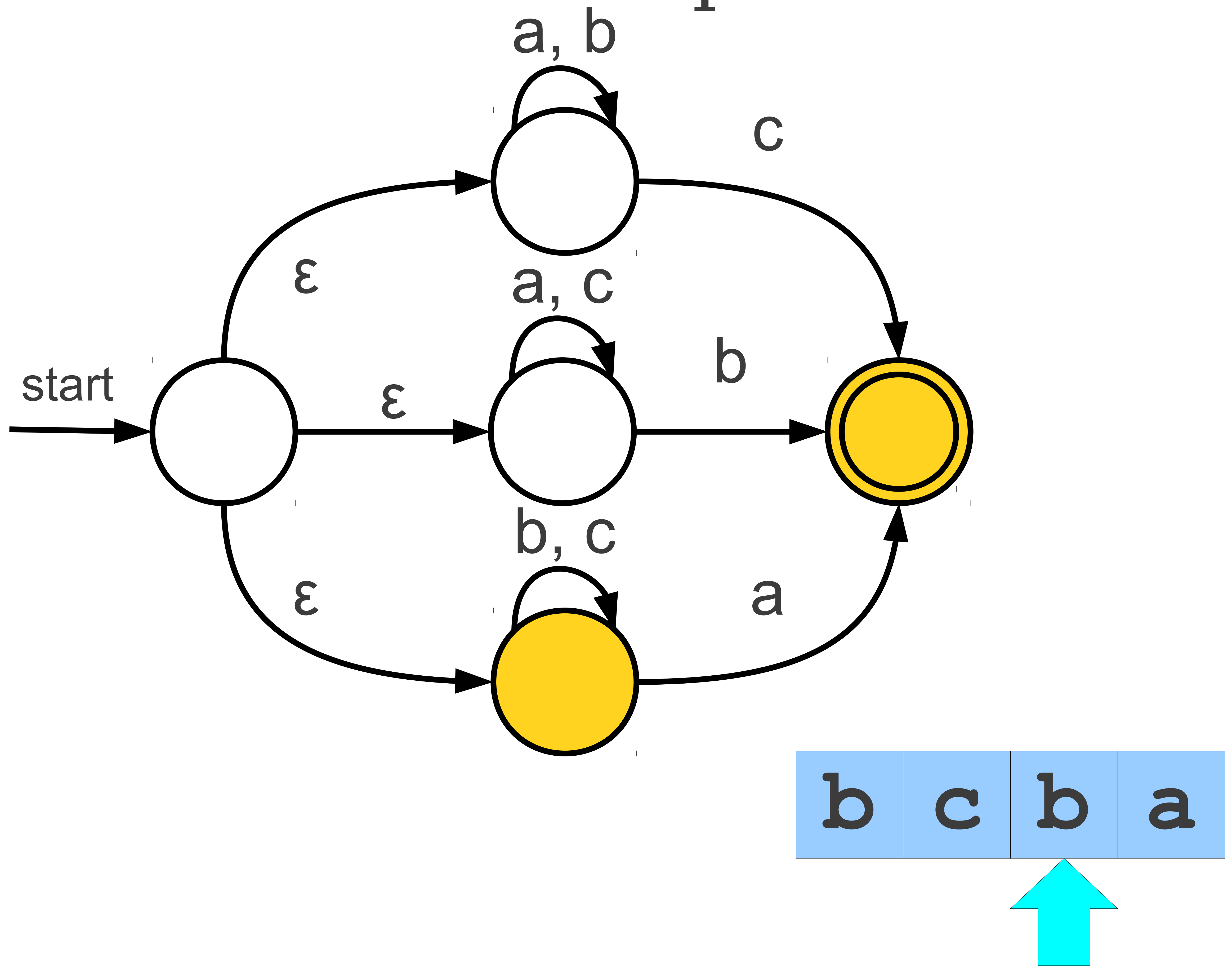

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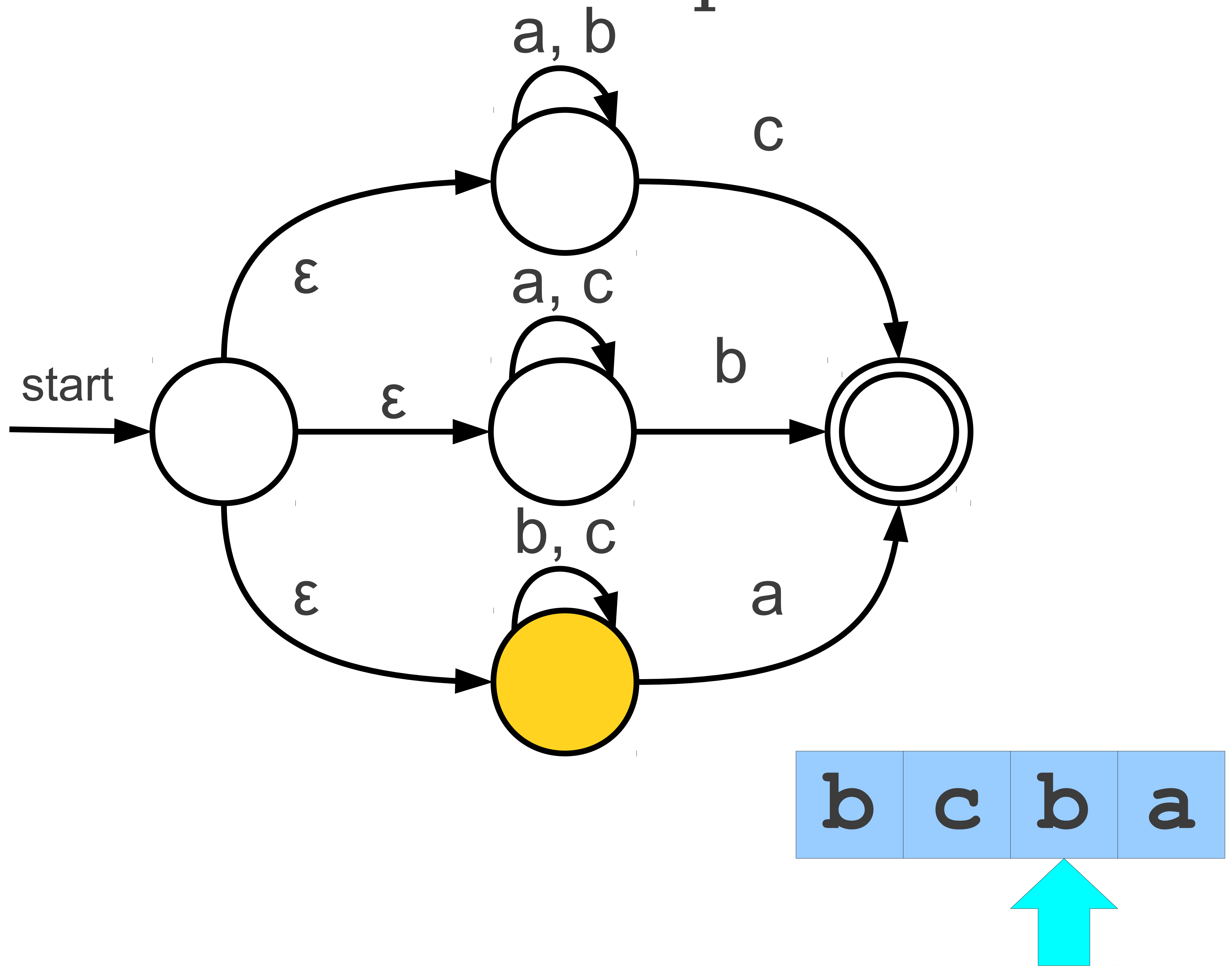

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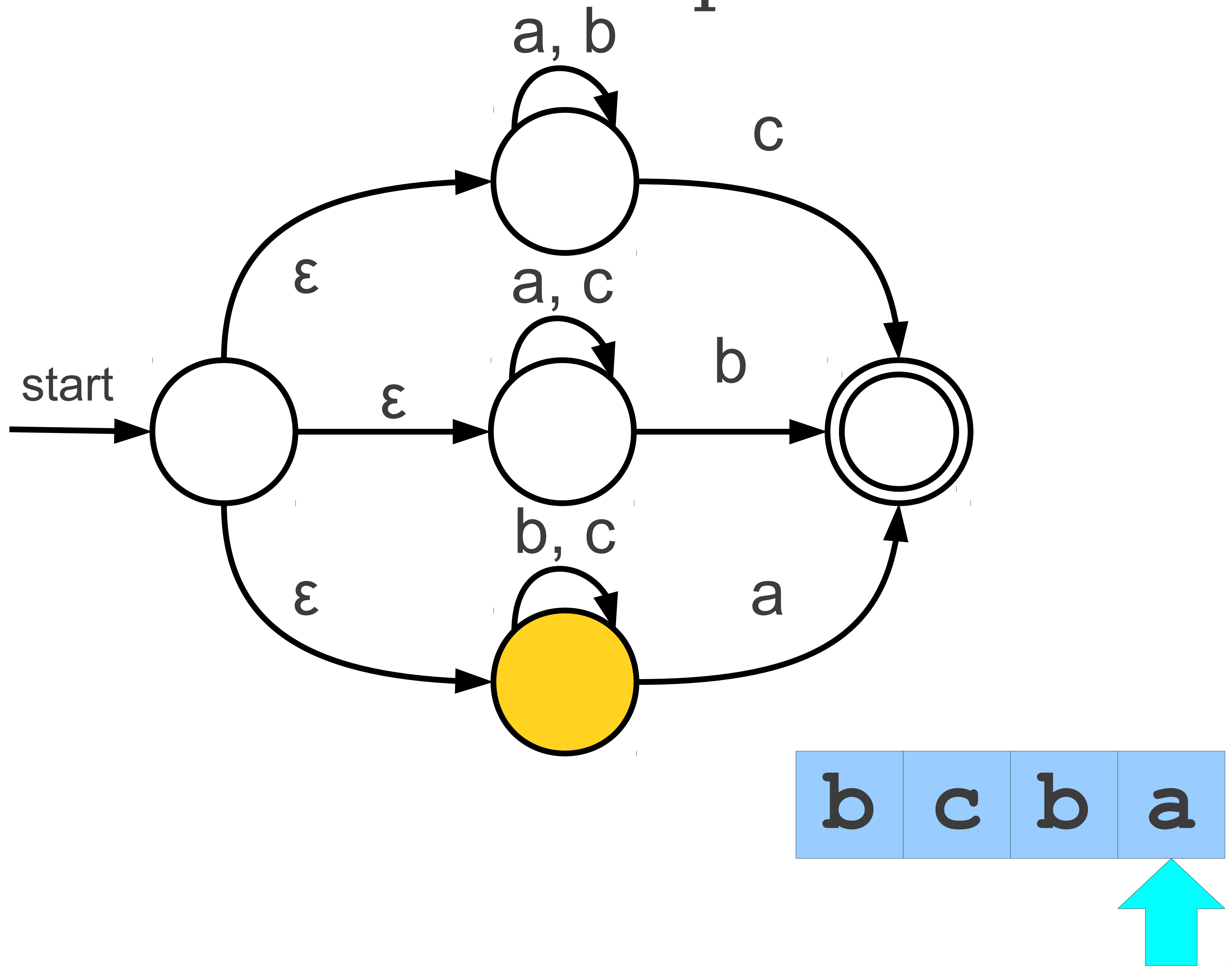

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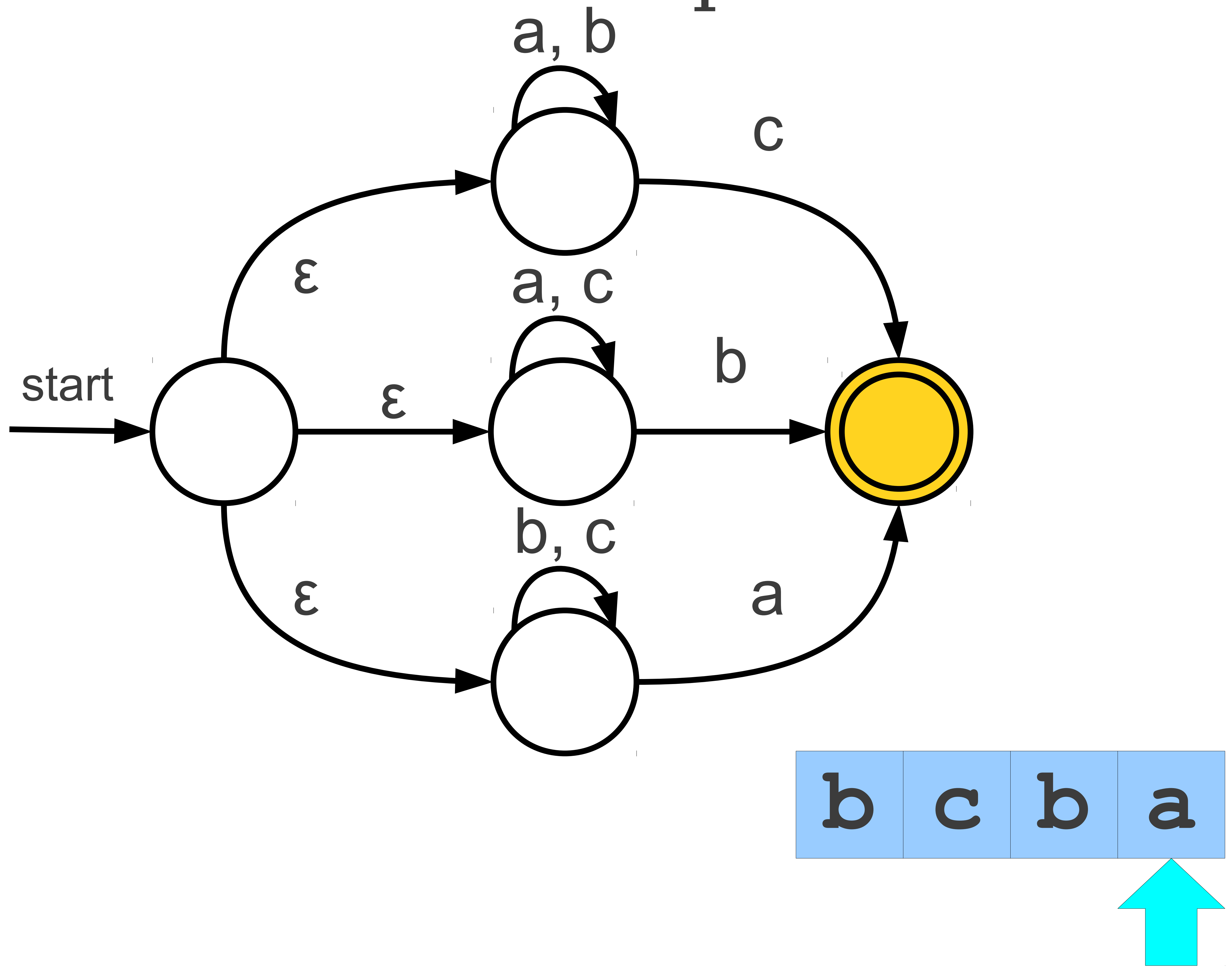

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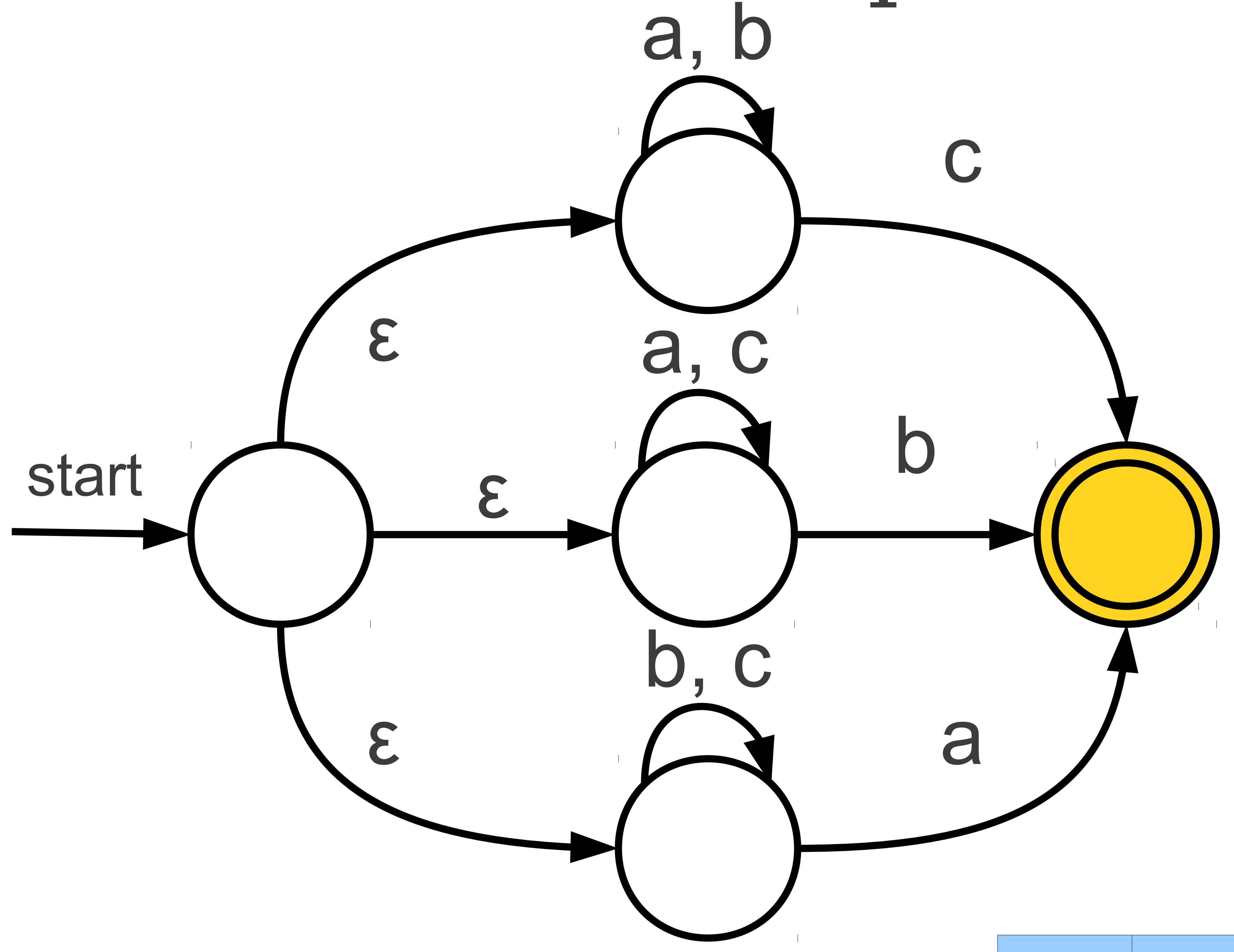


b	c	b	a

Finite State Automata

- Regular expressions = specification
- Finite automata = implementation
- A finite automaton consists of
  - An input alphabet Σ
  - A set of states S
  - A start state n
  - A set of accepting states $F \subseteq S$
  - A set of transitions δ
    - $state_k$ ----> $state_j$

Finite State Automata

- Transition

  $s1 \rightarrow^{a} s2$

- A character is read

  In state s1 on input “a” go to state s2

- If end of input and in accepting state

  Accept

- Otherwise

  Reject

DFA vs. NFA

- Deterministic Finite Automata (DFA)
  - One transition per input per state
  - No ε-moves
- Nondeterministic Finite Automata (NFA)
  - Can have multiple transitions for one input in a given state
  - Can have ε-moves

DFA vs. NFA

- NFAs and DFAs recognize the same set of languages (regular languages)
  - For a given NFA, there exists a DFA, and vice versa
- DFAs are faster to execute
  - There are no choices to consider
  - Tradeoff: simplicity
    - For a given language DFA can be exponentially larger than NFA.

Automating Lexical Analyzer (scanner) Construction

To convert a specification into code:

1 Write down the RE for the input language

2 Build a big NFA

3 Build the DFA that simulates the NFA

4 Systematically shrink the DFA

5 Turn it into code

Scanner generators

- Lex and Flex work along these lines
- Algorithms are well-known and well-understood

Alternative Approaches

- We'll go through the "classic" procedure above but some scanners use different approaches:
    - Brzozowski: use the "derivative" operation on languages to directly produce a DFA from a regexp
        - Advantage: simple to implement, extends easily to support regex conjunction, negation. Often used for regex interpreters
        - Disadvantage: computationally expensive to generate minimal DFAs

Automating Lexical Analyzer (scanner) Construction

RE→ NFA *(Thompson's construction)*

- Build an NFA for each term
- Combine them with ε-moves

NFA → DFA *(subset construction)*

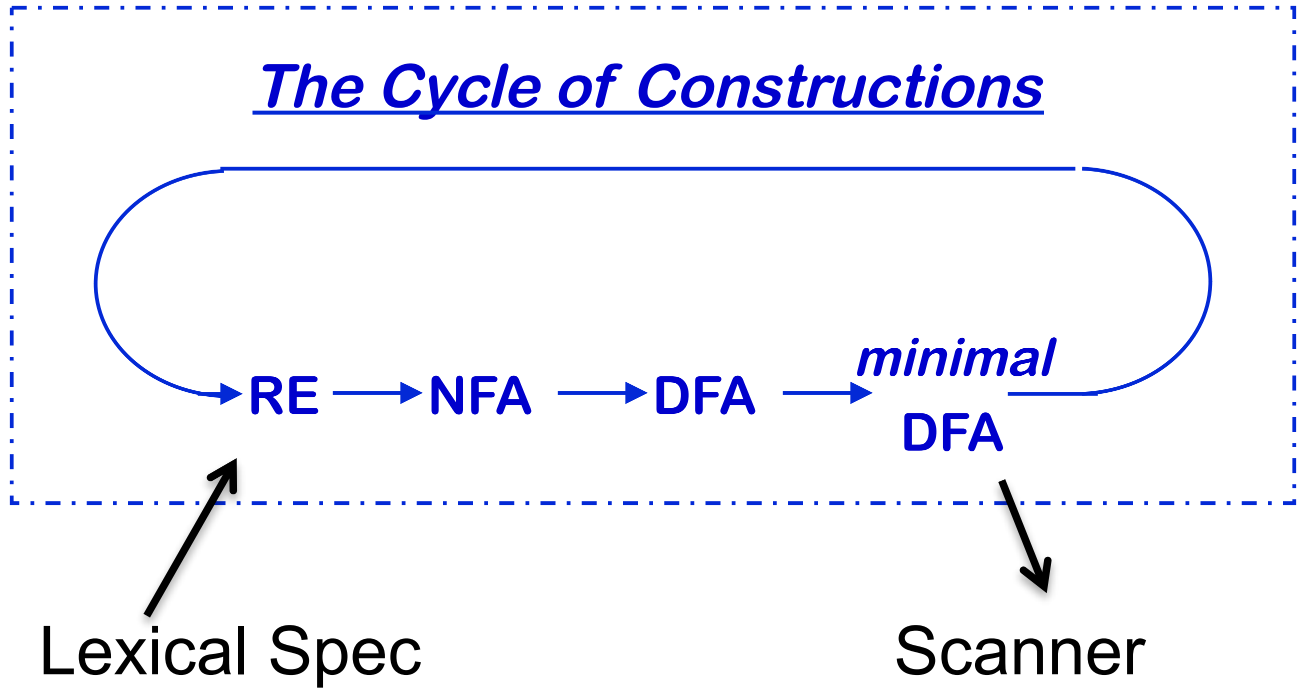
- Build the simulation

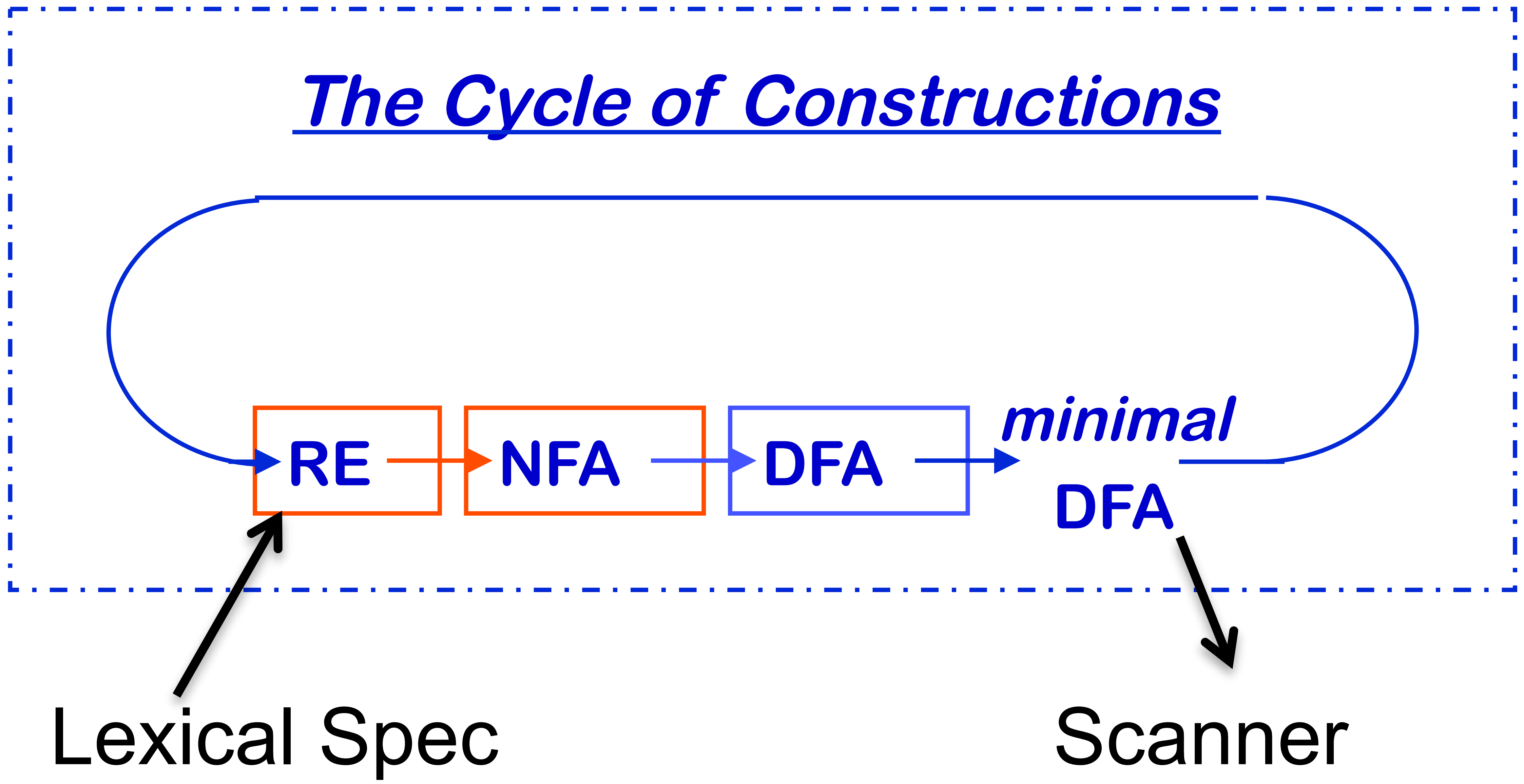
DFA → Minimal DFA

- Hopcroft's algorithm

DFA →RE *(Not part of the scanner construction)*

- All pairs, all paths problem
- Take the union of all paths from s_0 to an accepting state




The Cycle of Constructions
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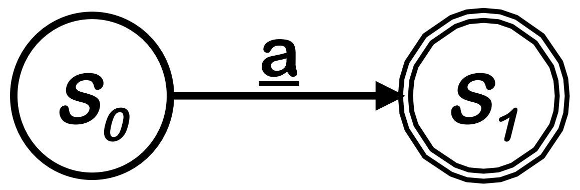
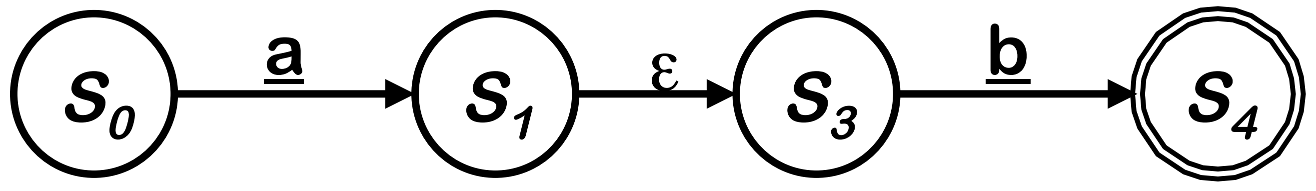
RE →NFA using Thompson's Construction

Key idea

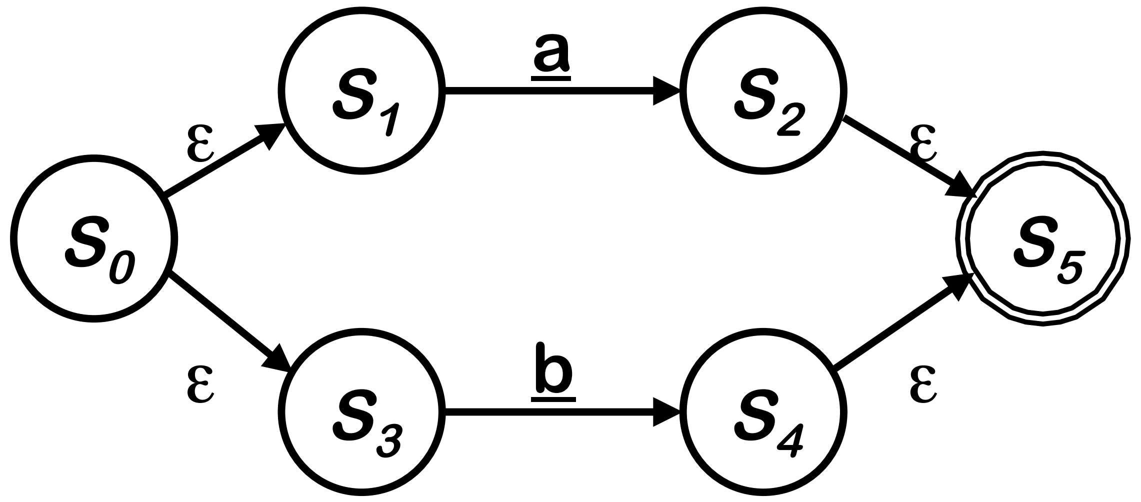
- NFA pattern for each symbol & each operator
- Join them with ε moves in precedence order



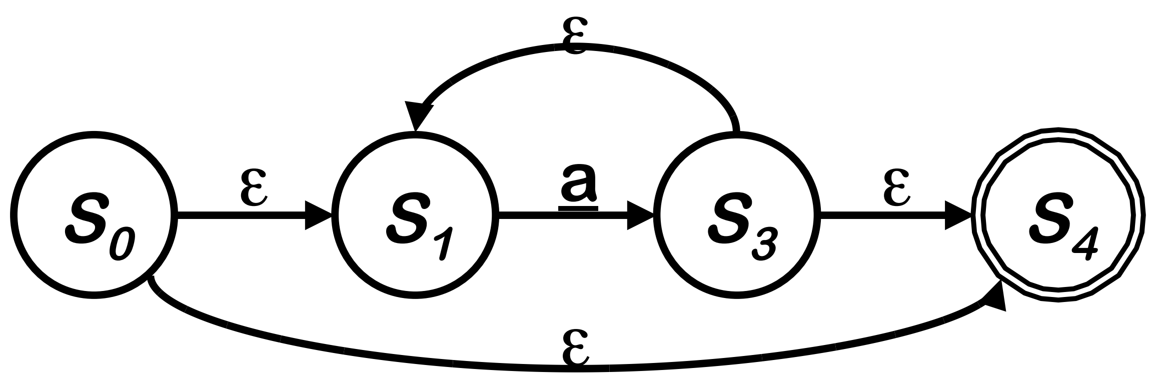
NFA for a



NFA for ab



NFA for a | b



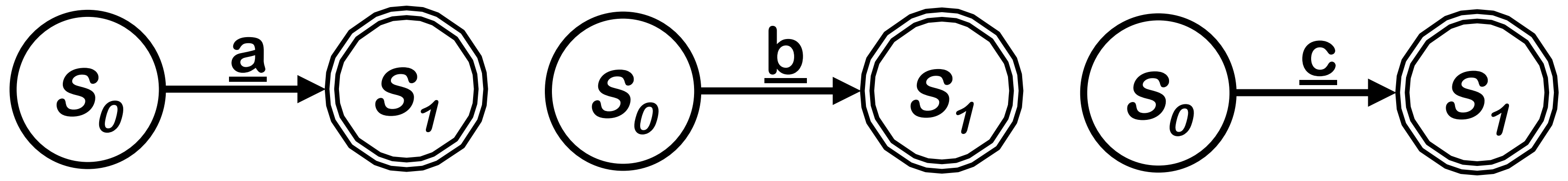
NFA for a*

Ken Thompson, CACM, 1968

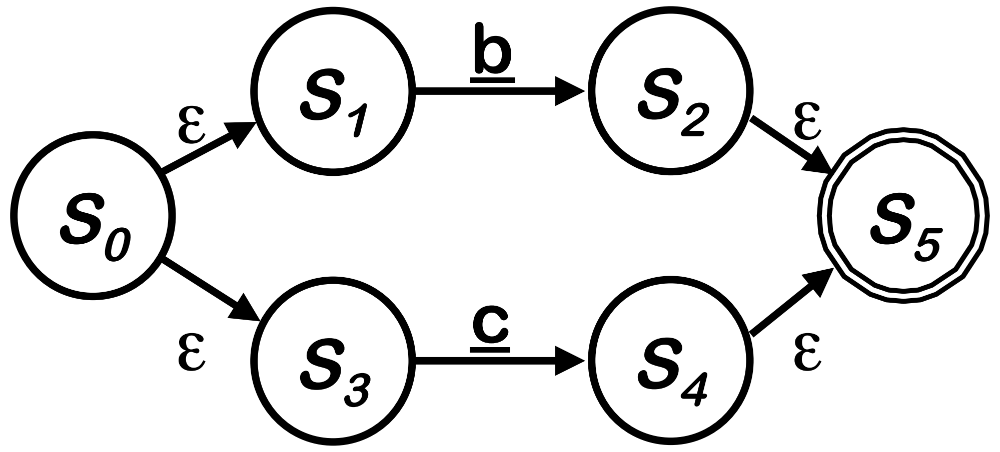
Example of Thompson’s Construction

Let’s try a (b | c)*

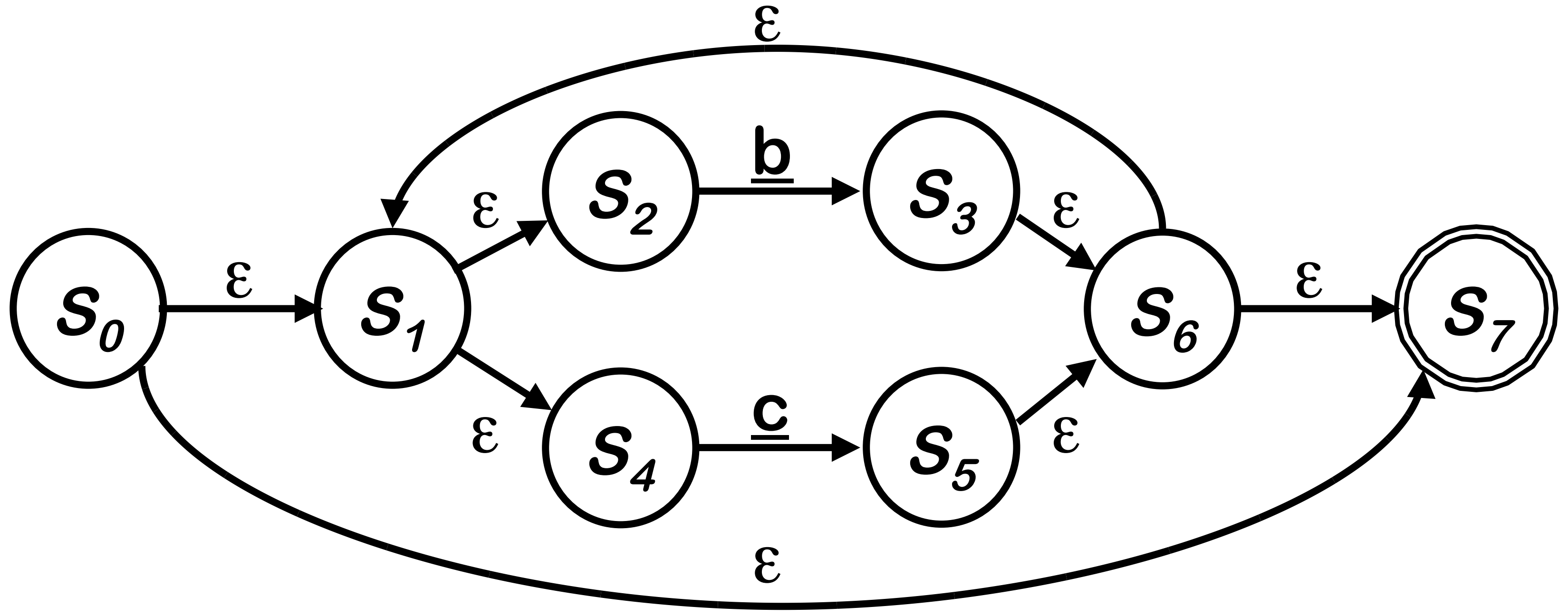
1. a, b, & c



2. b | c

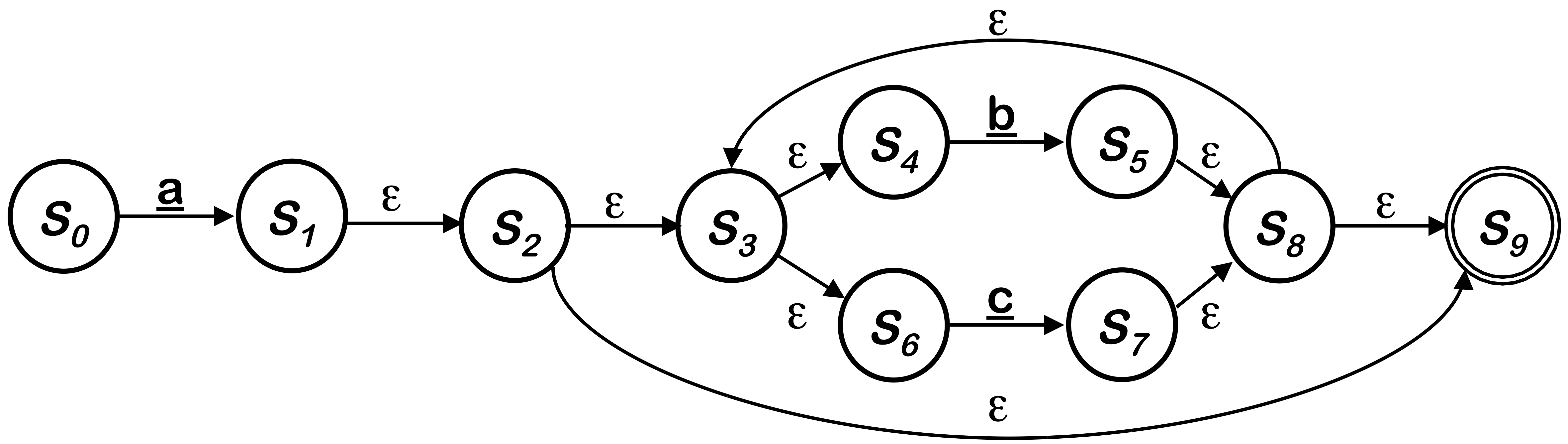


3. (b | c)*

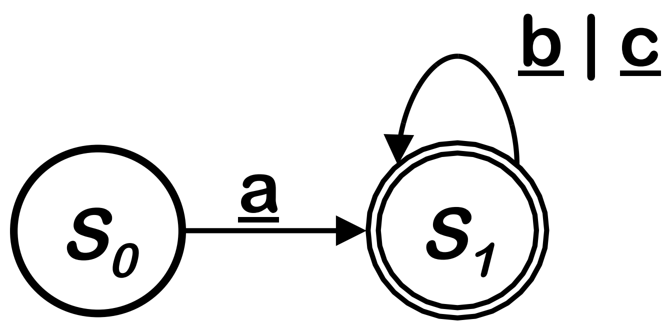


Example of Thompson's Construction *(con't)*

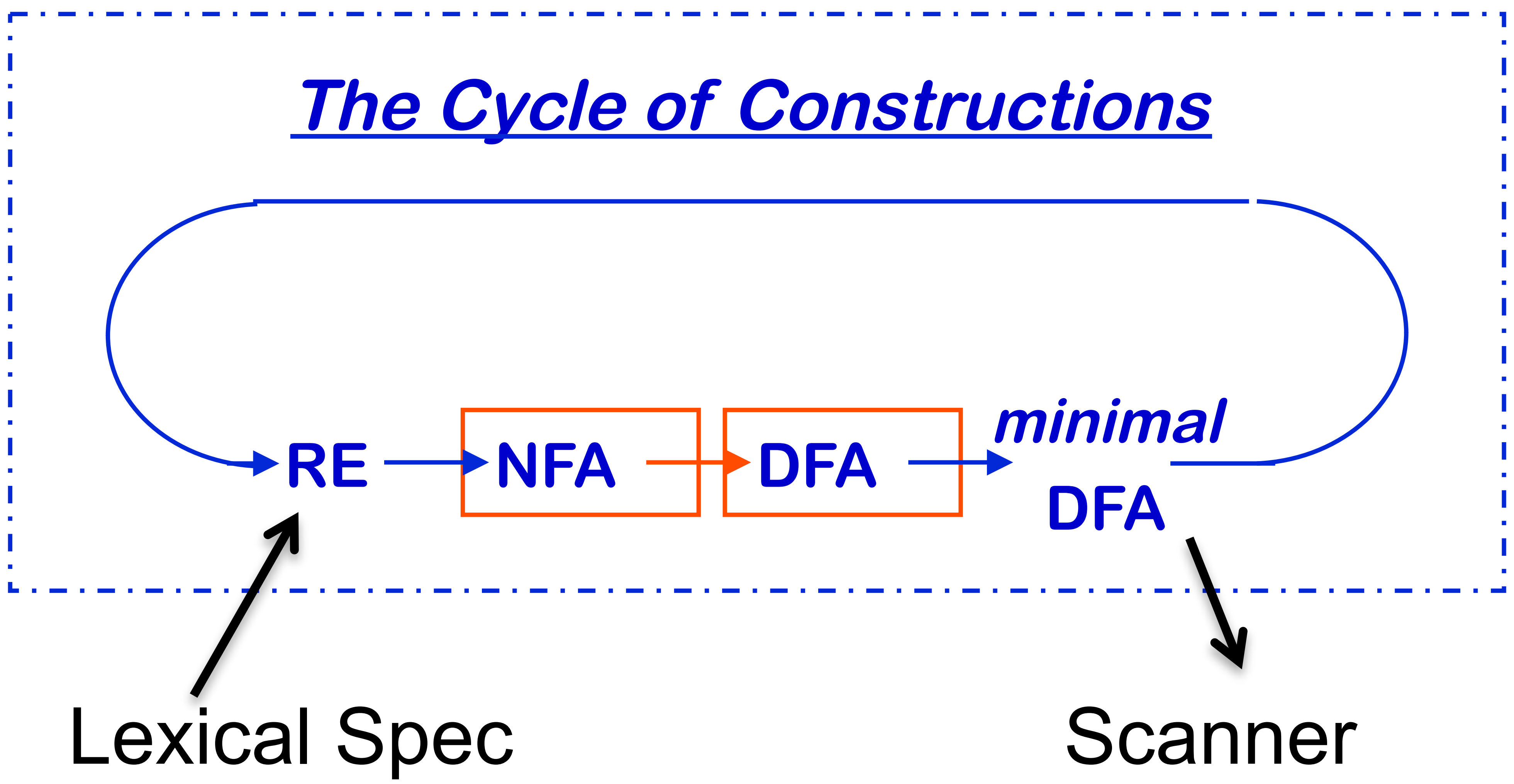
4. a (b | c)*



Of course, a human would design something simpler ...



But, we can automate production of the more complex one ...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NFA to DFA : Trick

- Simulate the NFA
- Each state of DFA

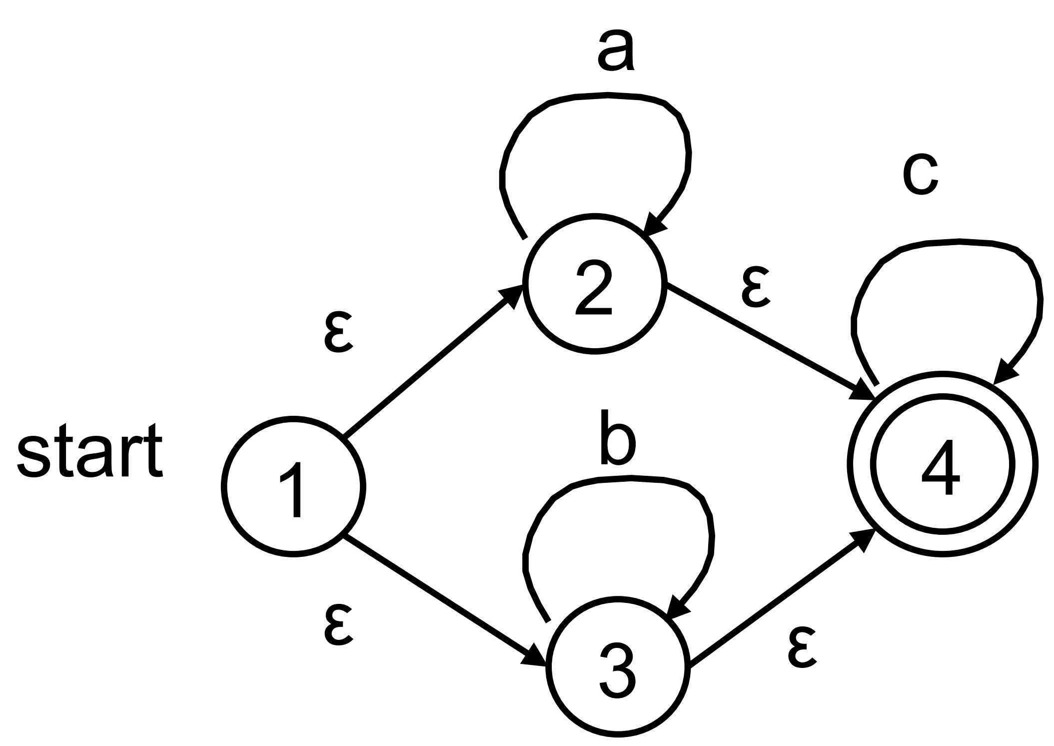
  = a non-empty subset of states of the NFA
- Start state

  = the set of NFA states reachable through e-moves from NFA start state
- Add a transition $S \rightarrow^{a} S'$ to DFA iff
  - S' is the set of NFA states reachable from any state in S after seeing the input a, considering ε-moves as well

NFA to DFA

- Remove the non-determinism
  – States with multiple outgoing edges due to same input
  – ε transitions

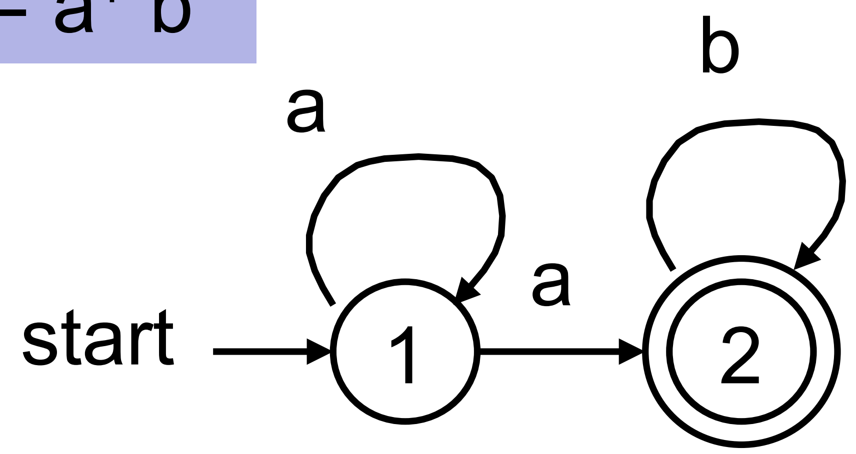
(a*| b*) c*



NFA to DFA (2)

- Multiple transitions
  - Solve by subset construction
  - Build new DFA based upon the set of states each representing a unique subset of states in NFA

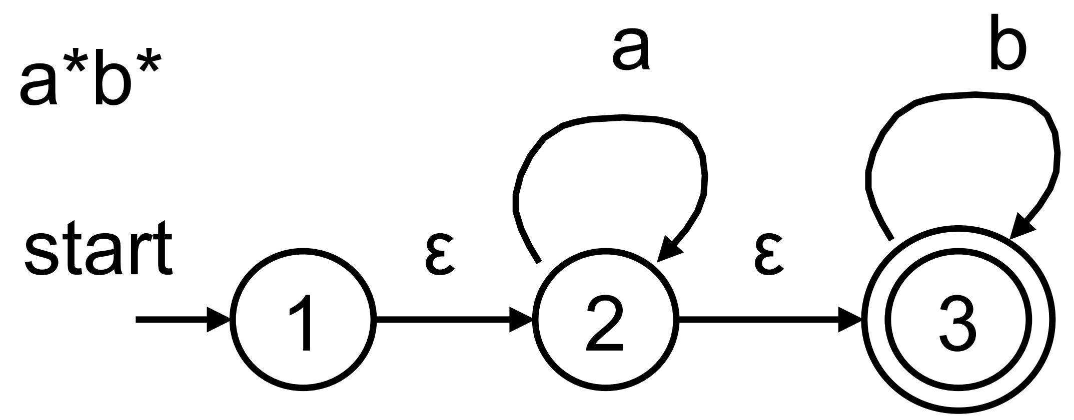
R= $a^+ b^*$



ε-closure(1) = {1} include state "1"
(1,a) → {1,2} include state "1/2"
(1,b) → ERROR

NFA to DFA (3)

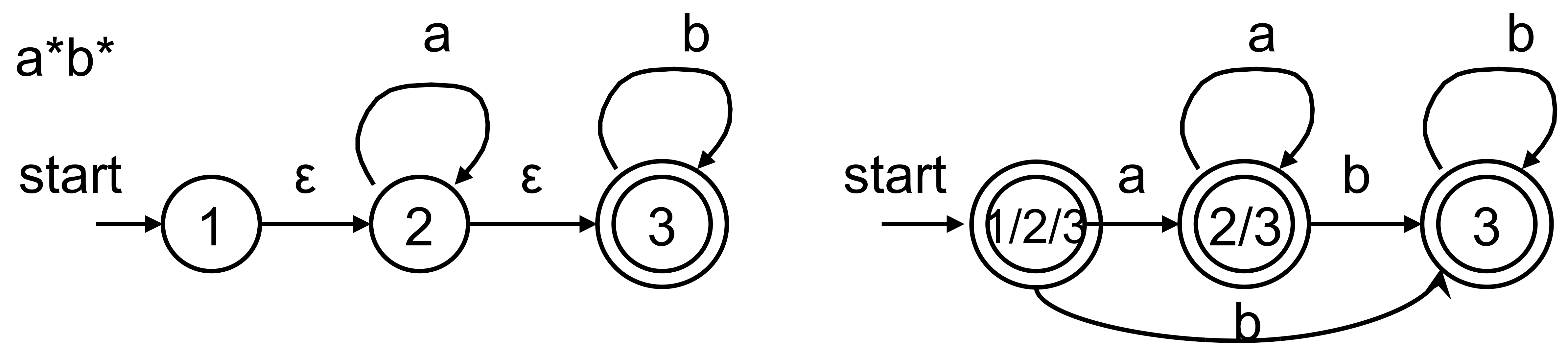
- ε transitions
  - Any state reachable by an ε transition is “part of the state”
  - ε-closure - Any state reachable from S by ε transitions is in the ε-closure; treat ε-closure as 1 big state, always include ε-closure as part of the state



1. ε-closure(1) = {1,2,3}; include1/2/3
2. Move(1/2/3, a) = {2, 3} + ε-closure(2,3) = {2,3} ; include 2/3
3. Move(1/2/3, b) = {3} + ε-closure(3) = {3} ; include state 3
4. Move(2/3, a) = {2} + ε-closure(2) = {2,3}
5. Move(2/3, b) = {3} + ε-closure(3) = {3}
6. Move(3, b) = {3} + ε-closure(3) = {3}

NFA to DFA (3)

- ε transitions
  - Any state reachable by an ε transition is “part of the state”
  - ε-closure - Any state reachable from S by ε transitions is in the ε-closure; treat ε-closure as 1 big state, always include ε-closure as part of the state



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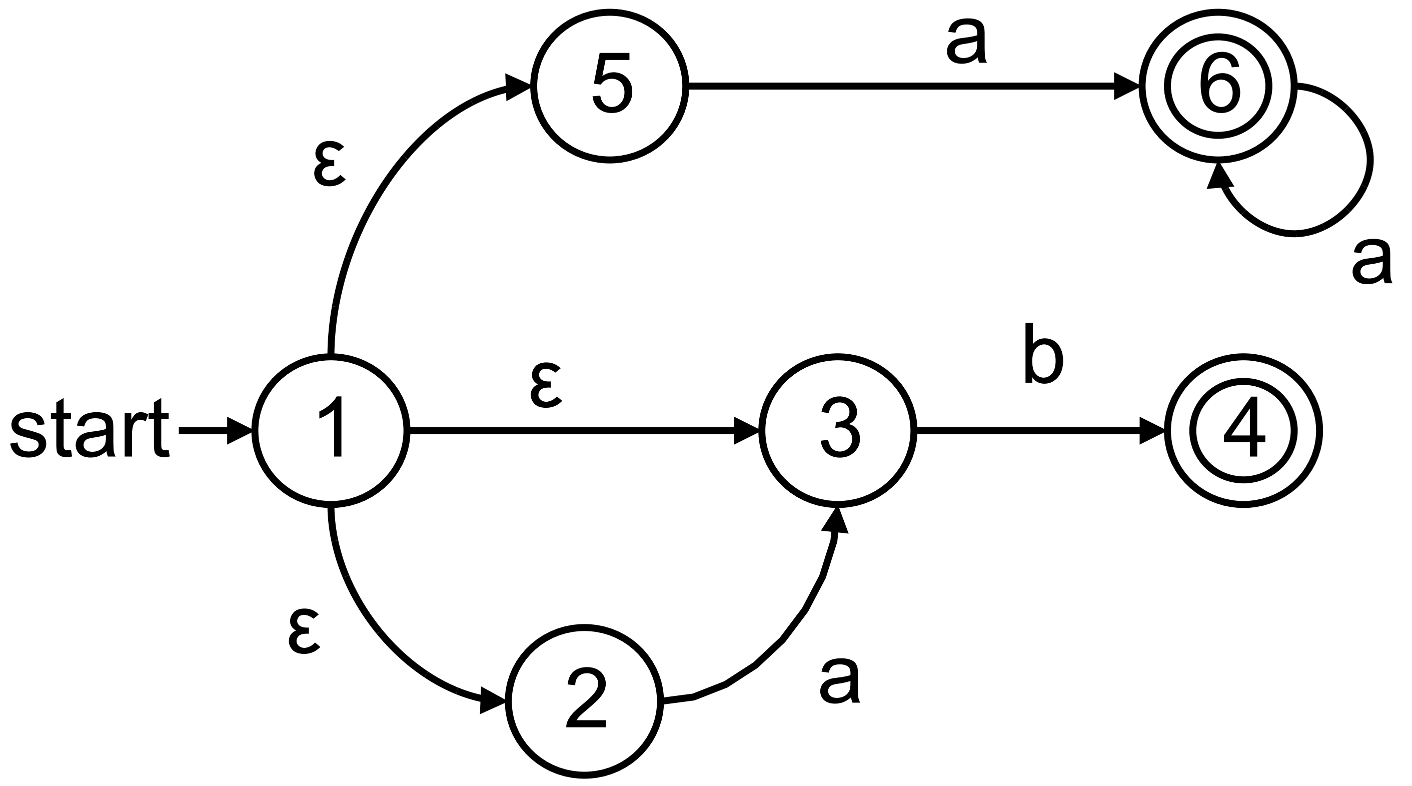
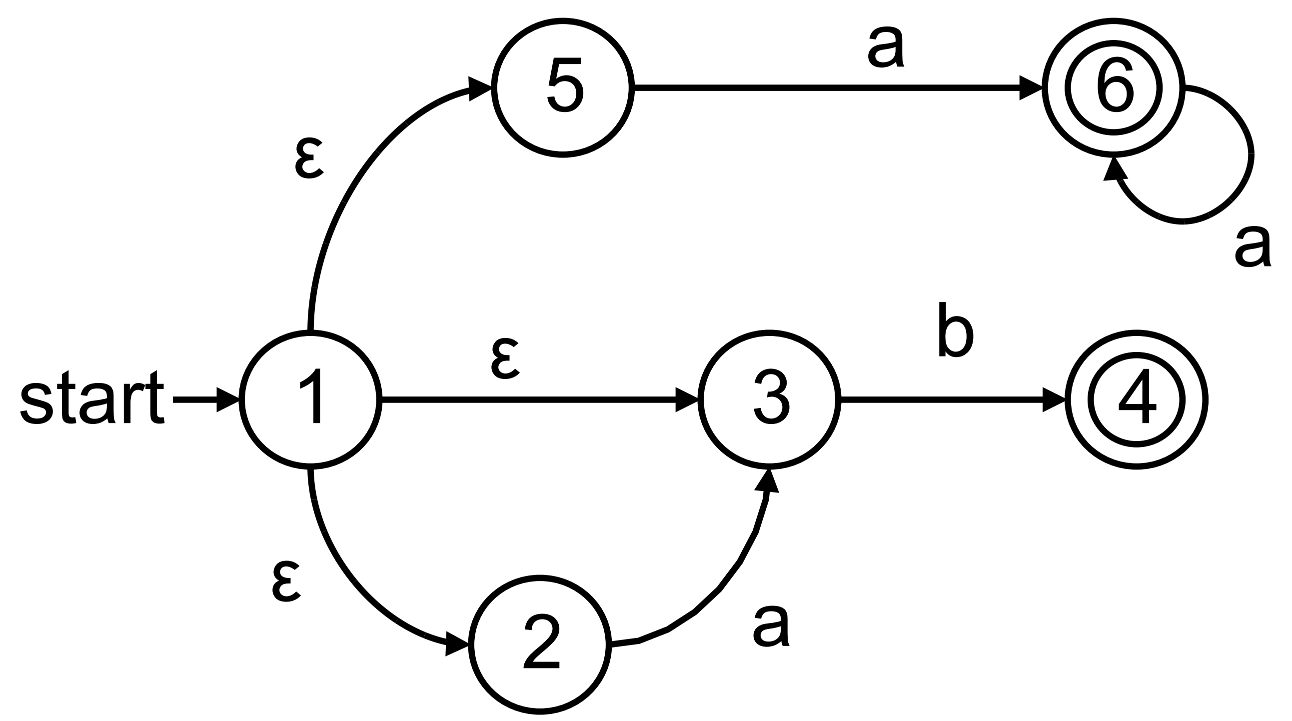
4. Move(2/3, a) = {2} + ε-closure(2) = {2,3}
5. Move(2/3, b) = {3} + ε-closure(3) = {3}
6. Move(3, b) = {3} + ε-closure(3) = {3}

NFA to DFA - Example



NFA to DFA - Example



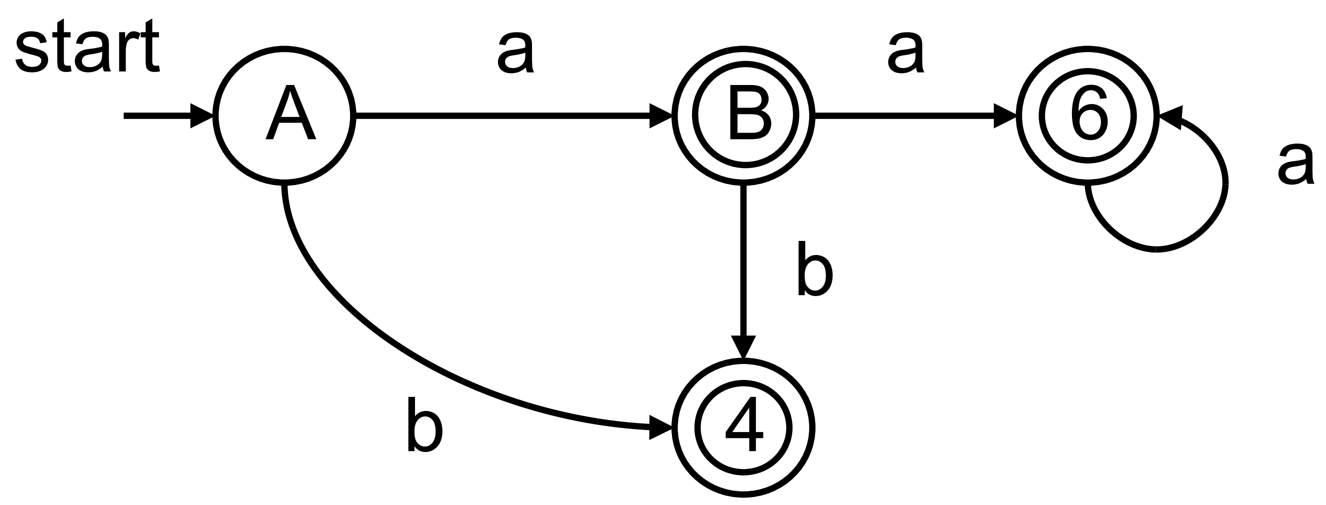
ε-closure(1) = {1, 2, 3, 5}

Create a new state A = {1, 2, 3, 5}

move(A, a) = {3, 6} + ε-closure(3,6) = {3,6}

Create B = {3,6}

move(A, b) = {4} + ε-closure(4) = {4}



move(B, a) = {6} + ε-closure(6) = {6}

move(B, b) = {4} + ε-closure(4) = {4}

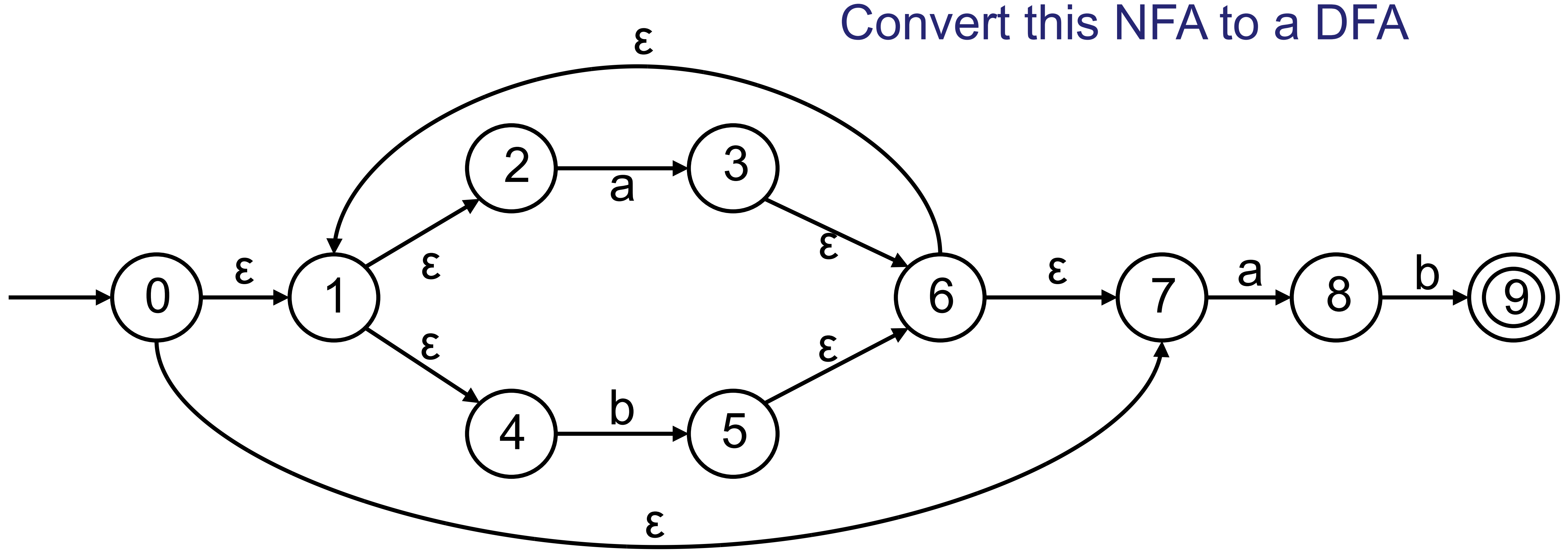
move(6, a) = {6} + ε-closure(6) = {6}

move(6, b) → ERROR

move(4, a|b) → ERROR

Class Problem

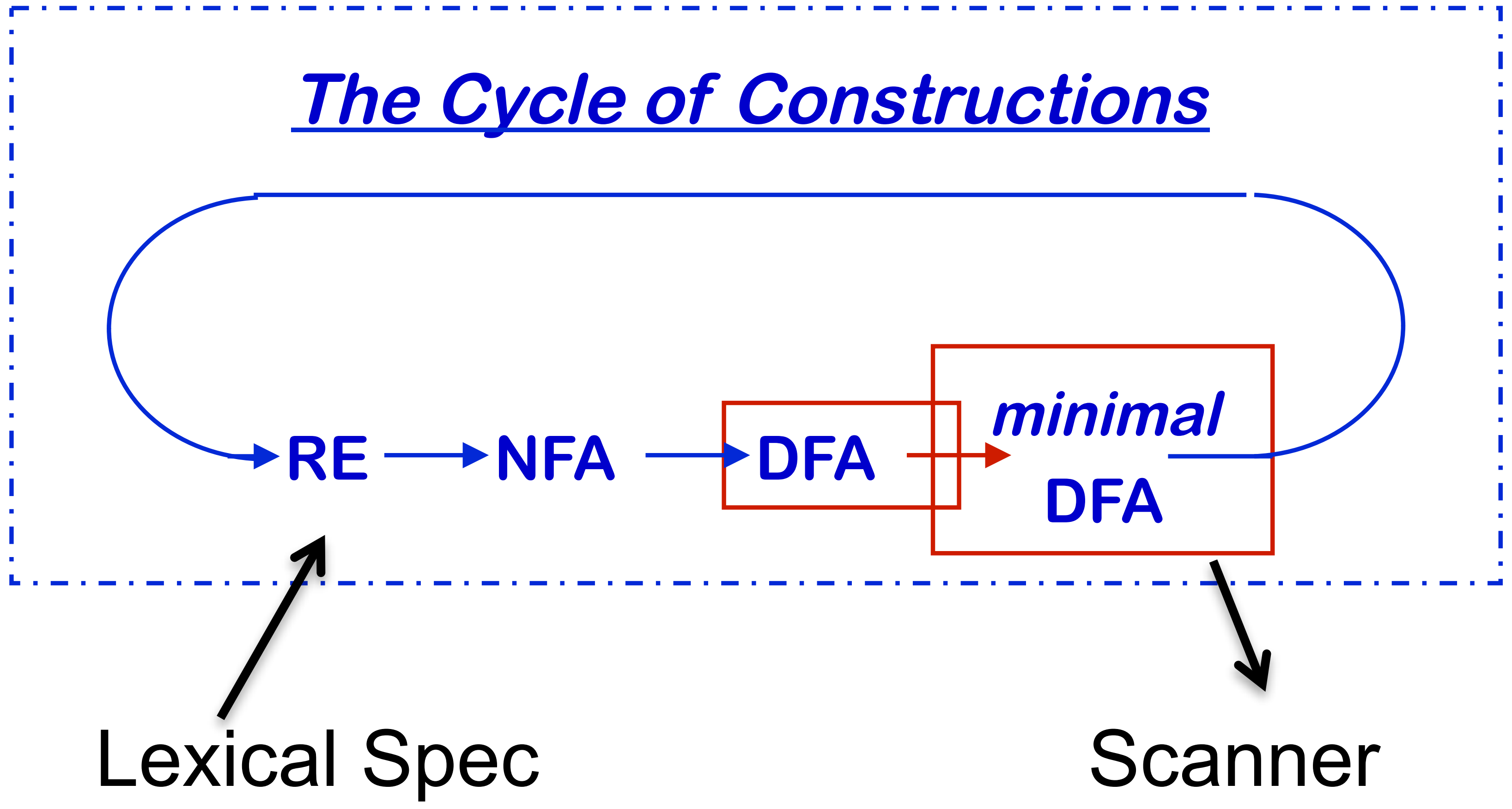
Convert this NFA to a DFA



NFA to DFA : cont..

- An NFA may be in many states at any time
- How many different states ?
- If there are N states, the NFA must be in some subset of those N states
- How many subsets are there?

  2^N - 1 = finitely many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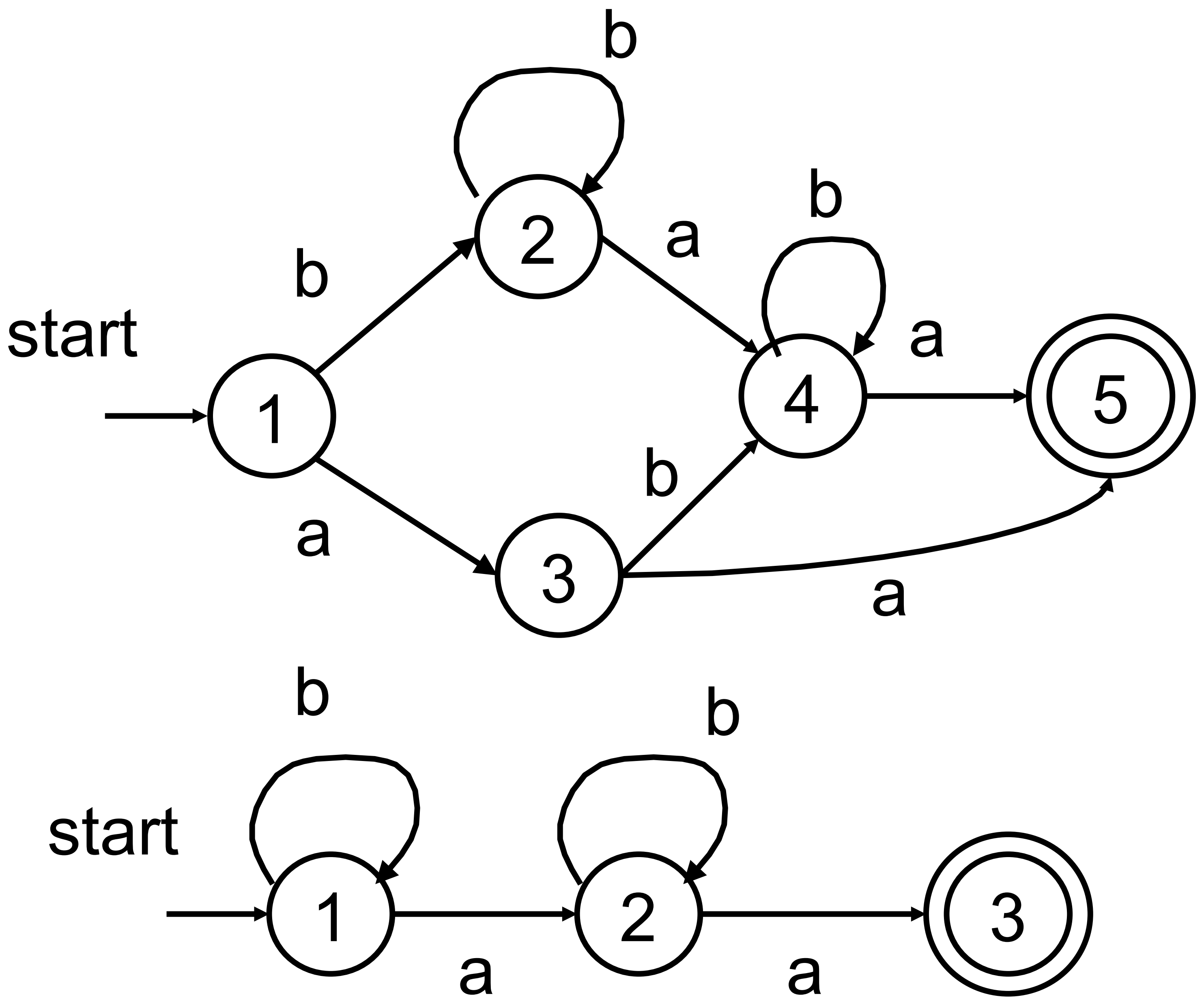
State Minimization

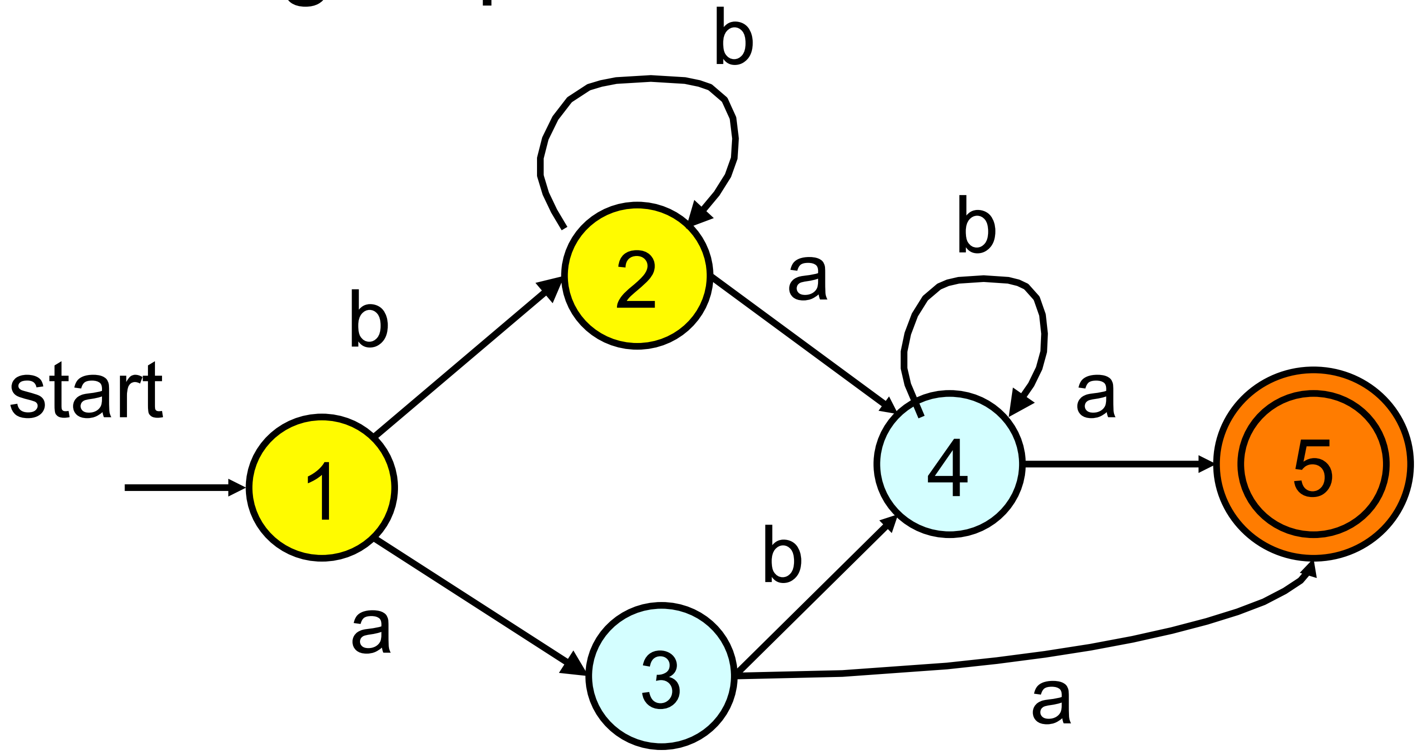
- Resulting DFA can be quite large
  - Contains redundant or equivalent states



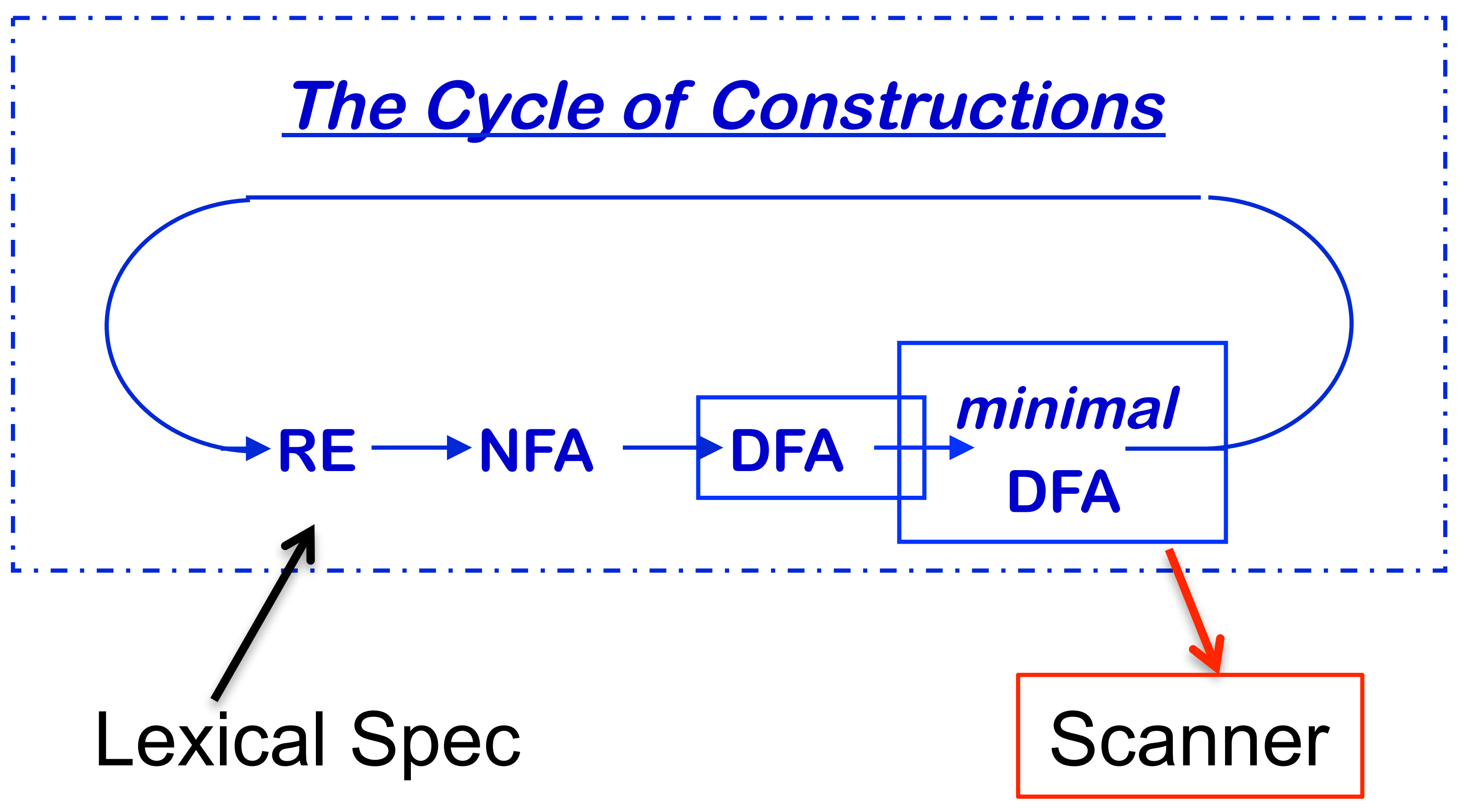
Both DFAs accept
b*ab*a

State Minimization (2)

- Idea – find groups of equivalent states and merge them
  - All transitions from states in group G1 go to states in another group G2
  - Construct minimized DFA such that there is 1 state for each group of states



Basic strategy: identify distinguishing transitions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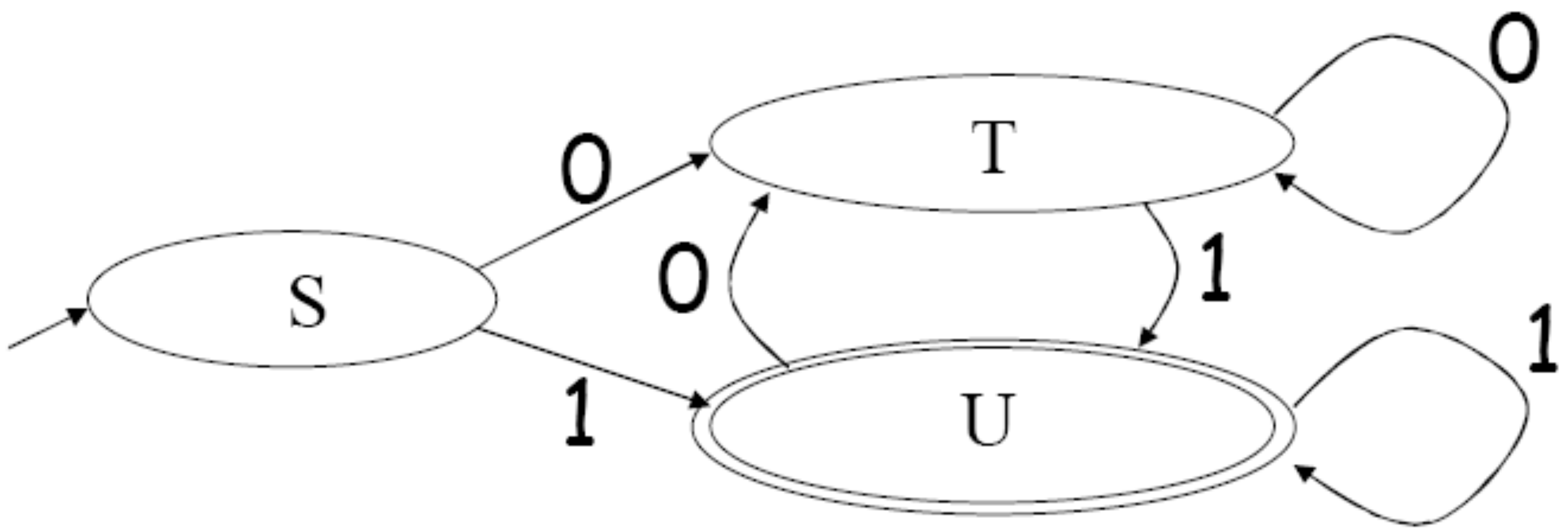
DFA Implementation

- A DFA can be implemented by a 2D table T
  - One dimension is “states”
  - Other dimension is “input symbol”
  - For every transition Si $\rightarrow^{a}$ Sk define T[i,a] = k
- DFA “execution”
  - If in state Si and input a, read T[i,a] = k and skip to state Sk
  - Very efficient

DFA Table Implementation : Example



	0	1
S	T	U
T	T	U
U	T	U

Implementation Cont ..

- NFA -> DFA conversion is at the heart of tools such as flex
- But, DFAs can be huge
- In practice, flex-like tools trade off speed for space in the choice of NFA and DFA representations

Lexer Generator

- Given regular expressions to describe the language (token types),
  - Step 1: Generates NFA that can recognize the regular language defined
    - existing algorithms
  - Step 2: Transforms NFA to DFA
    - existing algorithms
  - Tools: **lex, flex** for C

Challenges for Lexical Analyzer

- How do we determine which lexemes are associated with each token?
  - Regular expression to describe token type
- When there are multiple ways we could scan the input, how do we know which one to pick?
- How do we address these concerns efficiently?

Lexing Ambiguities

```
T_For            for
T_Identifier     [A-Za-z_][A-Za-z0-9_]*
```

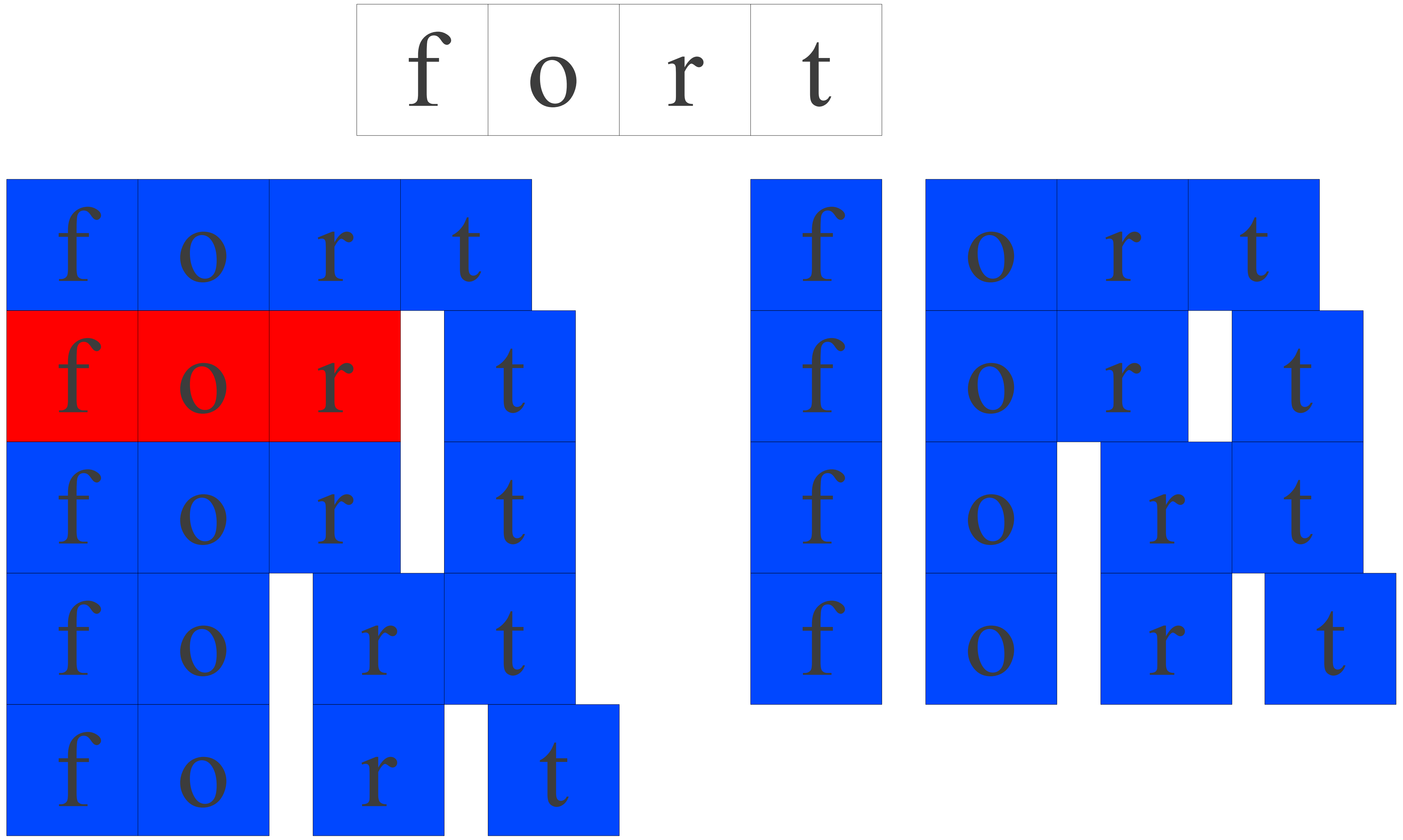
Lexing Ambiguities

T_For	for
T_Identifier	[A-Za-z_][A-Za-z0-9_]*

f	o	r	t

Lexing Ambiguities

T_For	for
T_Identifier	[A-Za-z_][A-Za-z0-9_]*



Conflict Resolution

- Assume all tokens are specified as regular expressions.
- Algorithm: **Left-to-right scan**.
- Tiebreaking rule one: **Maximal munch**.
  - Always match the longest possible prefix of the remaining text.

Lexing Ambiguities

T_For	for
T_Identifier	[A-Za-z_][A-Za-z0-9_]*

f	o	r	t

f	o	r	t

Implementing Maximal Munch

- Given a set of regular expressions, how can we use them to implement maximum munch?

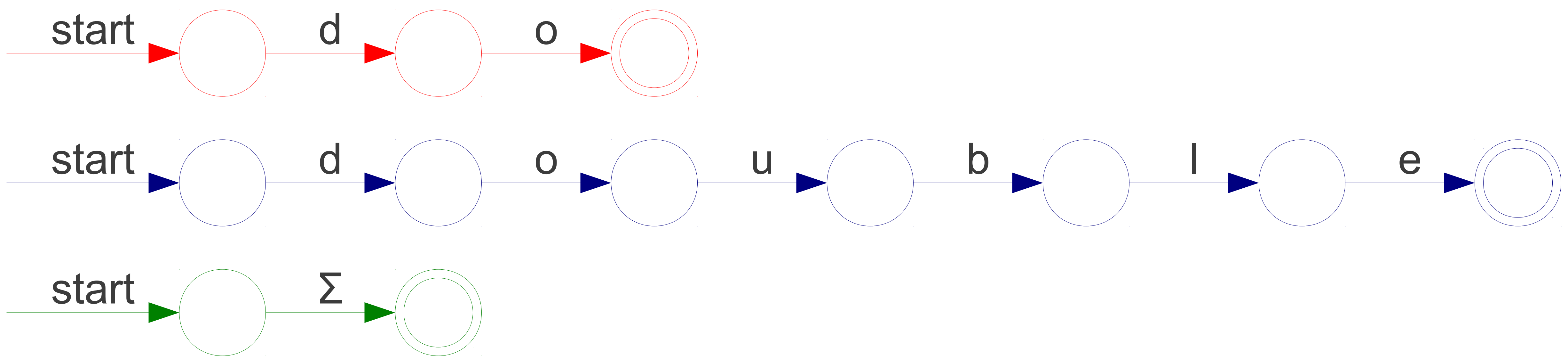
- Example

Implementing Maximal Munch

```
T_Do         do
T_Double     double
T_Mystery    [A-Za-z]
```

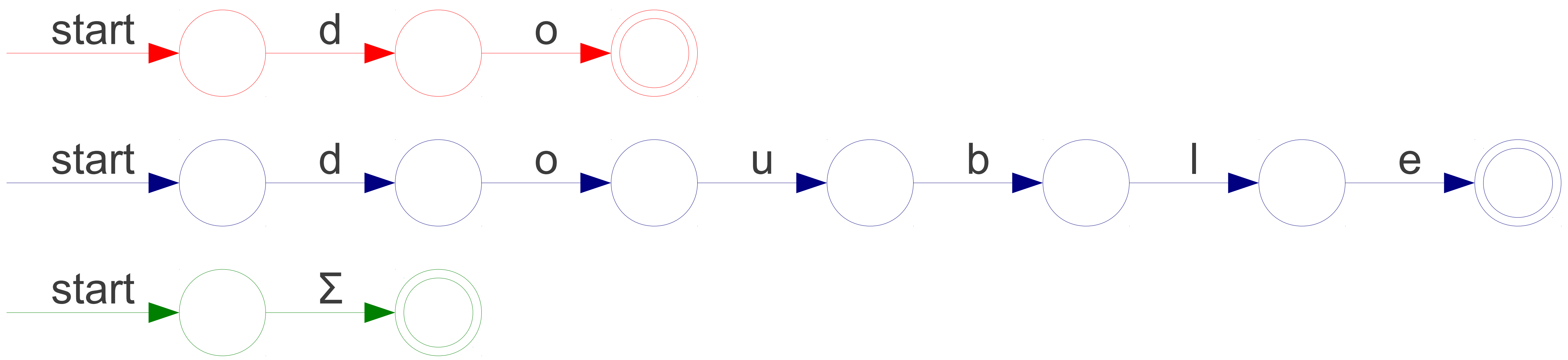
Implementing Maximal Munch

```
T_Do          do
T_Double      double
T_Mystery     [A-Za-z]
```



Implementing Maximal Munch

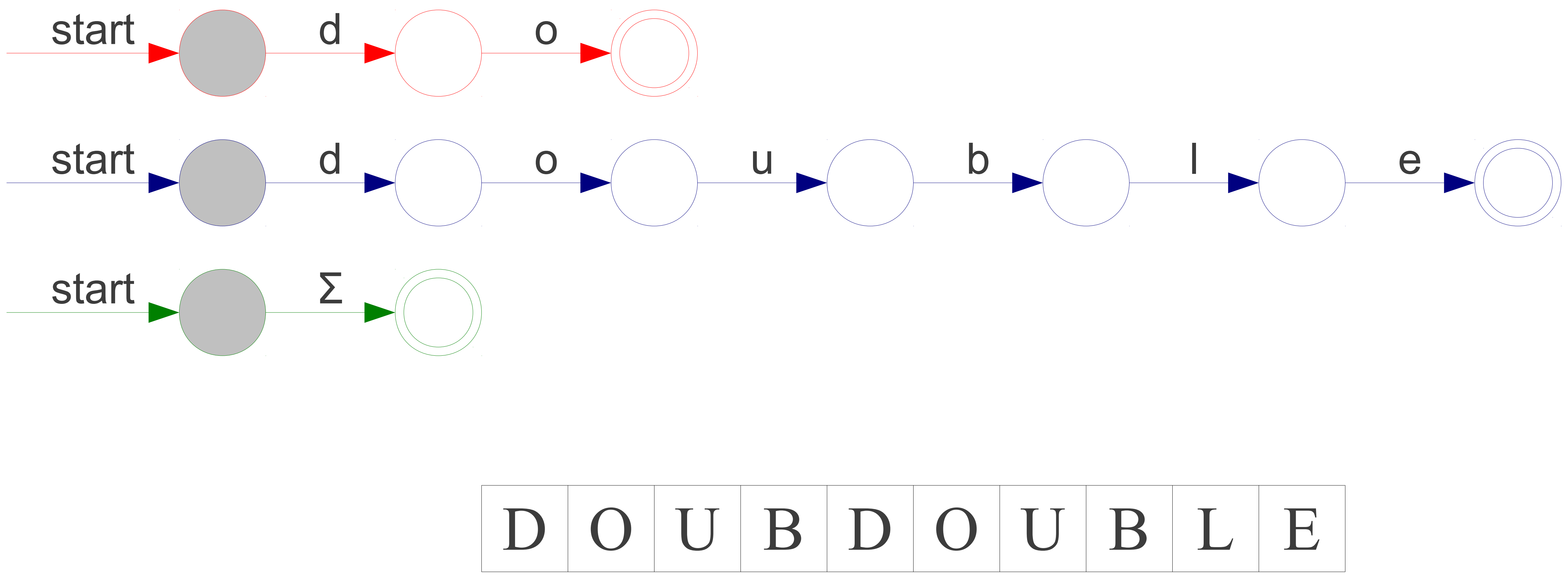
T_Do	do
T_Double	double
T_Mystery	[A-Za-z]



D	O	U	B	D	O	U	B	L	E

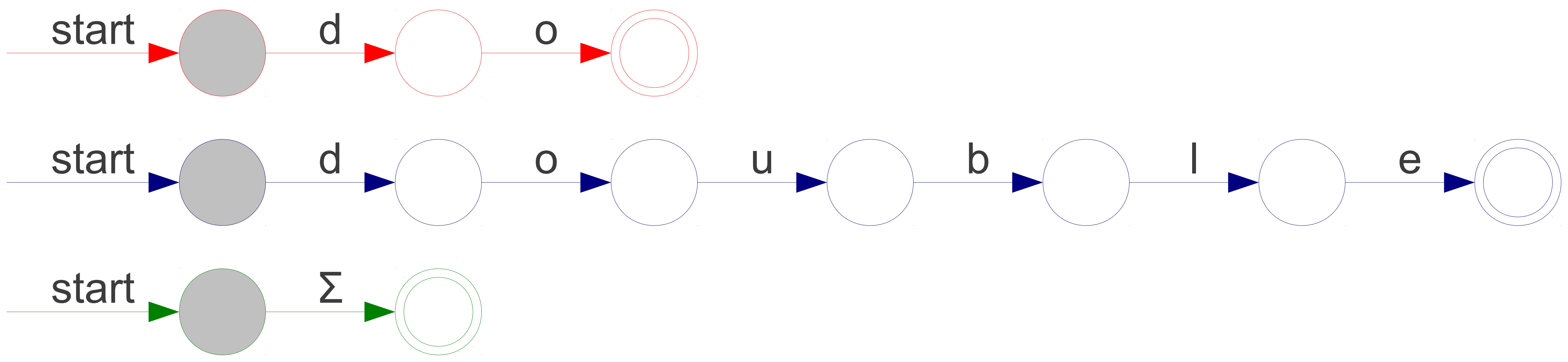
Implementing Maximal Munch

T_Do	do
T_Double	double
T_Mystery	[A-Za-z]



Implementing Maximal Munch

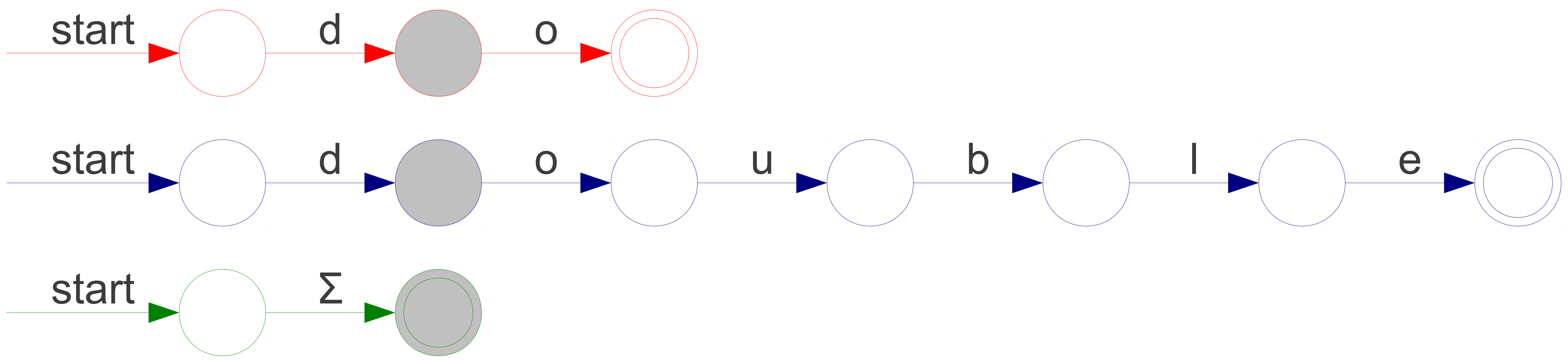
T_Do	do
T_Double	double
T_Mystery	[A-Za-z]

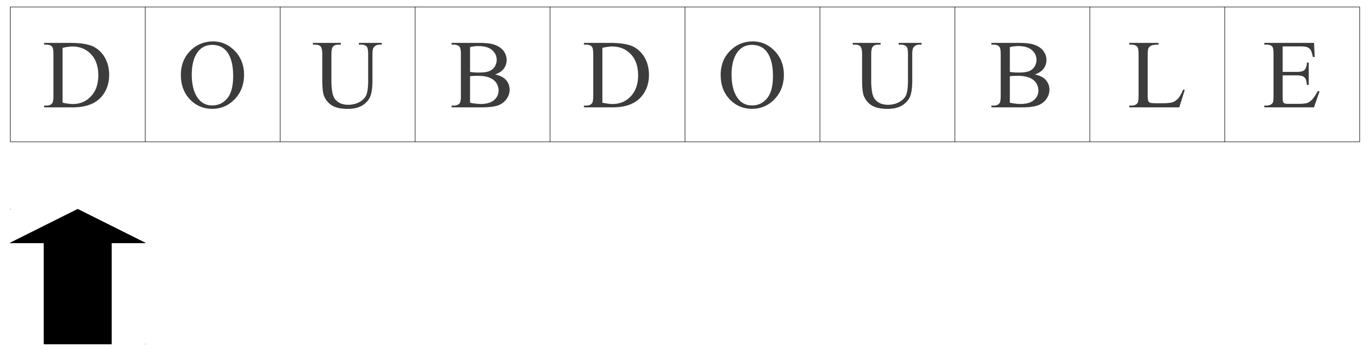


D	O	U	B	D	O	U	B	L	E

Implementing Maximal Munch

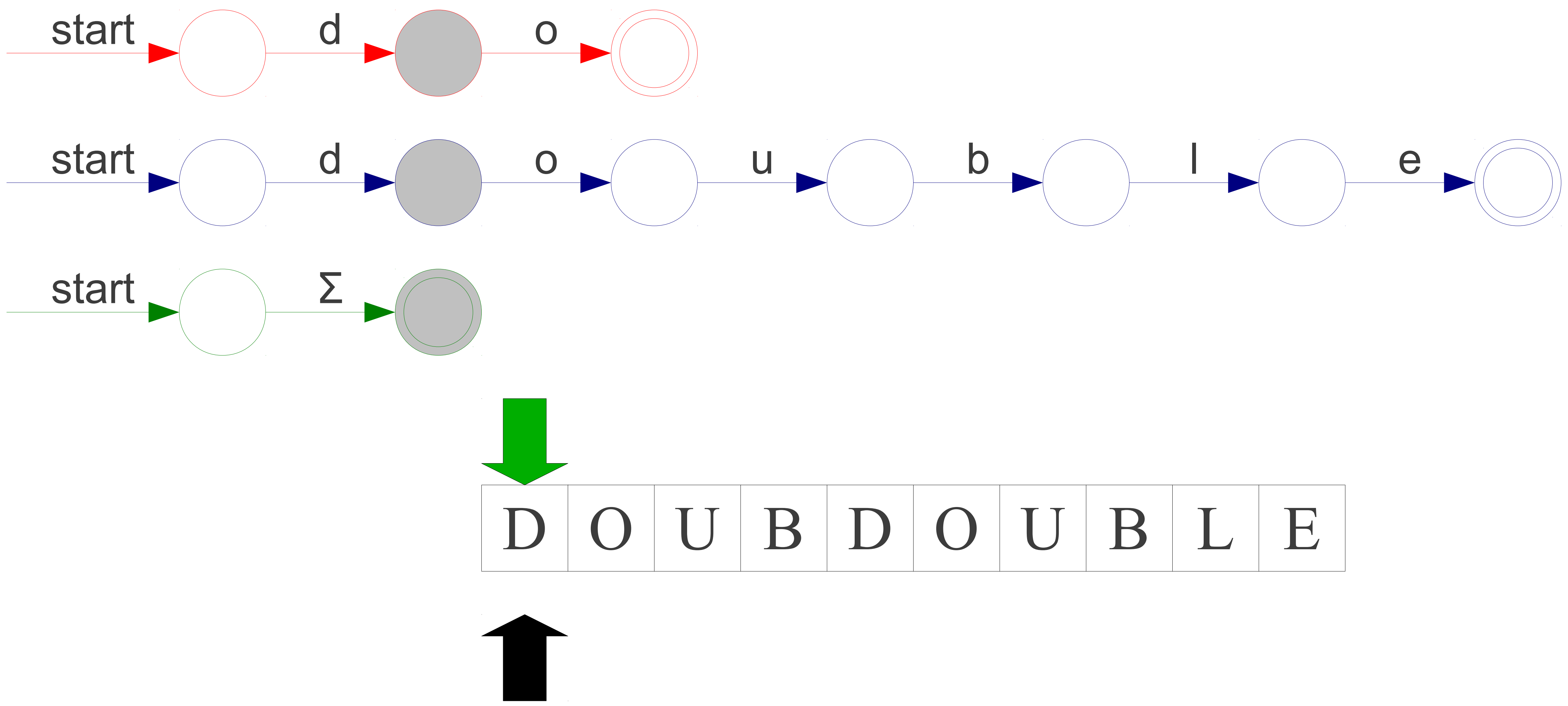
T_Do	do
T_Double	double
T_Mystery	[A-Za-z]





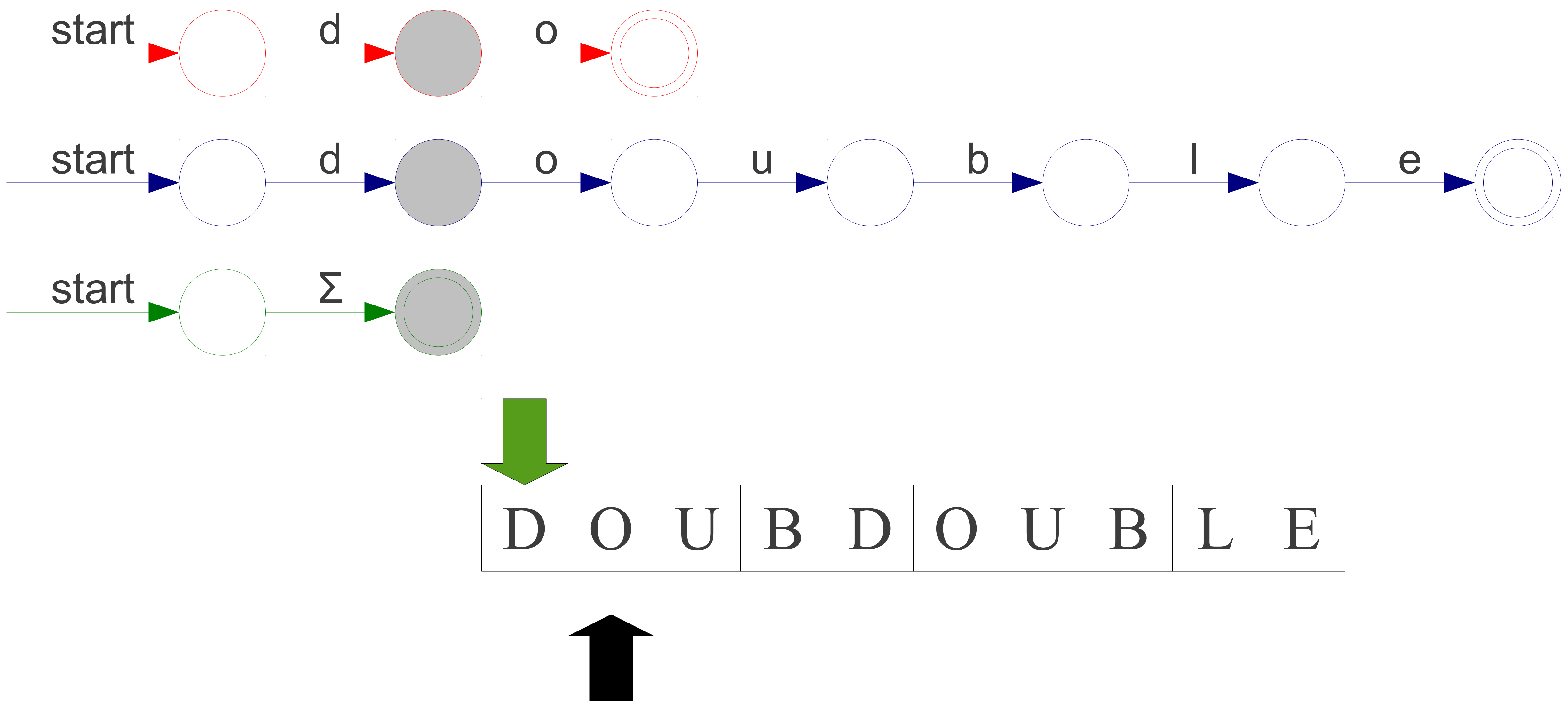
Implementing Maximal Munch

T_Do	do
T_Double	double
T_Mystery	[A-Za-z]



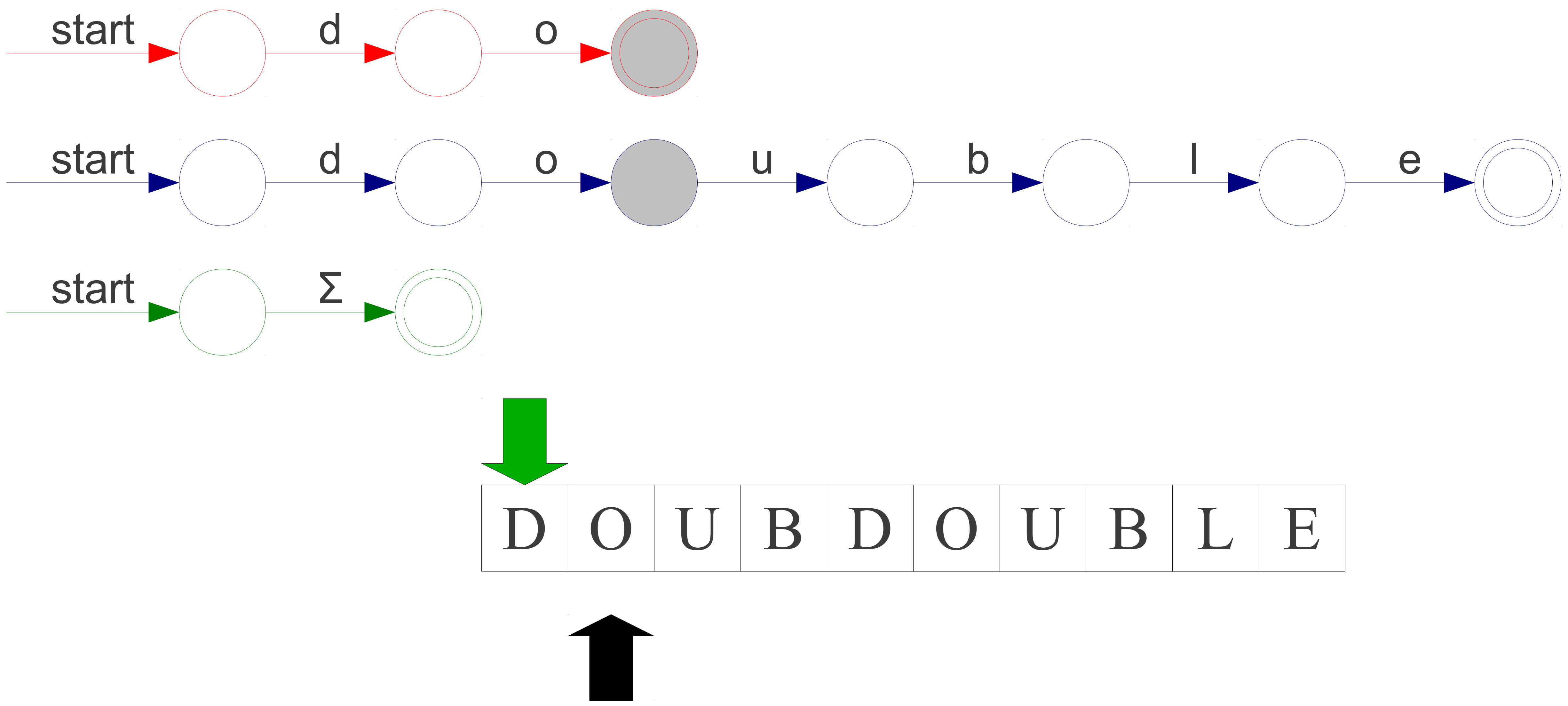
Implementing Maximal Munch

T_Do	do
T_Double	double
T_Mystery	[A-Za-z]



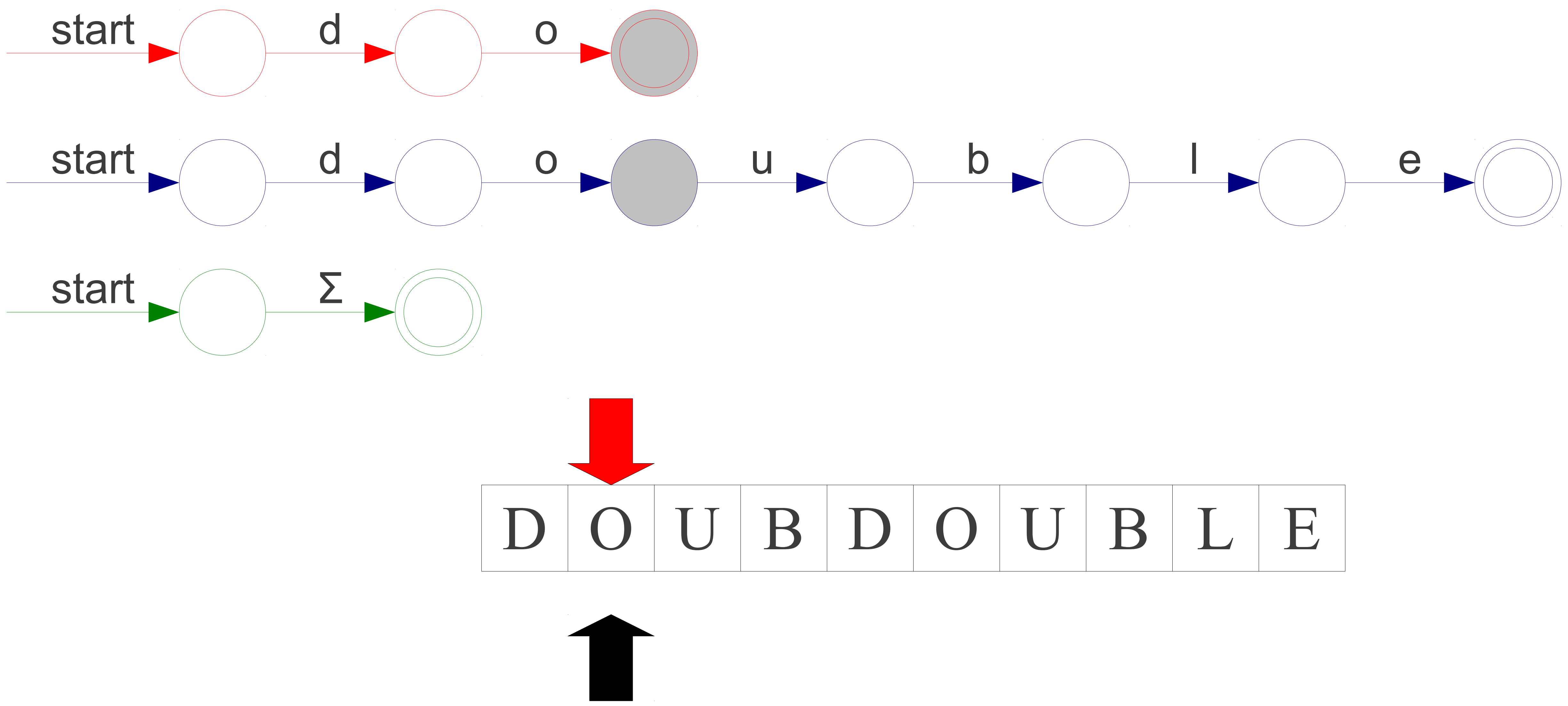
Implementing Maximal Munch

T_Do	do
T_Double	double
T_Mystery	[A-Za-z]



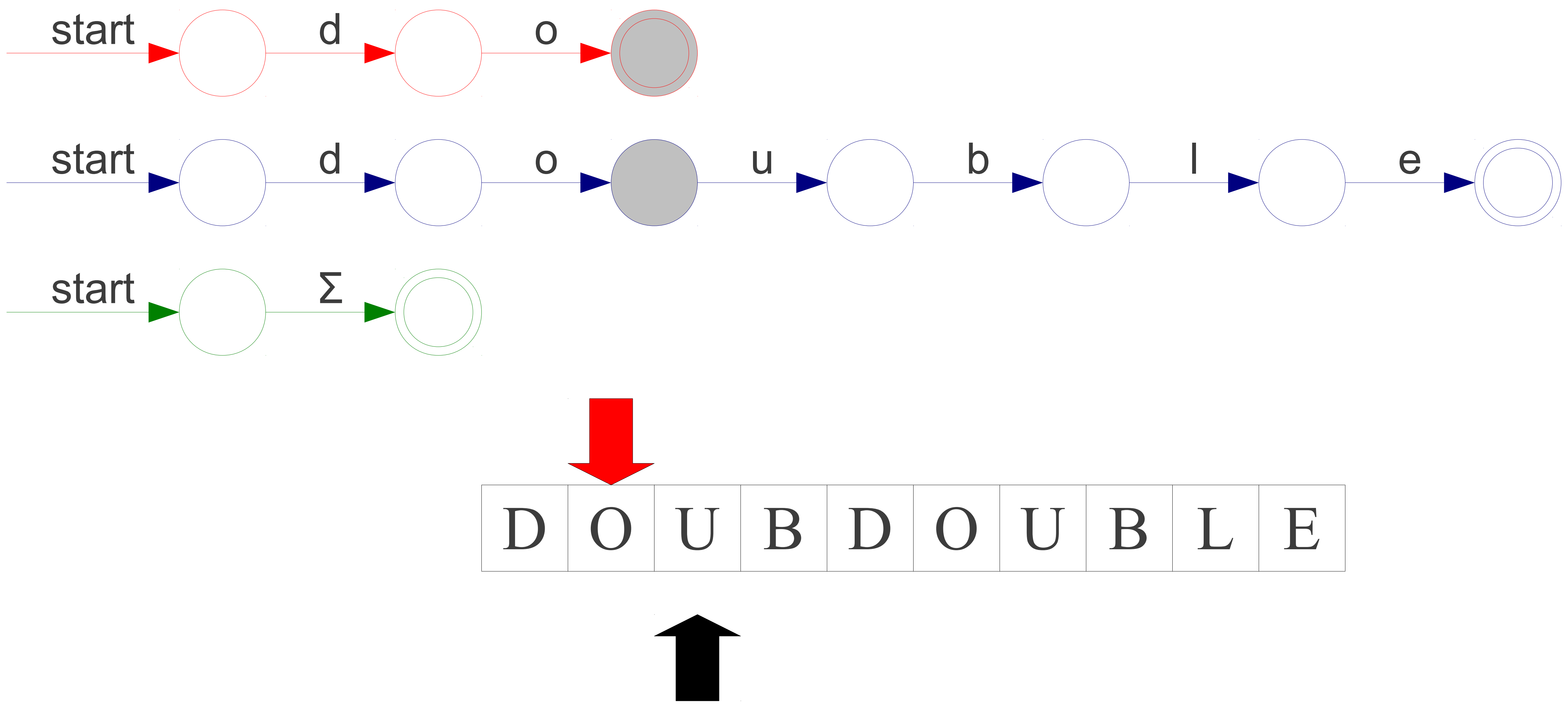
Implementing Maximal Munch

T_Do	do
T_Double	double
T_Mystery	[A-Za-z]



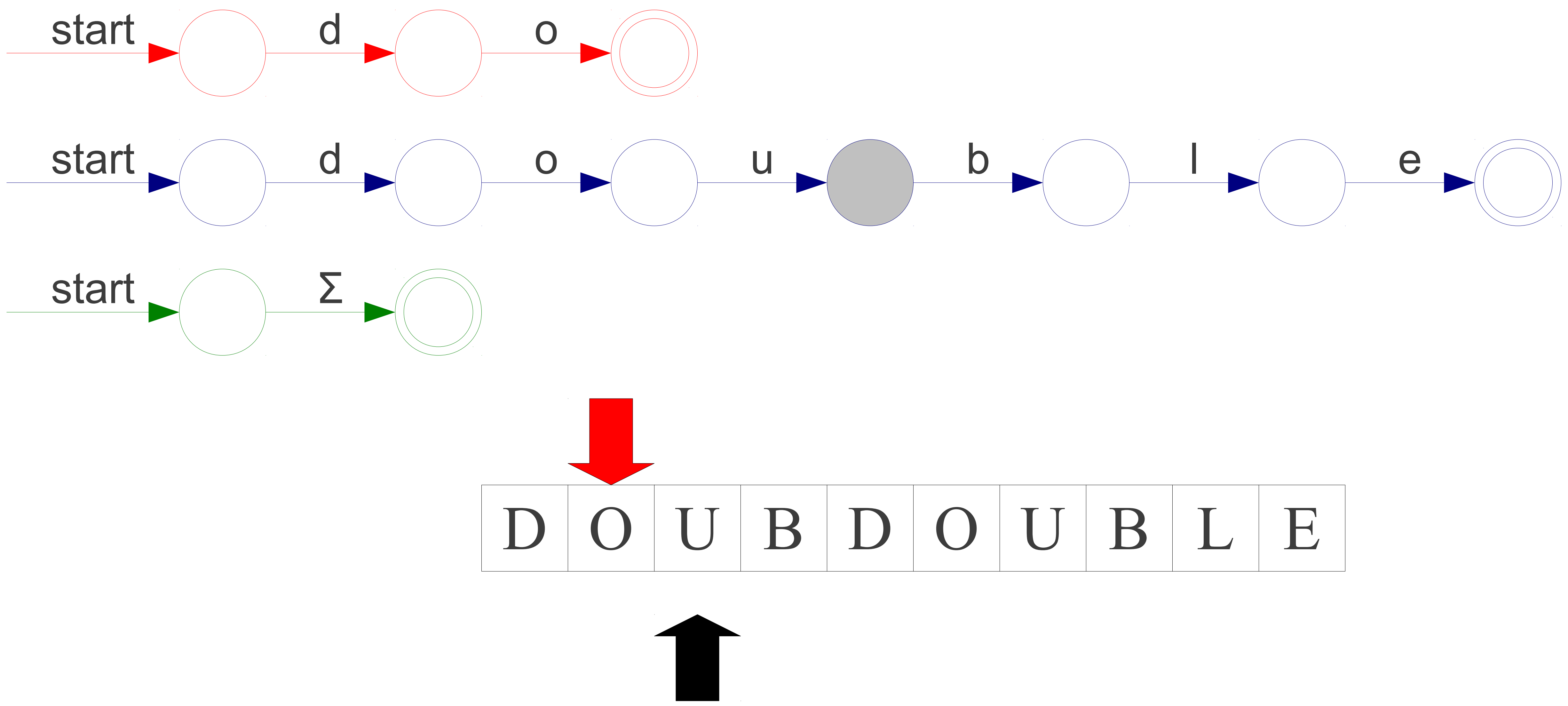
Implementing Maximal Munch

T_Do   do
T_Double   double
T_Mystery   [A-Za-z]



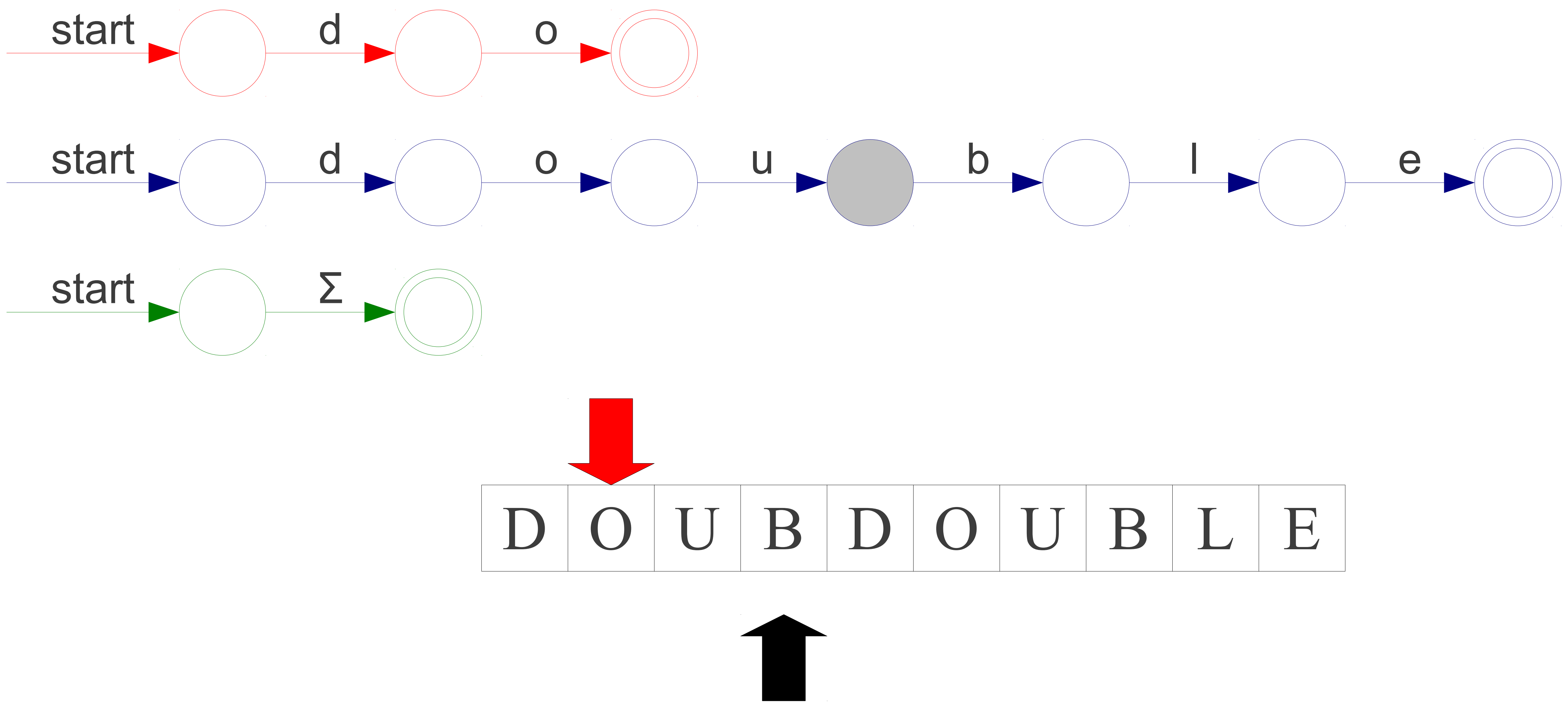
Implementing Maximal Munch

```
T_Do          do
T_Double      double
T_Mystery     [A-Za-z]
```



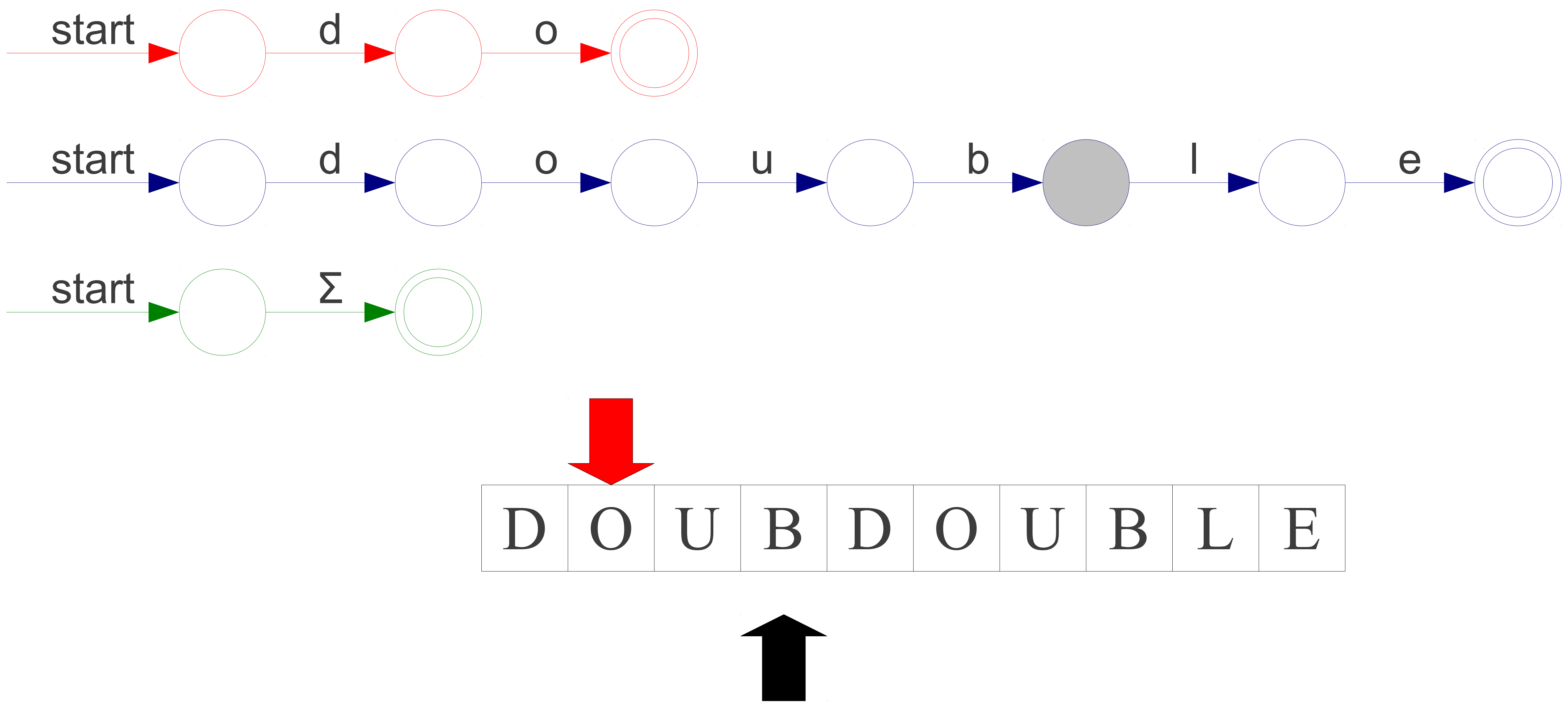
Implementing Maximal Munch

T_Do	do
T_Double	double
T_Mystery	[A-Za-z]



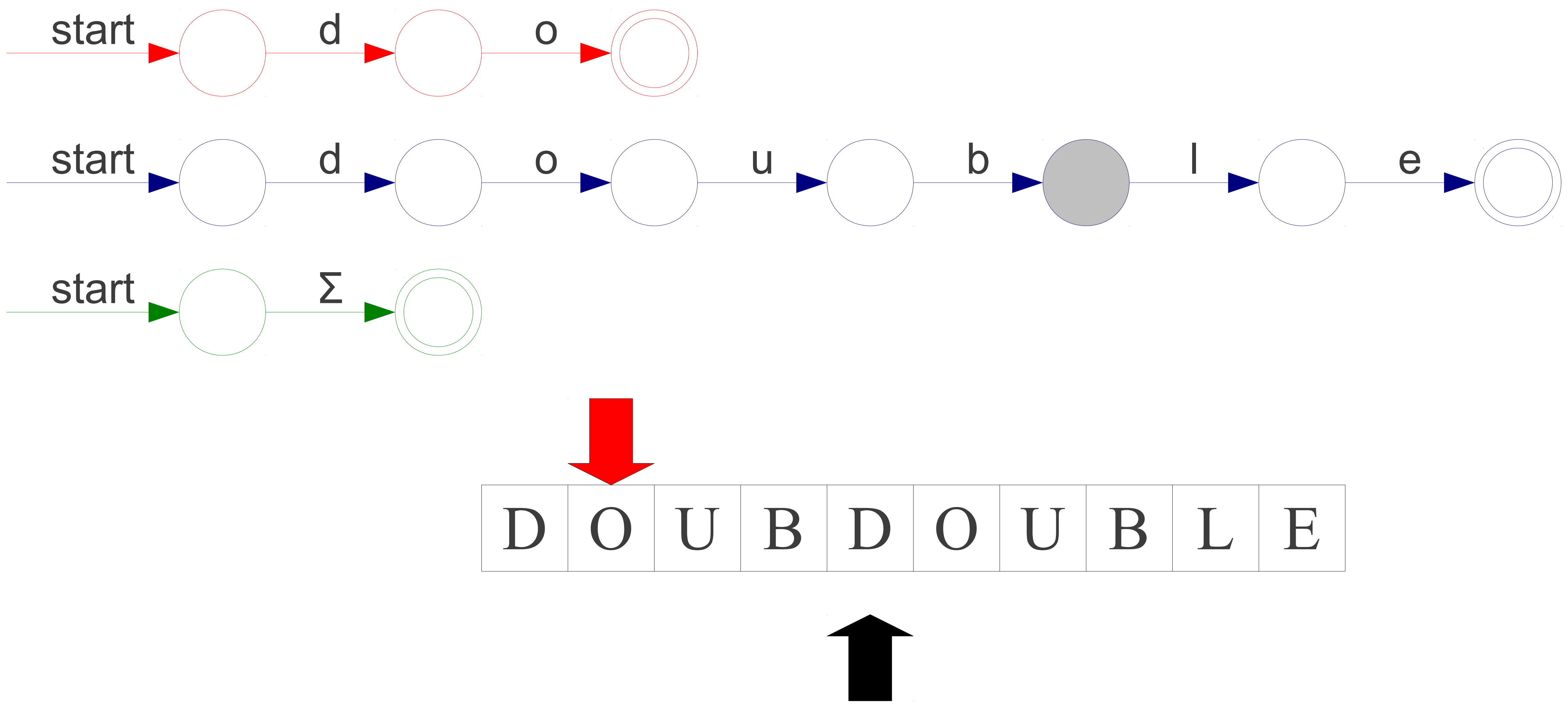
Implementing Maximal Munch

T_Do	do
T_Double	double
T_Mystery	[A-Za-z]



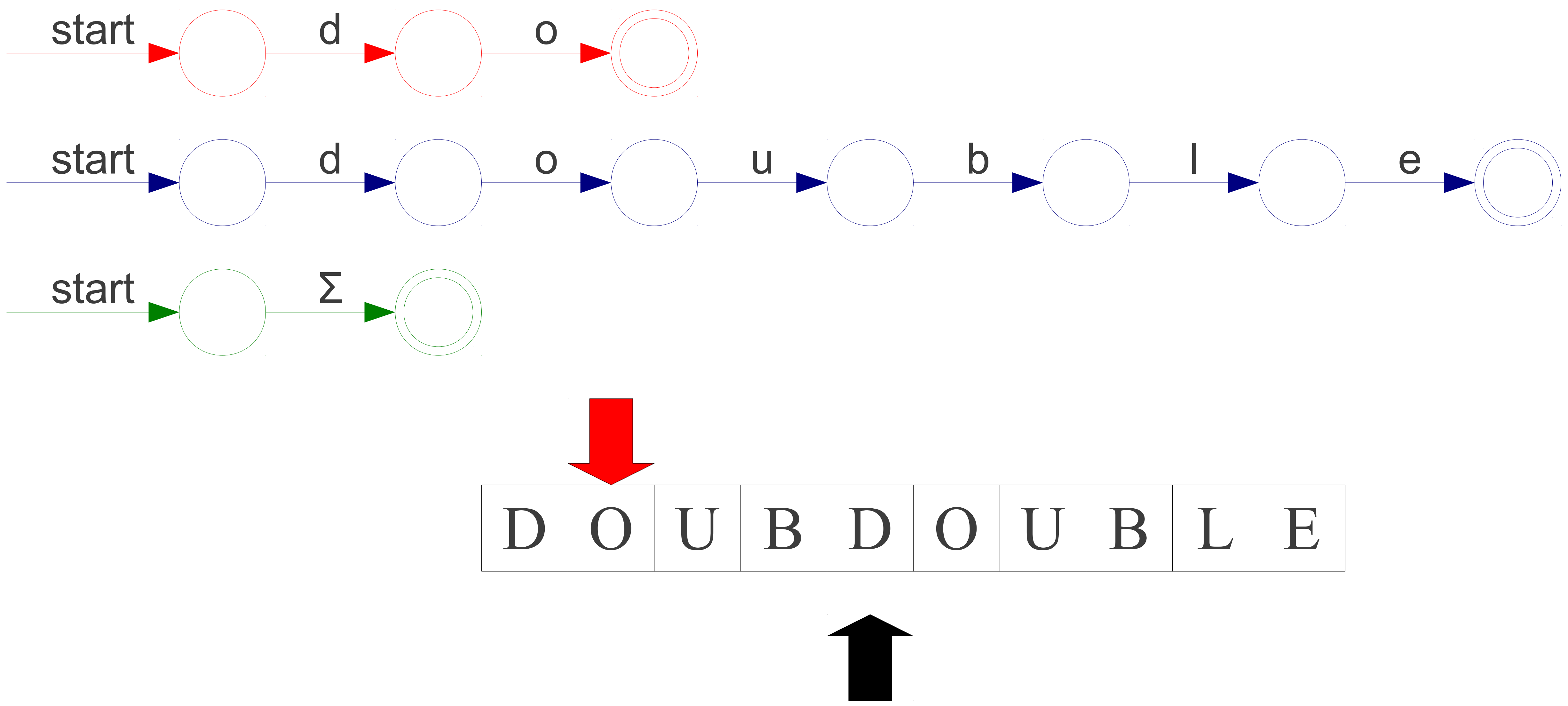
Implementing Maximal Munch

T_Do	do
T_Double	double
T_Mystery	[A-Za-z]



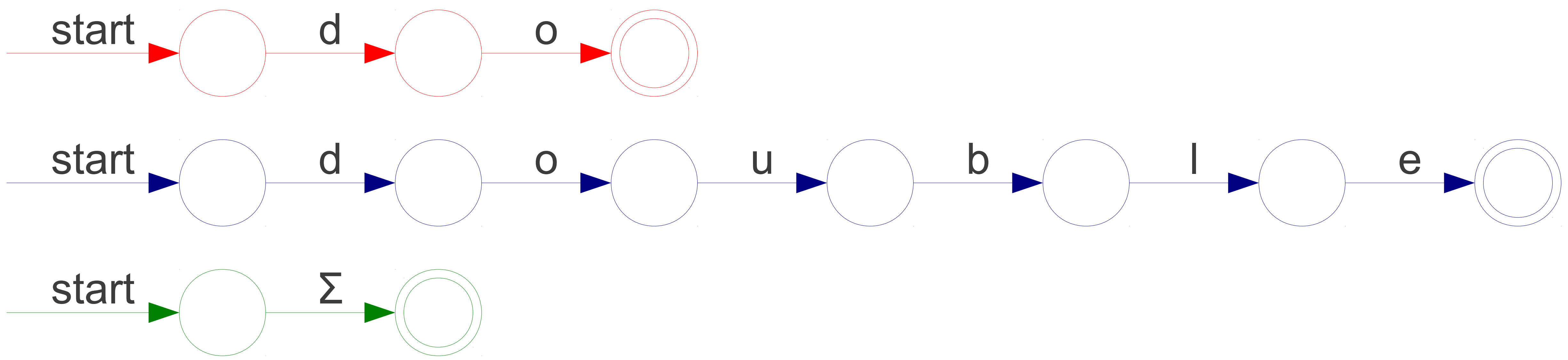
Implementing Maximal Munch

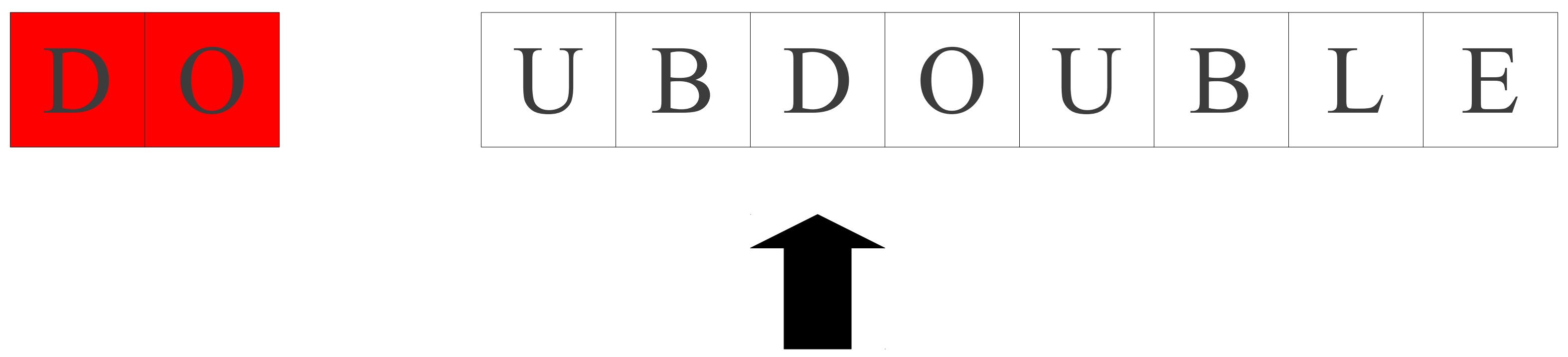
T_Do	do
T_Double	double
T_Mystery	[A-Za-z]



Implementing Maximal Munch

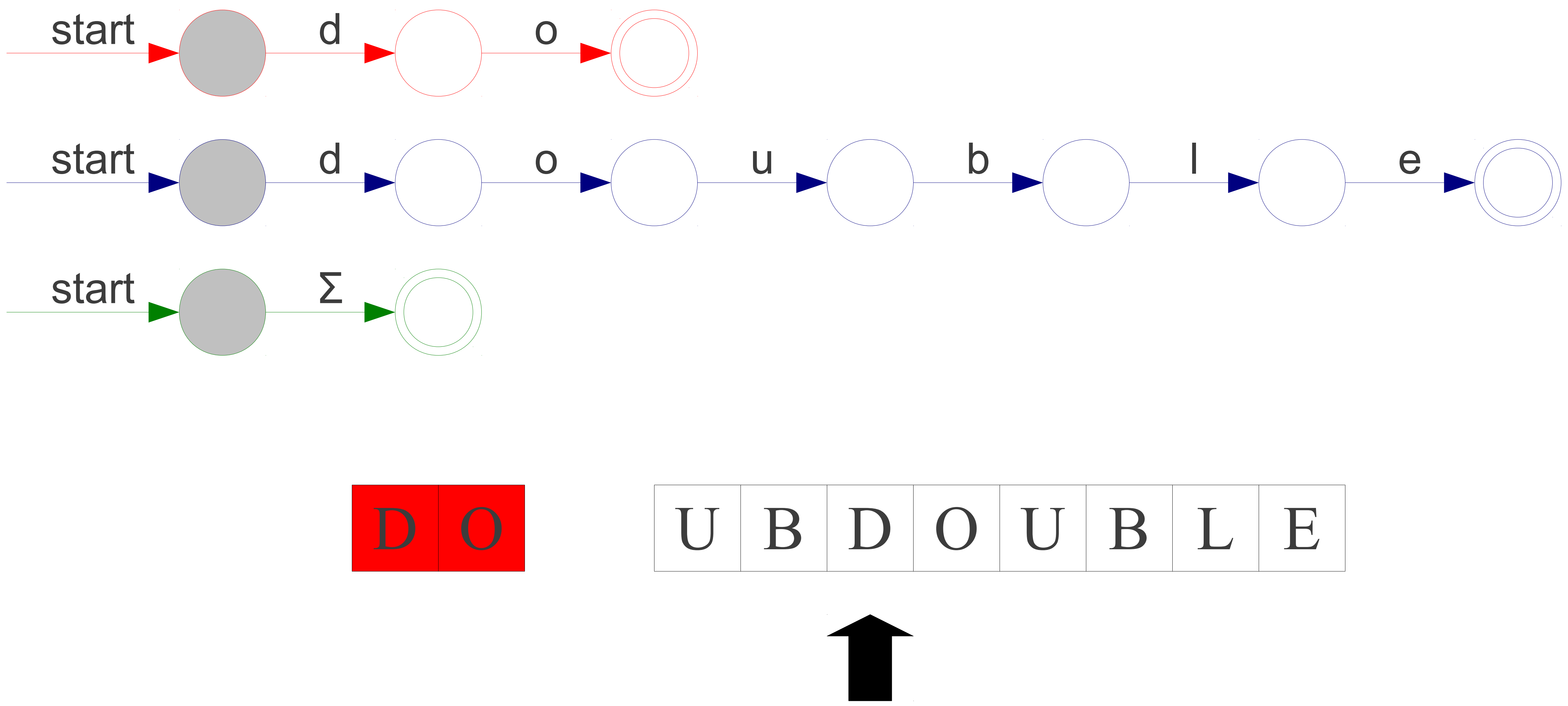
T_Do	do
T_Double	double
T_Mystery	[A-Za-z]





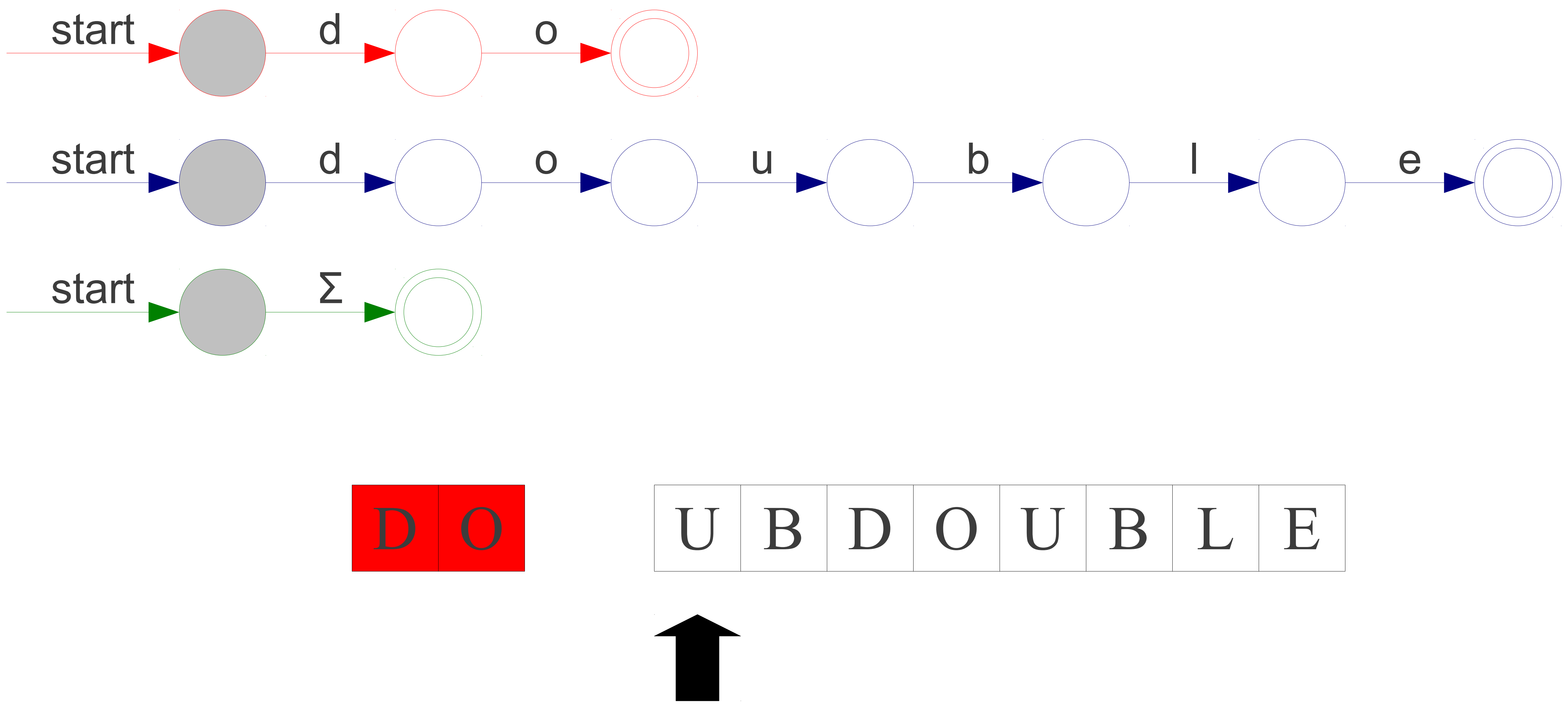
Implementing Maximal Munch

T_Do	do
T_Double	double
T_Mystery	[A-Za-z]



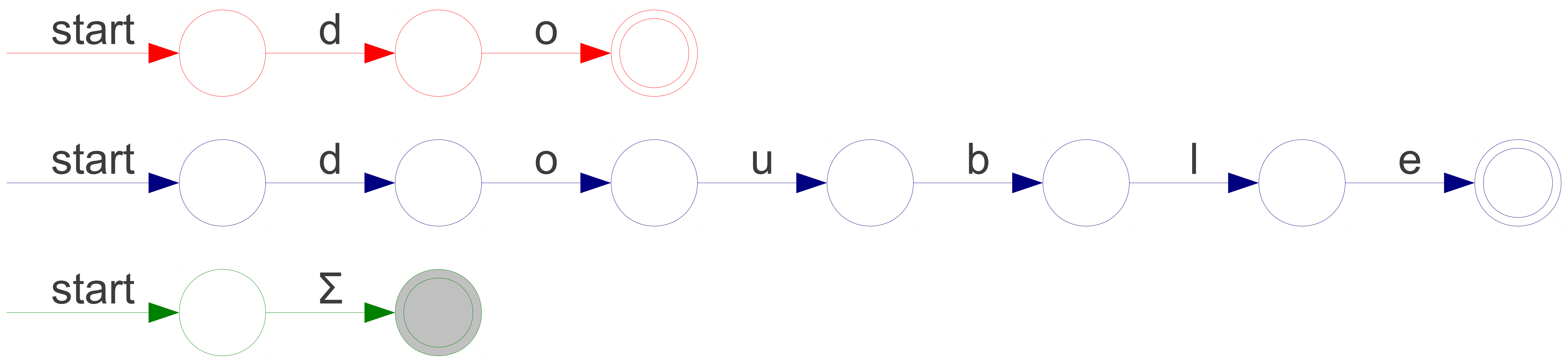
Implementing Maximal Munch

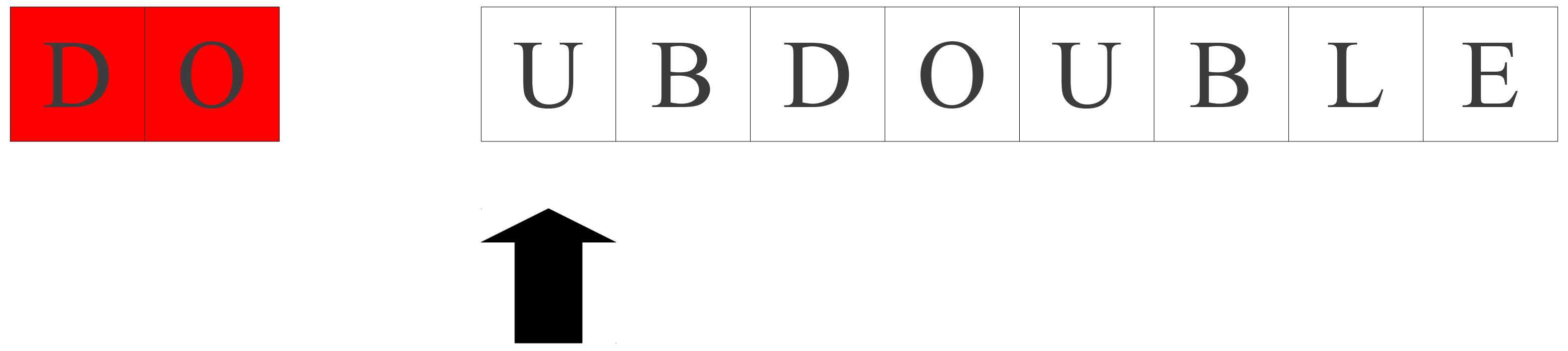
```
T_Do          do
T_Double      double
T_Mystery     [A-Za-z]
```



Implementing Maximal Munch

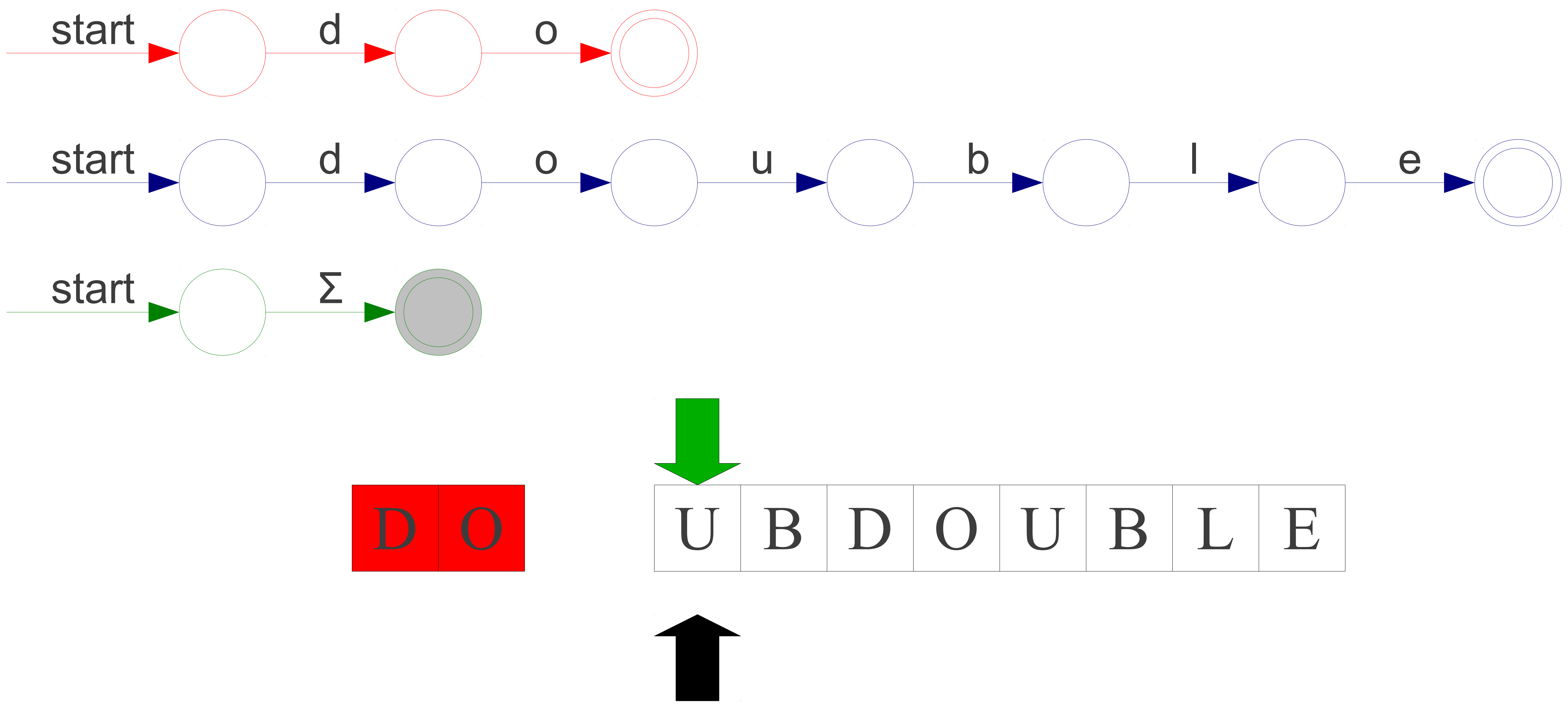
T_Do	do
T_Double	double
T_Mystery	[A-Za-z]





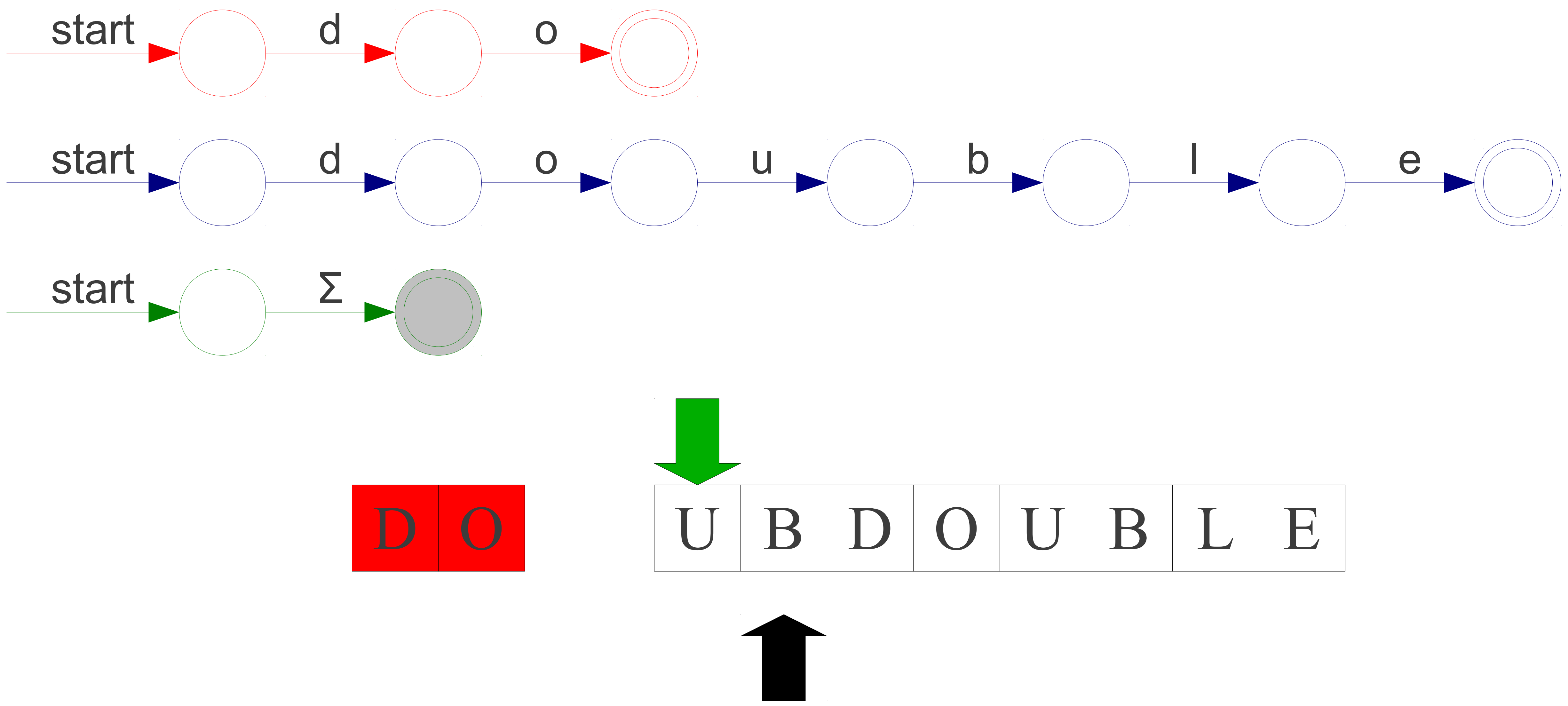
Implementing Maximal Munch

T_Do	do
T_Double	double
T_Mystery	[A-Za-z]



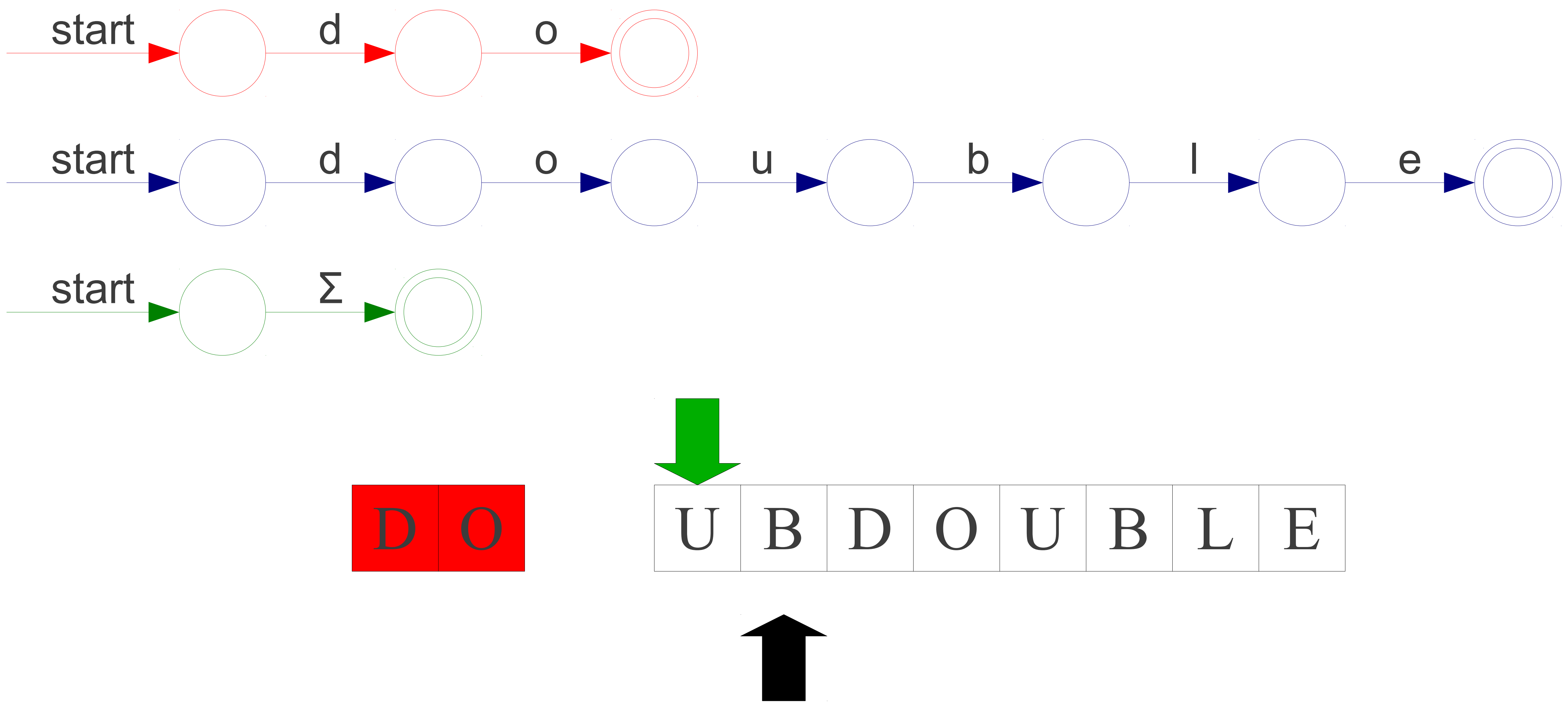
Implementing Maximal Munch

T_Do	do
T_Double	double
T_Mystery	[A-Za-z]



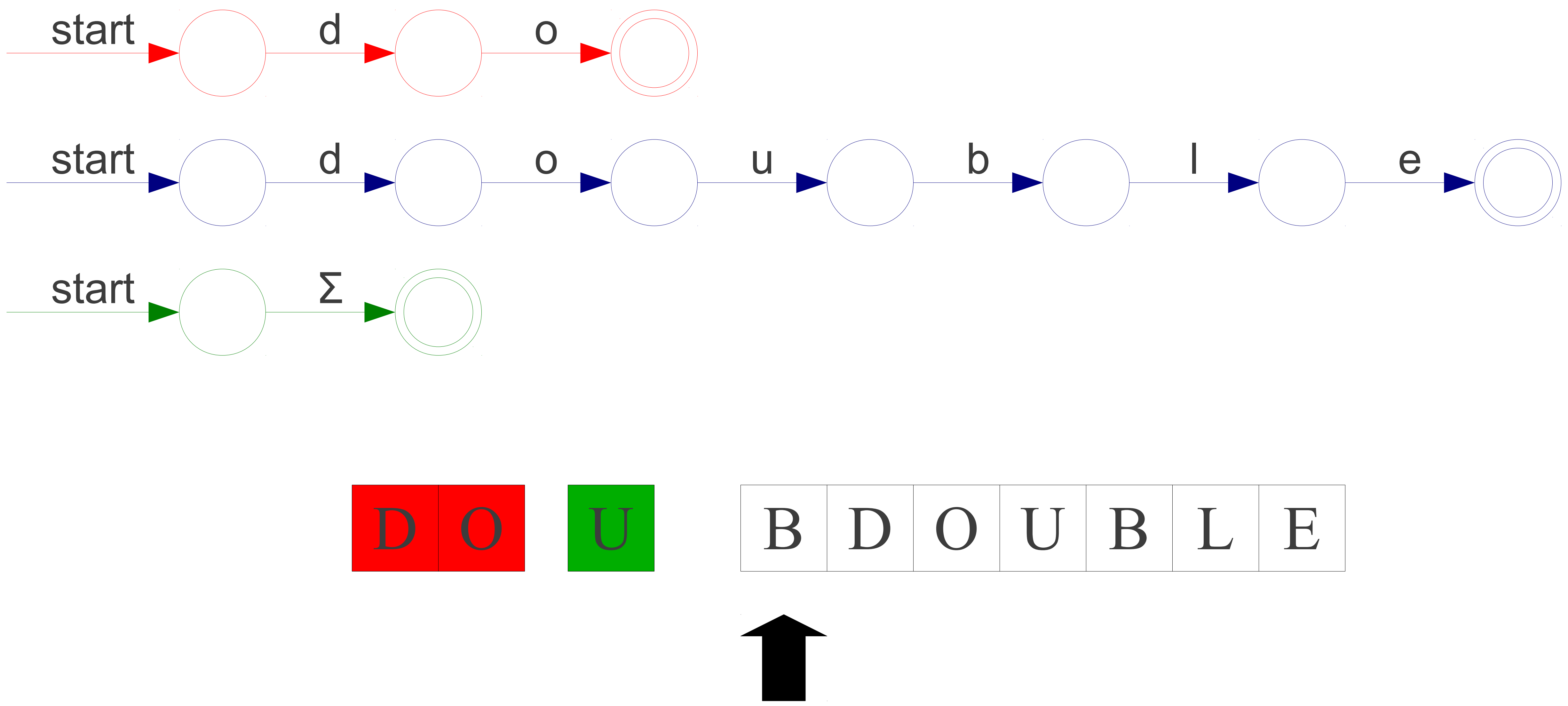
Implementing Maximal Munch

T_Do	do
T_Double	double
T_Mystery	[A-Za-z]



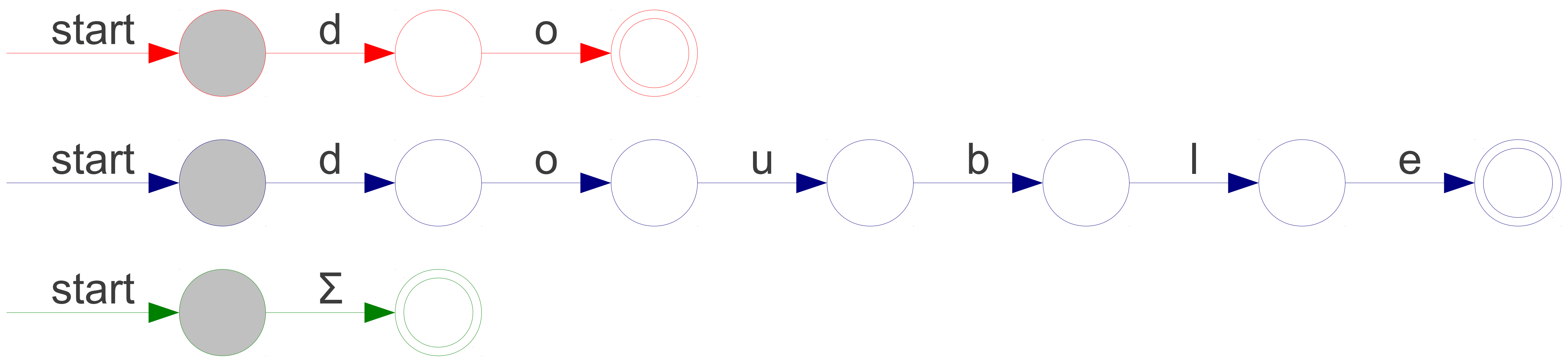
Implementing Maximal Munch

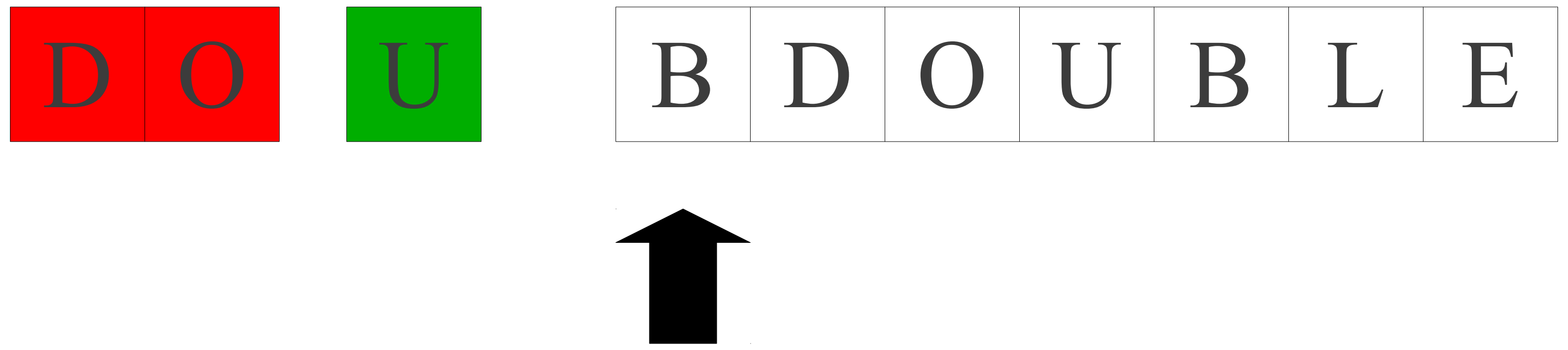
T_Do	do
T_Double	double
T_Mystery	[A-Za-z]



Implementing Maximal Munch

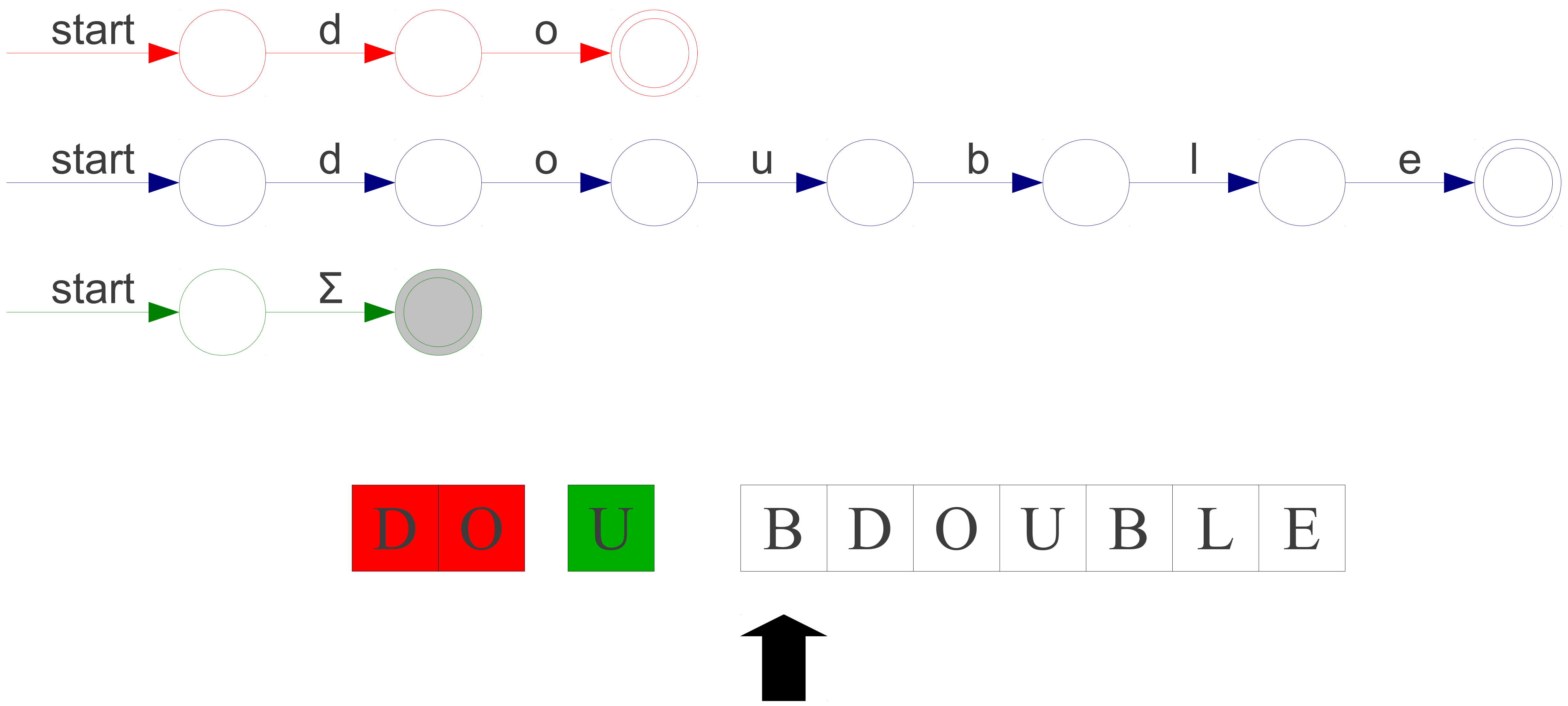
T_Do	do
T_Double	double
T_Mystery	[A-Za-z]





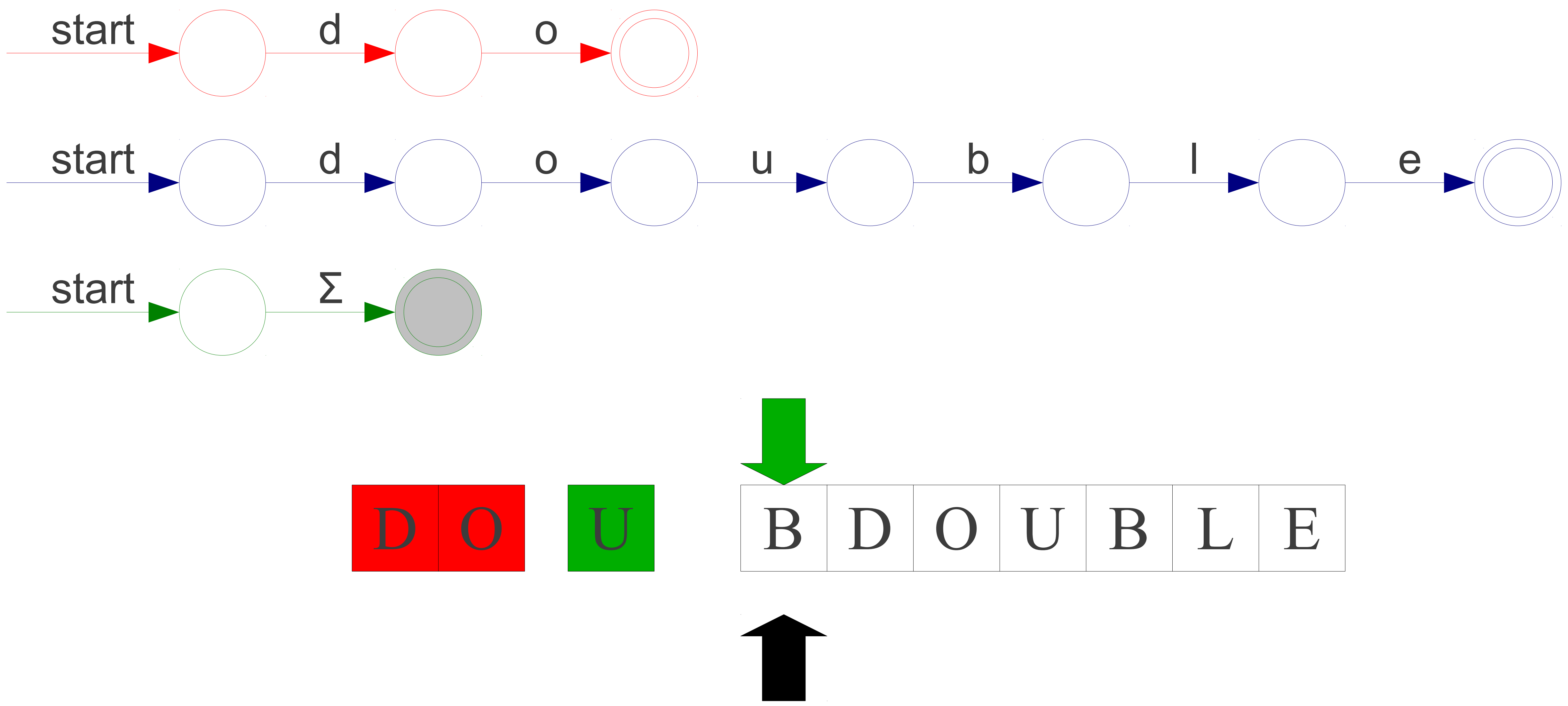
Implementing Maximal Munch

T_Do	do
T_Double	double
T_Mystery	[A-Za-z]



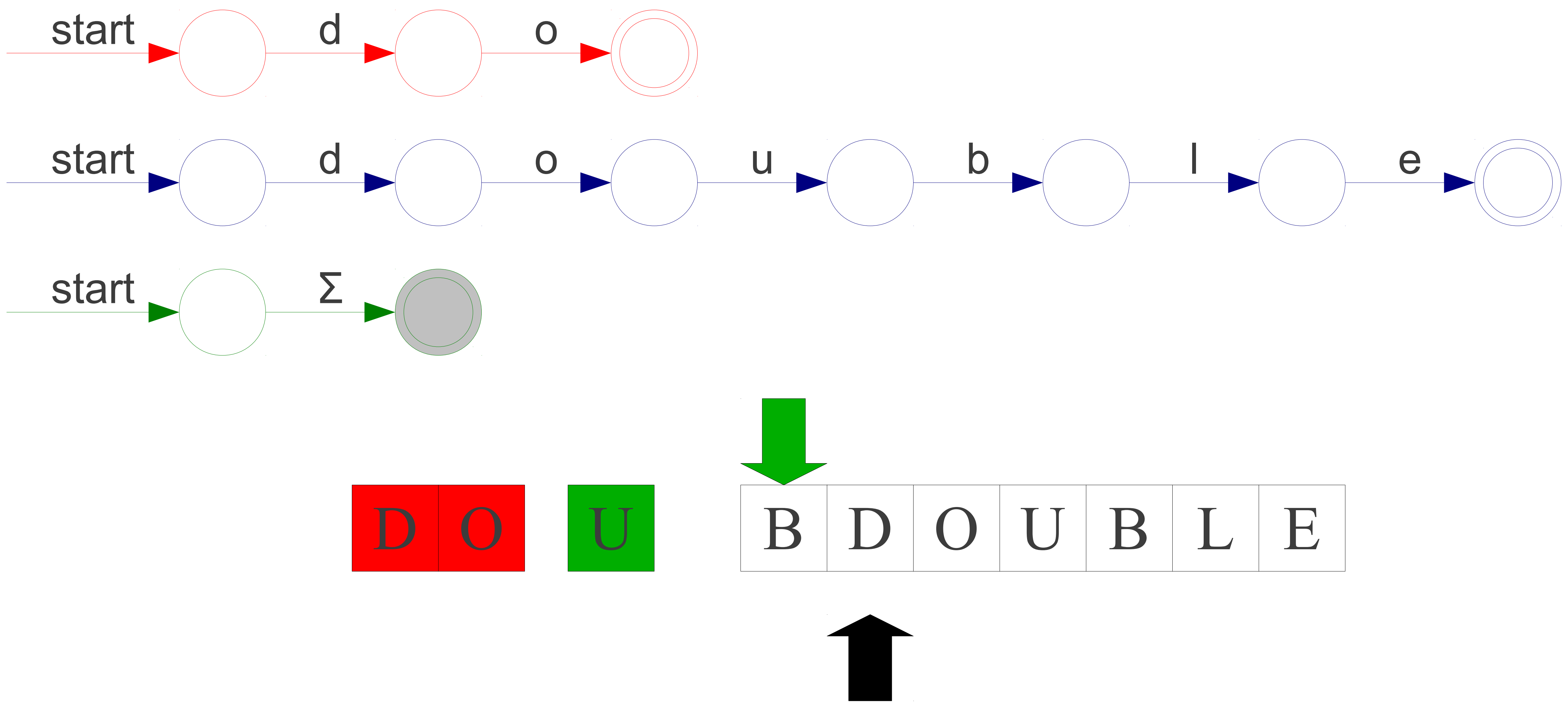
Implementing Maximal Munch

T_Do	do
T_Double	double
T_Mystery	[A-Za-z]



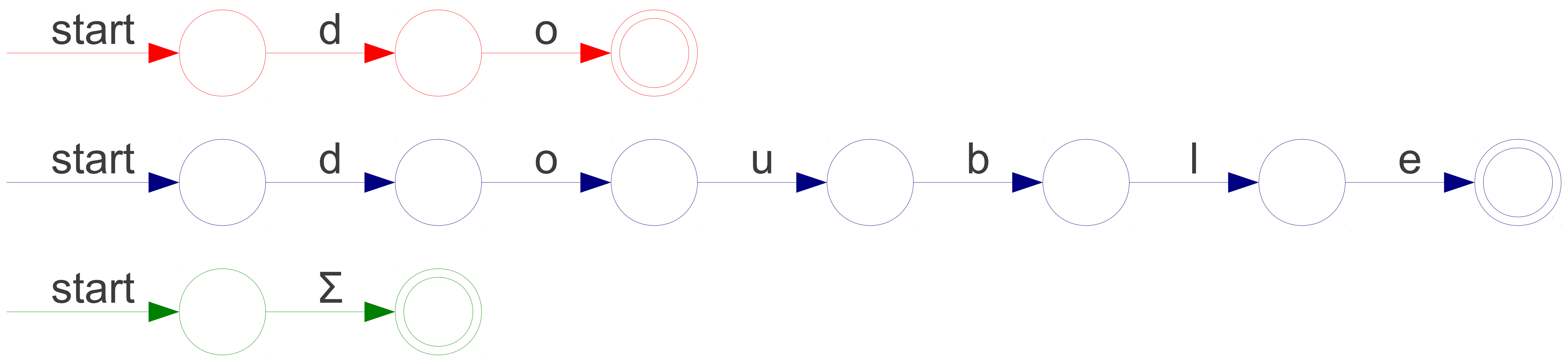
Implementing Maximal Munch

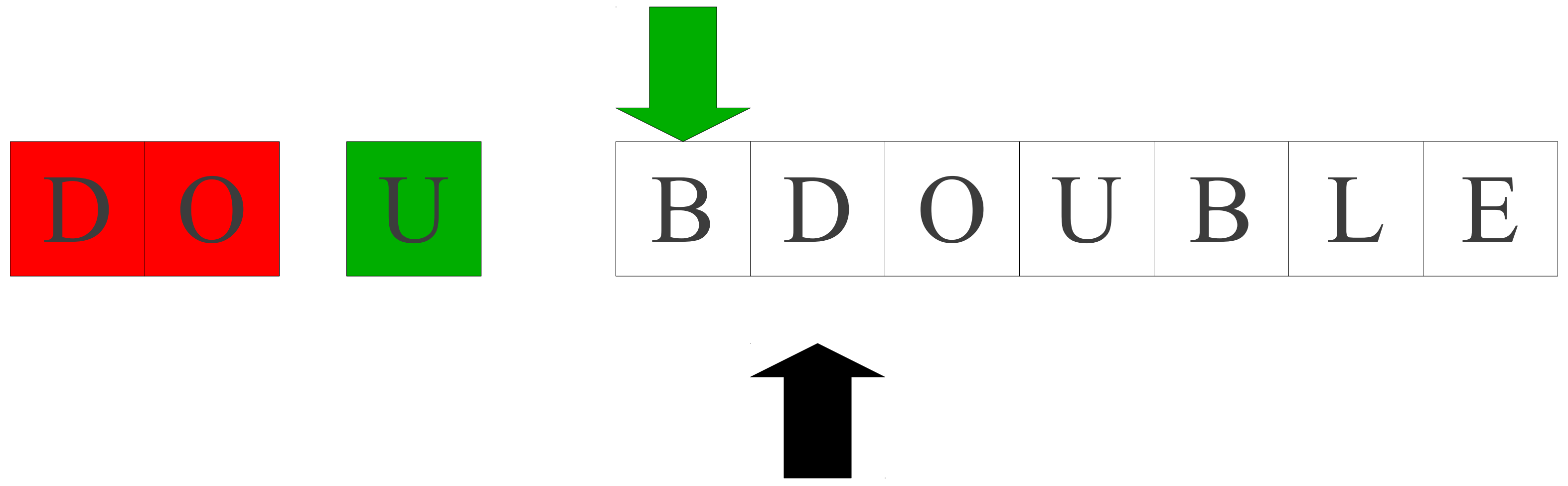
T_Do	do
T_Double	double
T_Mystery	[A-Za-z]



Implementing Maximal Munch

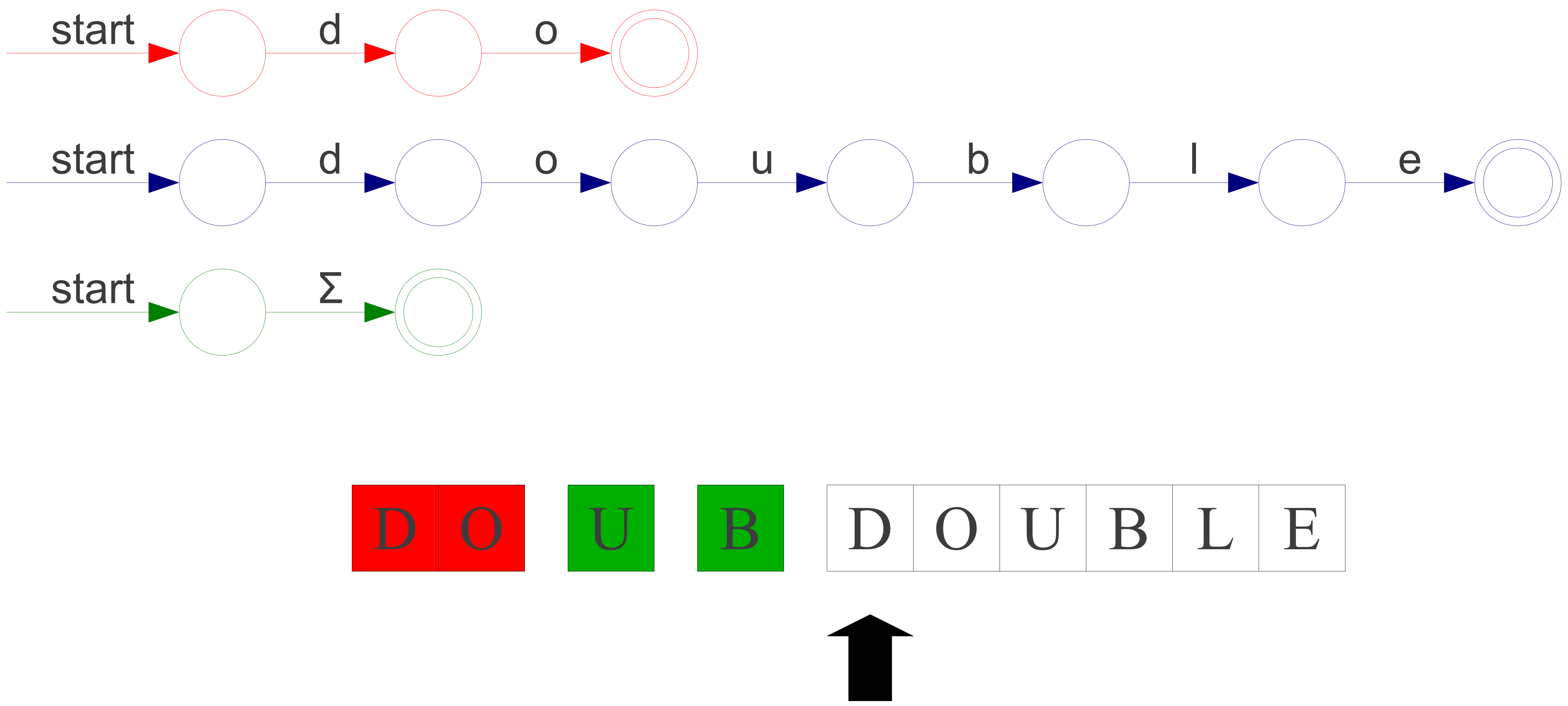
T_Do	do
T_Double	double
T_Mystery	[A-Za-z]





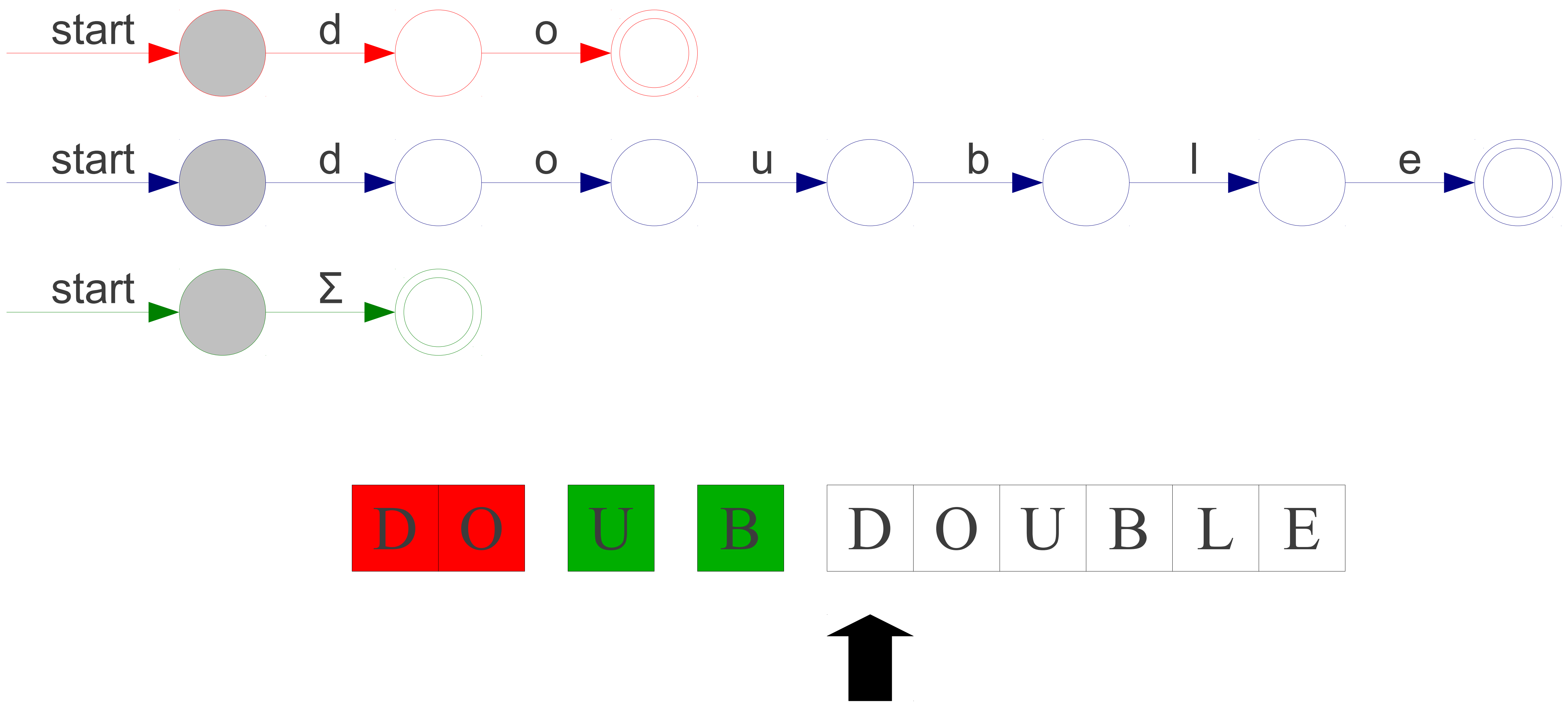
Implementing Maximal Munch

T_Do	do
T_Double	double
T_Mystery	[A-Za-z]



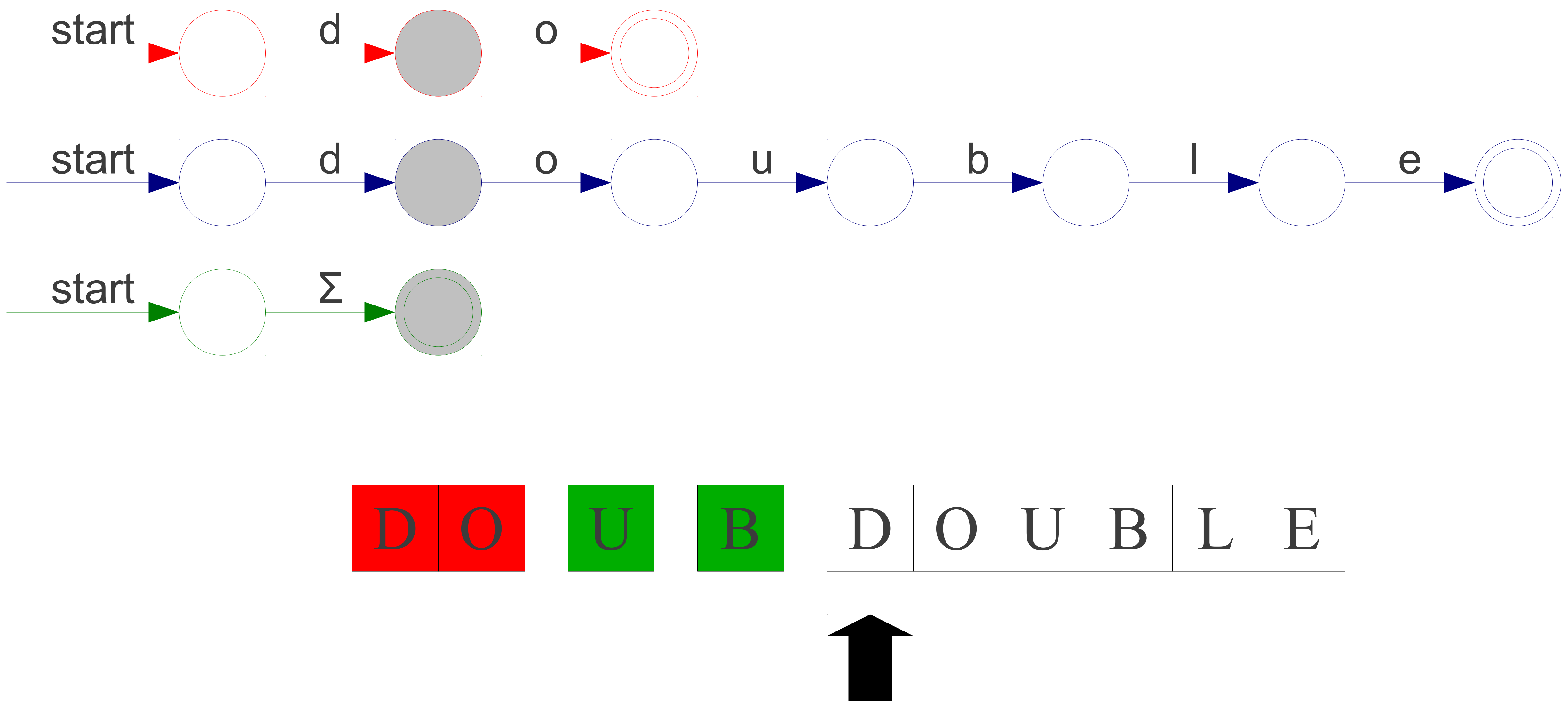
Implementing Maximal Munch

`T_Do`	`do`
`T_Double`	`double`
`T_Mystery`	`[A-Za-z]`



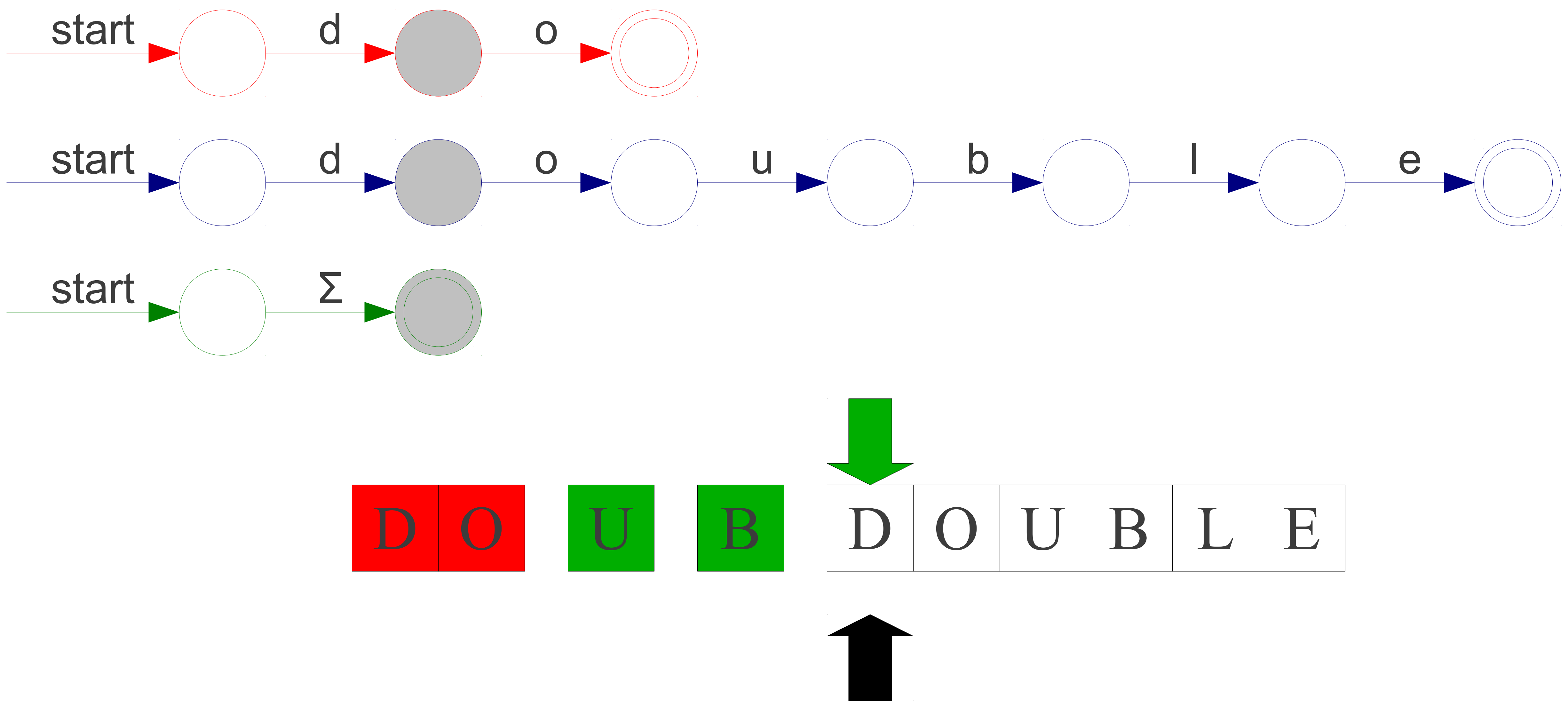
Implementing Maximal Munch

T_Do	do
T_Double	double
T_Mystery	[A-Za-z]



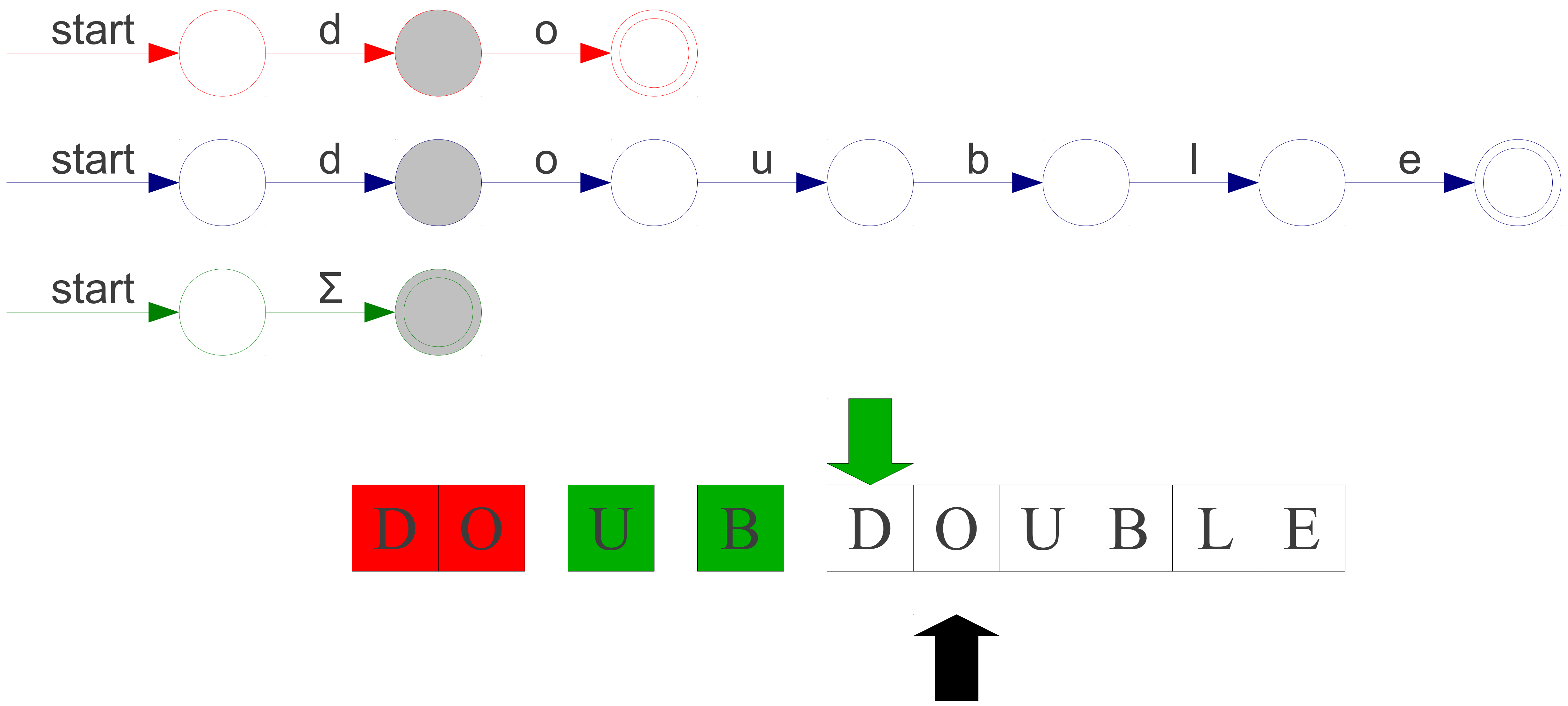
Implementing Maximal Munch

```
T_Do         do
T_Double     double
T_Mystery    [A-Za-z]
```



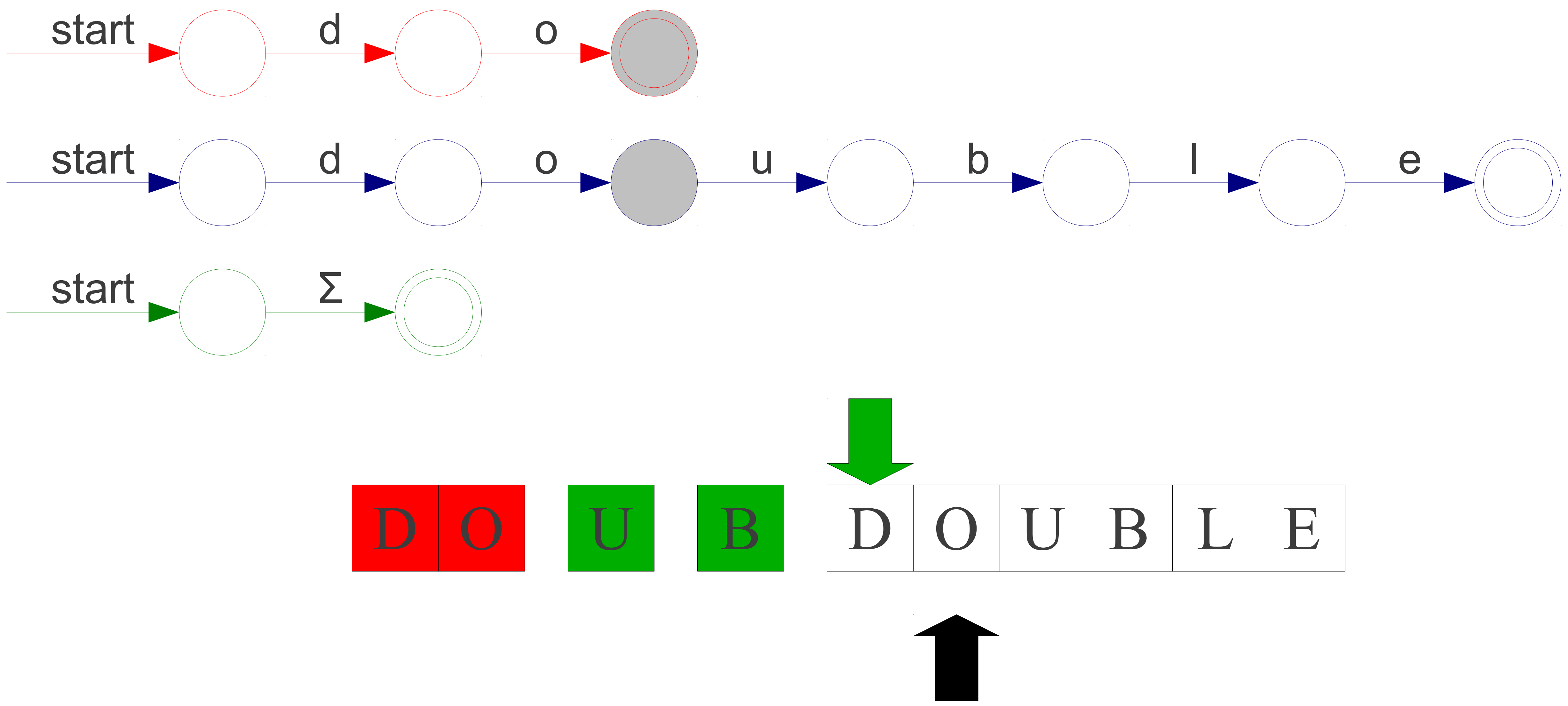
Implementing Maximal Munch

T_Do	do
T_Double	double
T_Mystery	[A-Za-z]



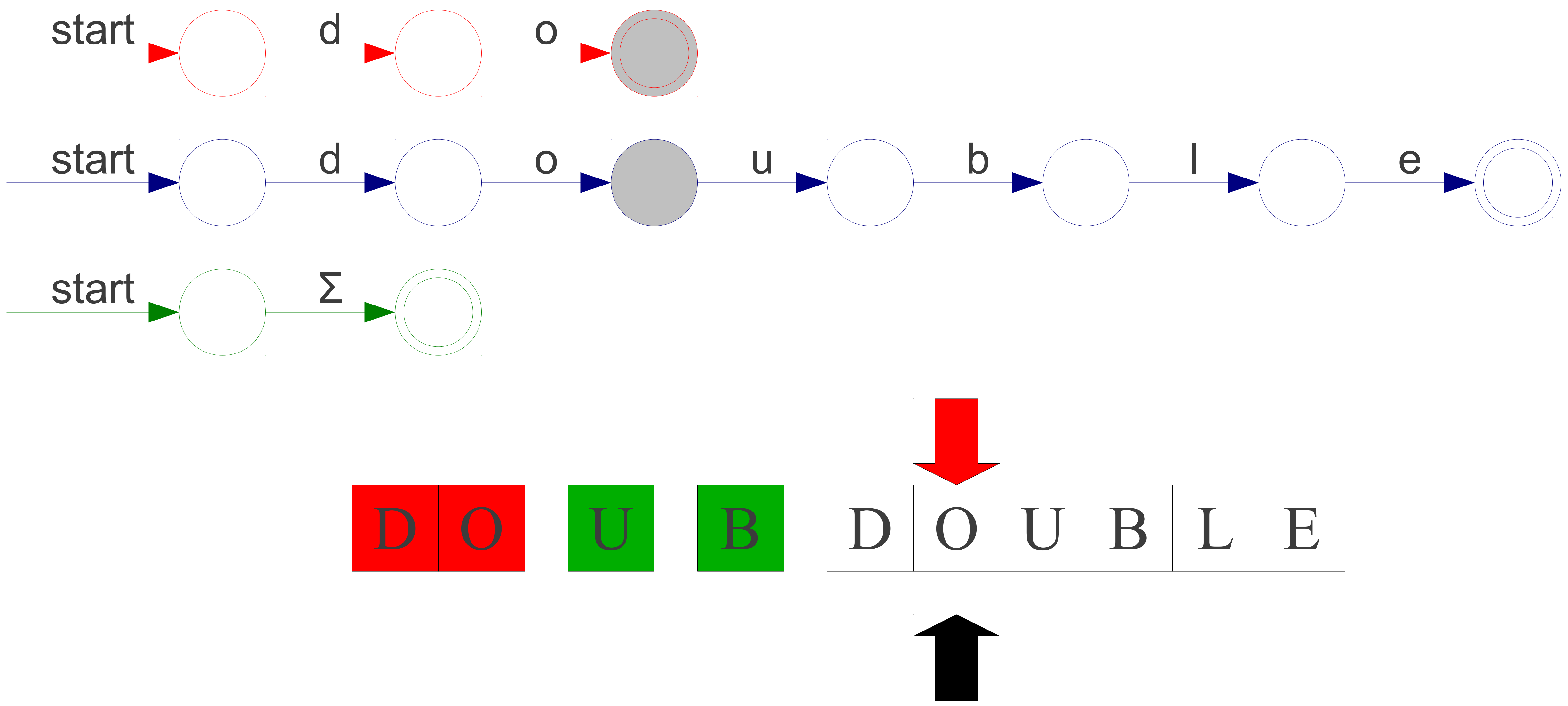
Implementing Maximal Munch

T_Do	do
T_Double	double
T_Mystery	[A-Za-z]



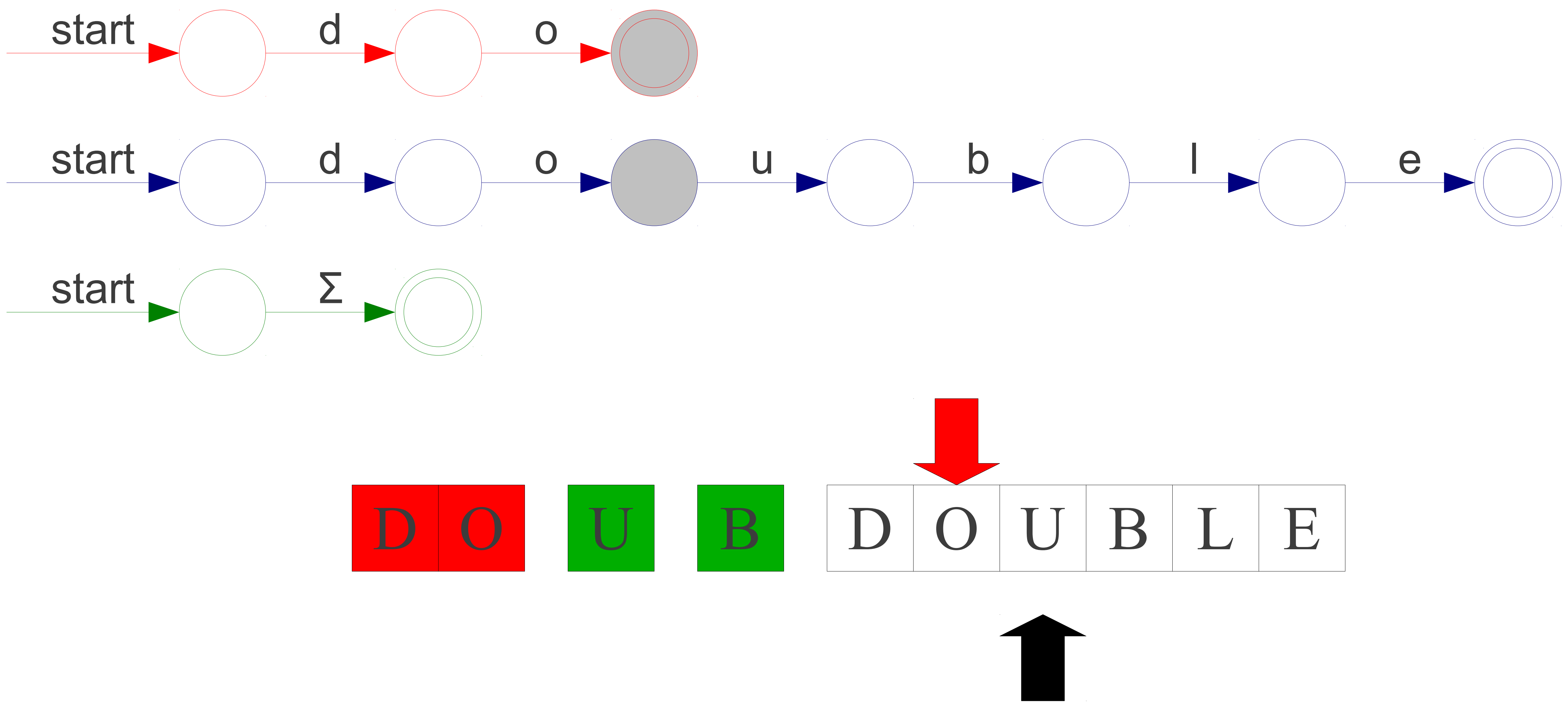
Implementing Maximal Munch

T_Do	do
T_Double	double
T_Mystery	[A-Za-z]



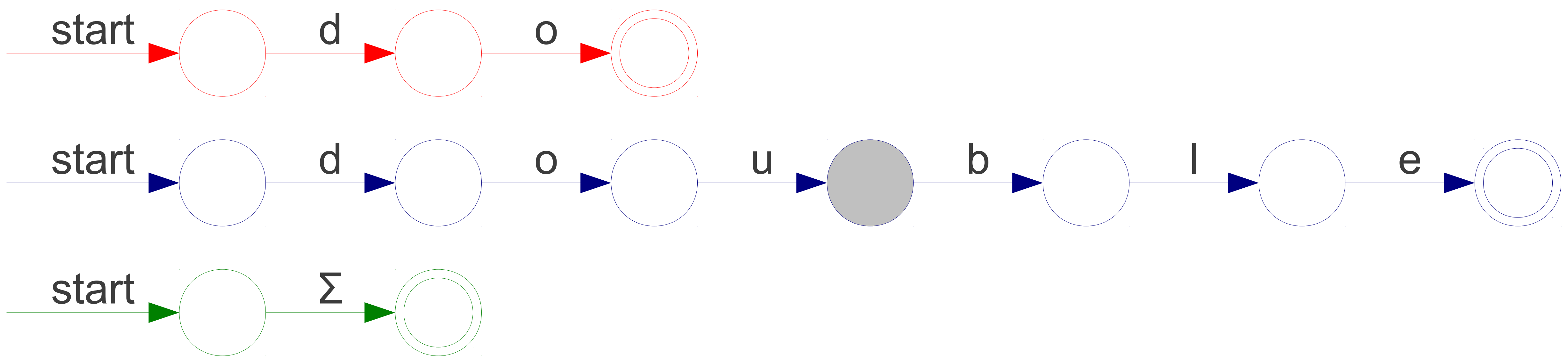
Implementing Maximal Munch

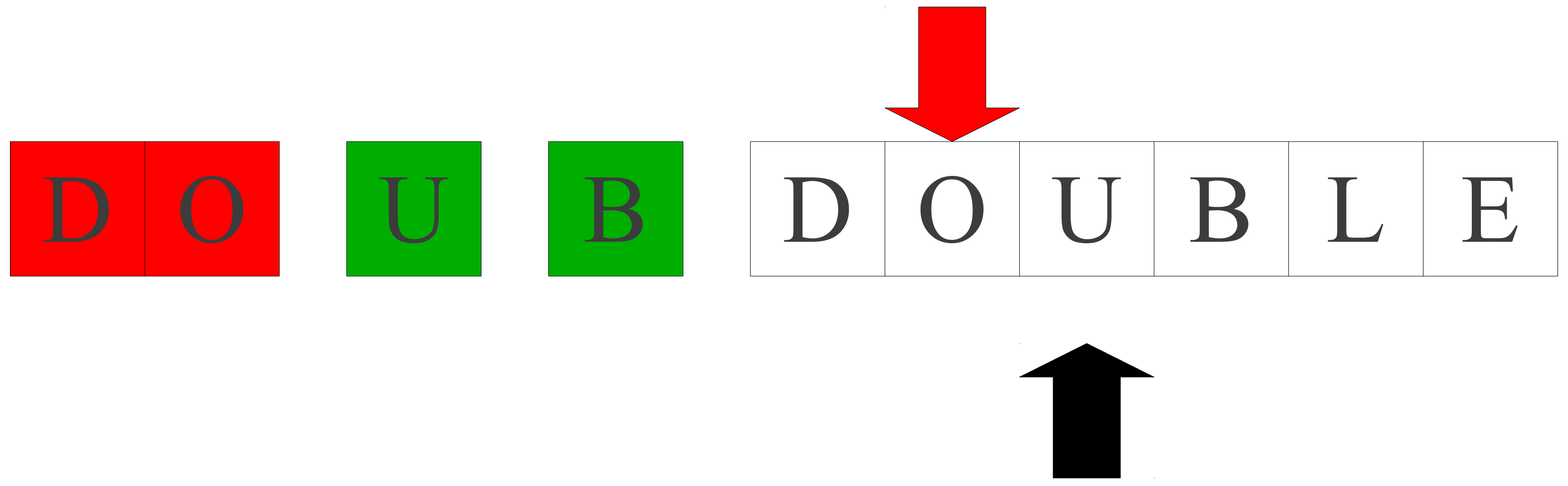
```
T_Do        do
T_Double    double
T_Mystery   [A-Za-z]
```



Implementing Maximal Munch

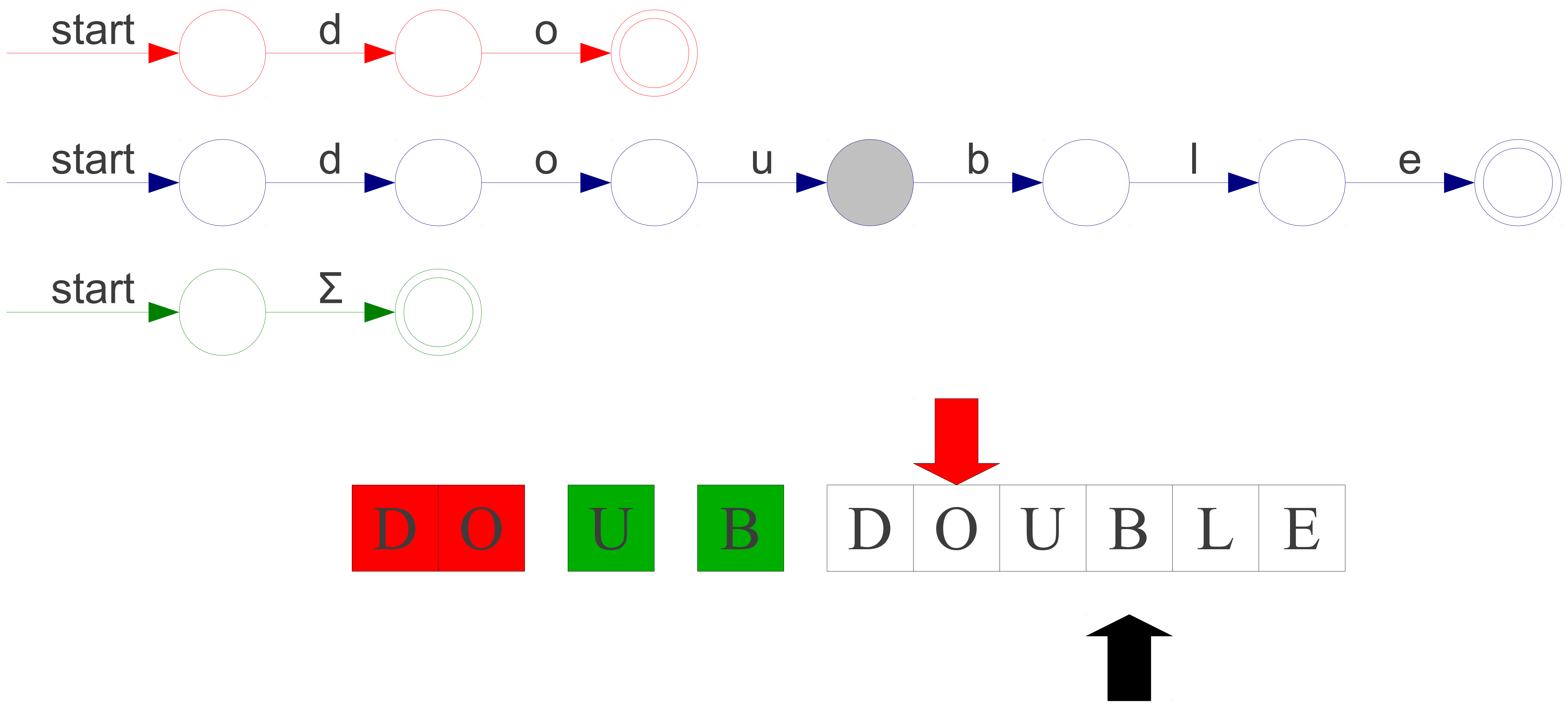
T_Do	do
T_Double	double
T_Mystery	[A-Za-z]





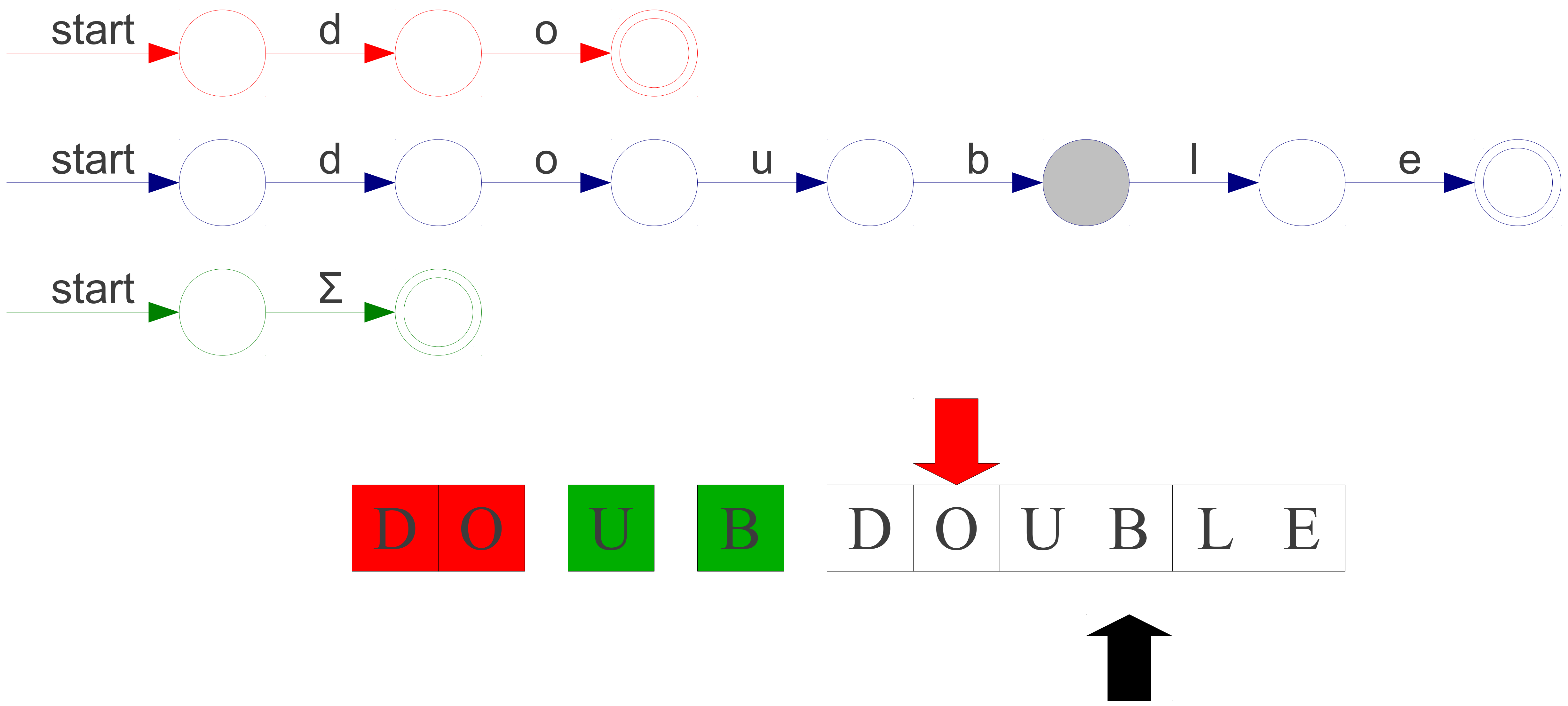
Implementing Maximal Munch

```
T_Do          do
T_Double      double
T_Mystery     [A-Za-z]
```



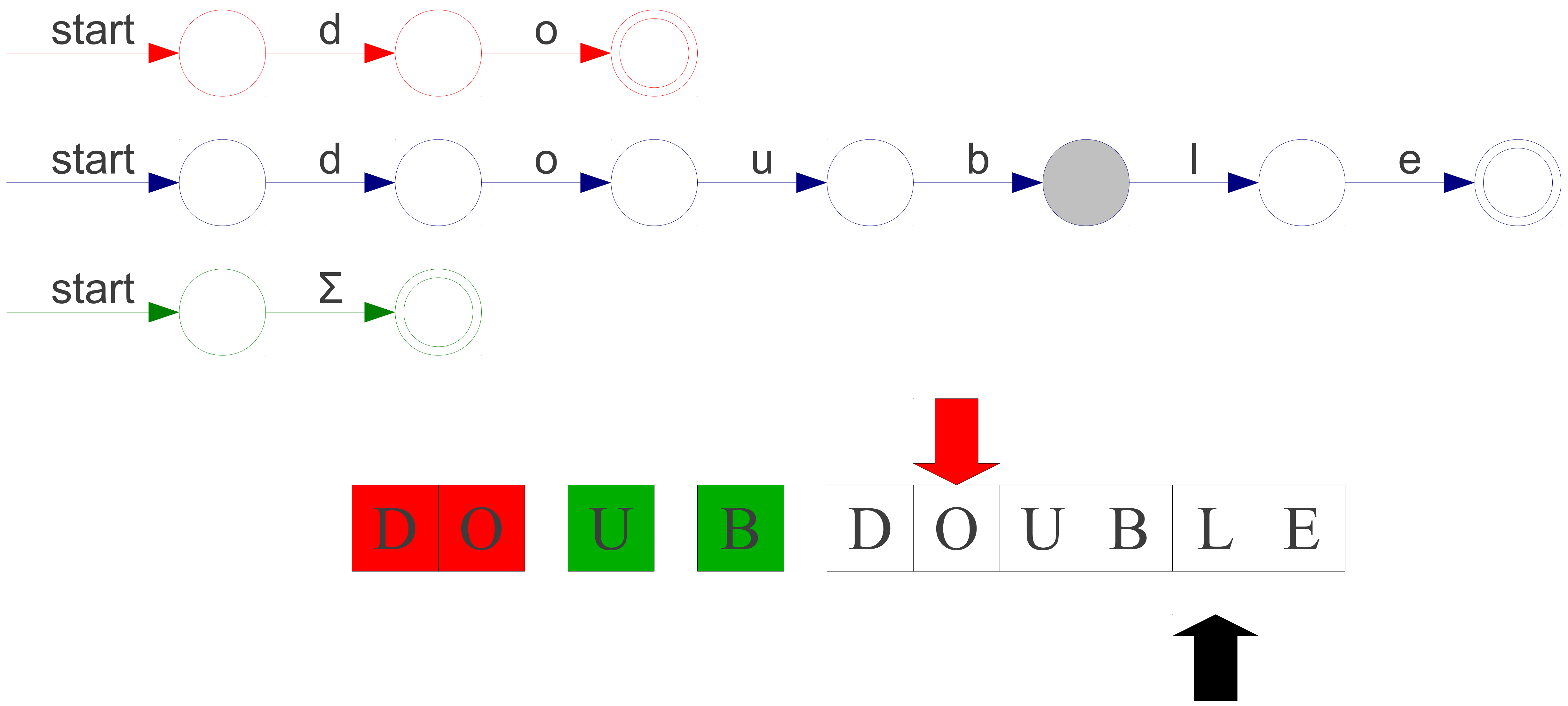
Implementing Maximal Munch

T_Do	do
T_Double	double
T_Mystery	[A-Za-z]



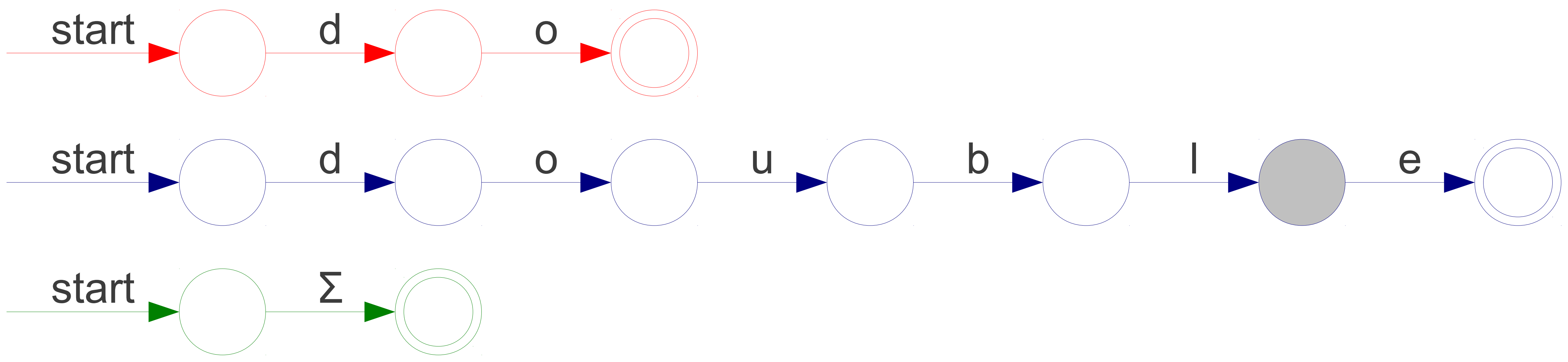
Implementing Maximal Munch

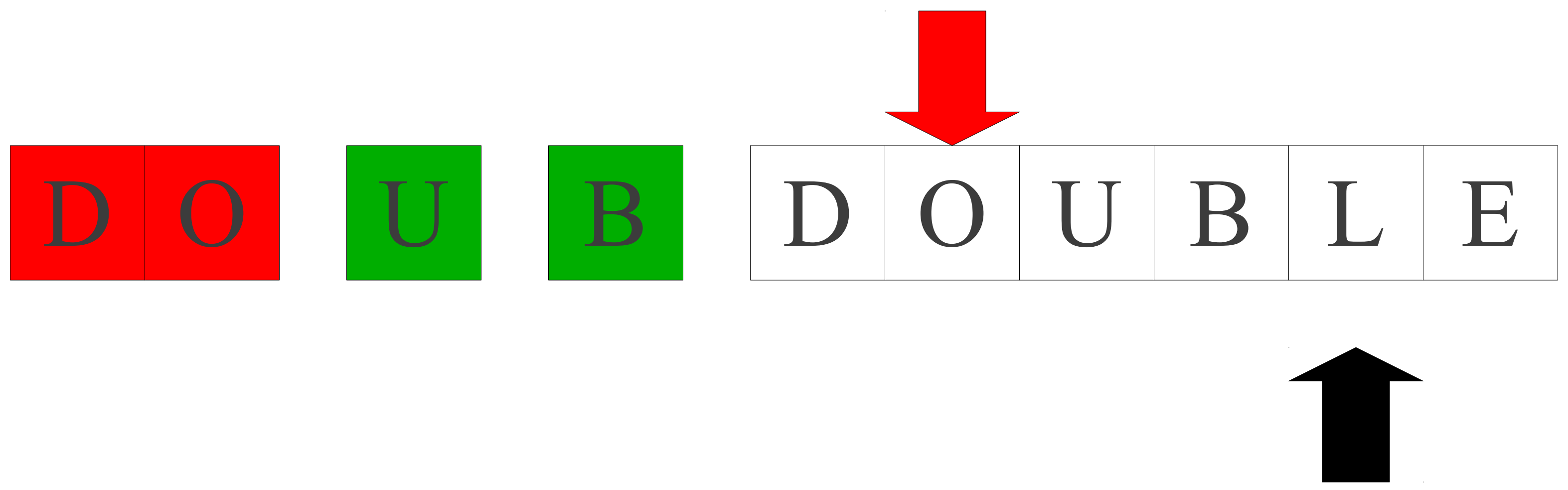
T_Do	do
T_Double	double
T_Mystery	[A-Za-z]



Implementing Maximal Munch

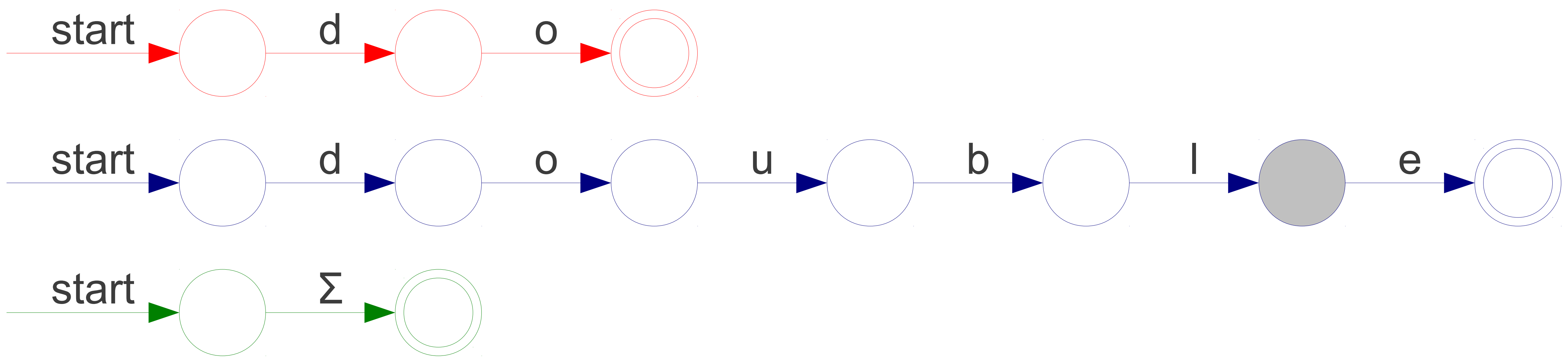
T_Do	do
T_Double	double
T_Mystery	[A-Za-z]

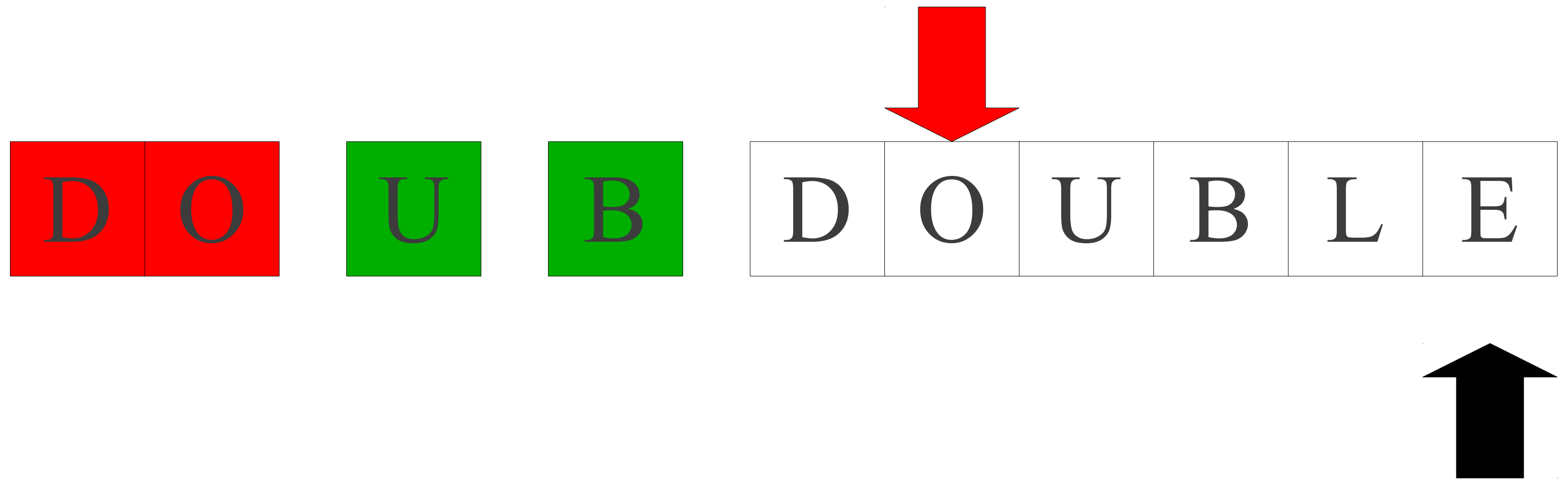




Implementing Maximal Munch

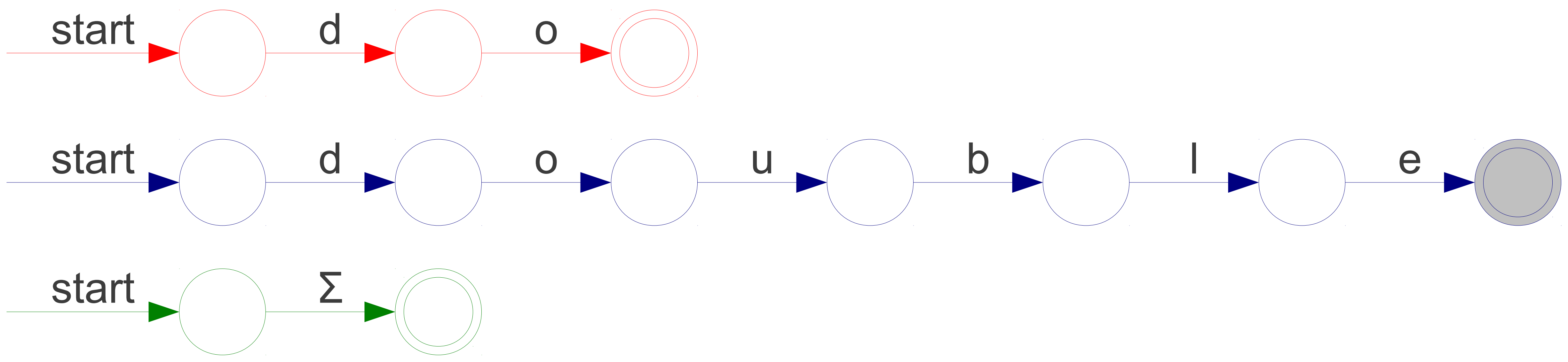
T_Do	do
T_Double	double
T_Mystery	[A-Za-z]

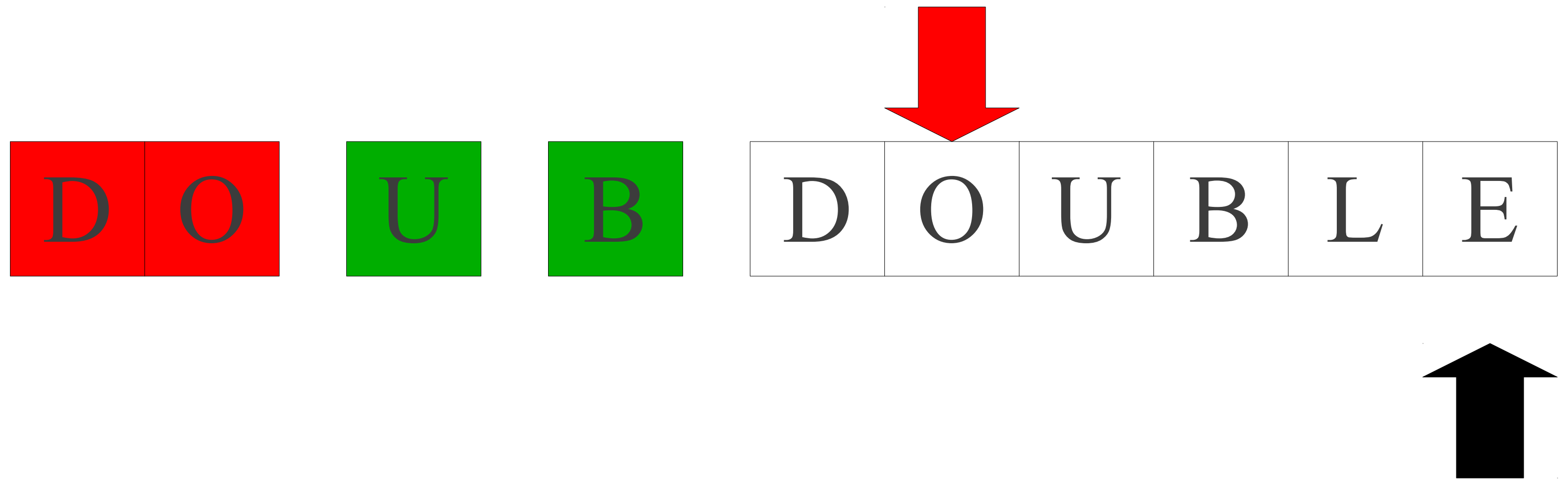




Implementing Maximal Munch

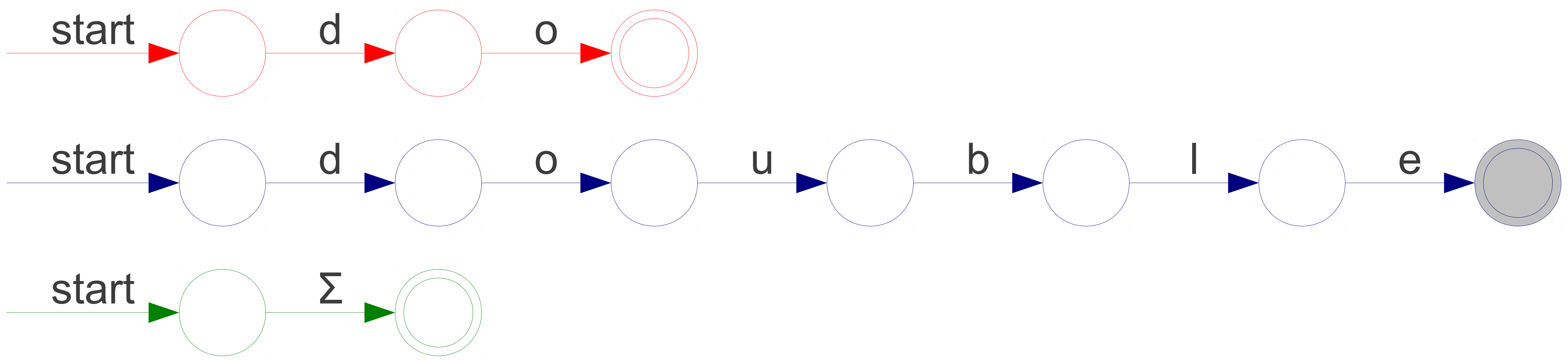
T_Do	do
T_Double	double
T_Mystery	[A-Za-z]

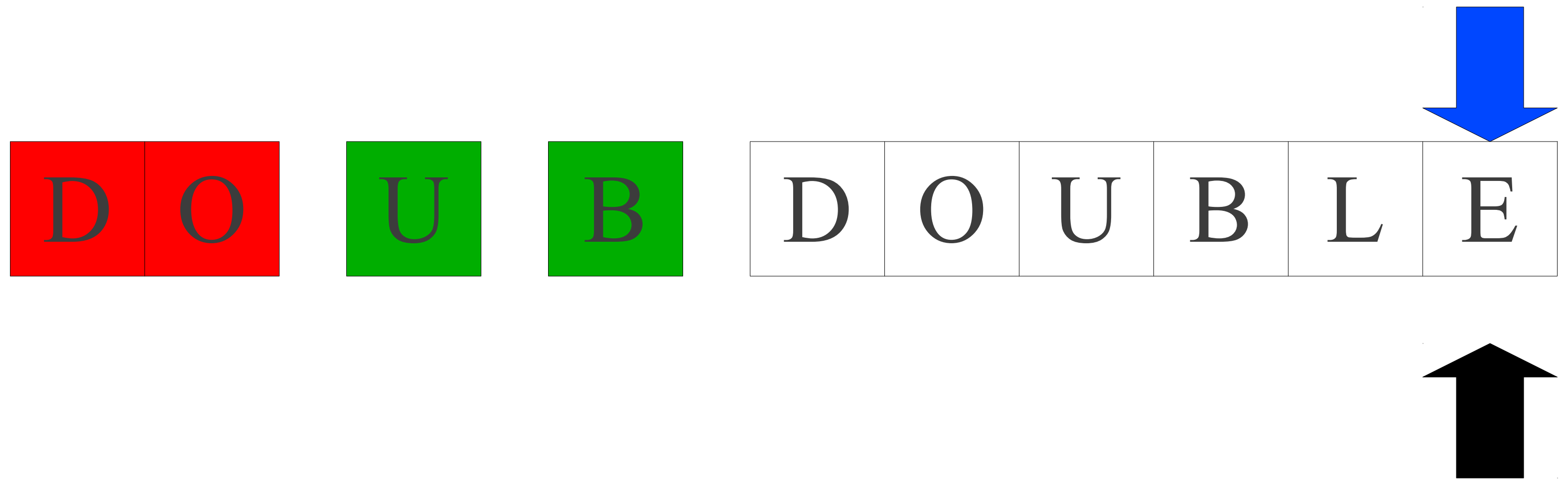




Implementing Maximal Munch

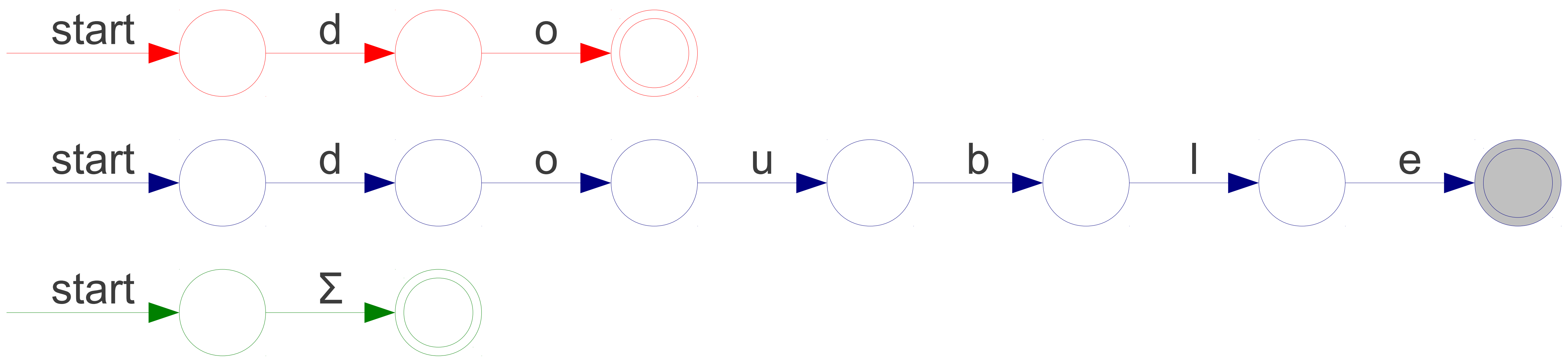
T_Do	do
T_Double	double
T_Mystery	[A-Za-z]

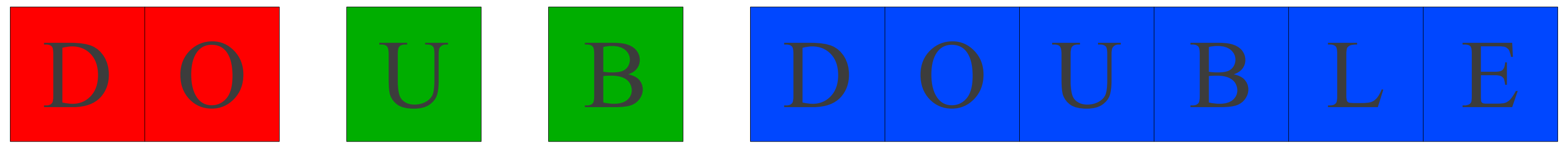




Implementing Maximal Munch

```
T_Do          do
T_Double      double
T_Mystery     [A-Za-z]
```





Other Conflicts

```
T_Do         do
T_Double     double
T_Identifier [A-Za-z_][A-Za-z0-9_]*
```

d	o	u	b	l	e

More Tiebreaking

- When two regular expressions apply, choose the one with the greater “priority.”
- Simple priority system: **pick the rule that was defined first.**

Other Conflicts

```
T_Do         do
T_Double     double
T_Identifier [A-Za-z_][A-Za-z0-9_]*
```

d	o	u	b	l	e

d	o	u	b	l	e
d	o	u	b	l	e

Other Conflicts

```
T_Do         do
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```

d	o	u	b	l	e

d	o	u	b	l	e

Lexer Generators as Compilers for Regexes

Source Language: Regexes + associated Token-construction code

Target Language: C or the lang the rest of your compiler is written in

Intermediate Representations: DFAs, NFAs
Passes: NFA -> DFA determinization
Optimization: DFA minimization
Can be mathematically proven to be correct, "optimal"