Graph Coloring
Register Allocation

October 20, 2021

Register Allocation

The compiler needs to decide which variables to store in which registers, which registers to “spill” onto the stack

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3. Graph Coloring: based on conflict information, assign registers to variables so that conflicting vars get different registers
4. Spilling: if graph coloring fails, pick a variable to put on the stack and retry

Example

```
def f(a, b):
  let x = a + b in
  let y = g(x)  in
  let z = x * y in
  h(x, z)
end
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x, y conflict
x, z conflict

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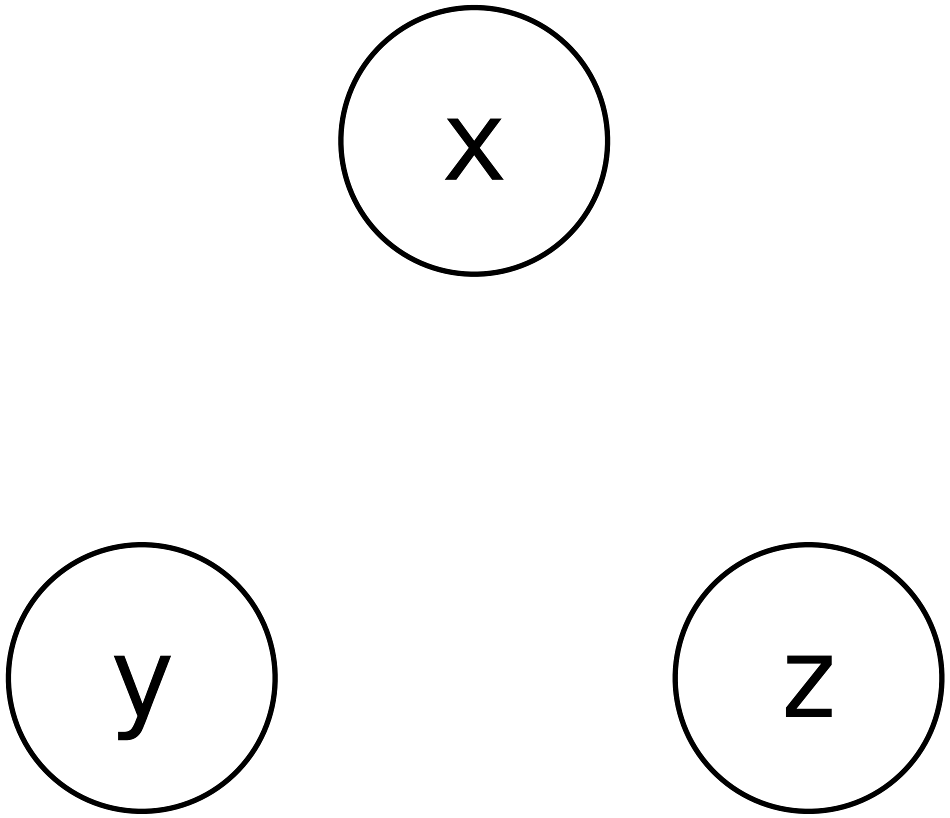

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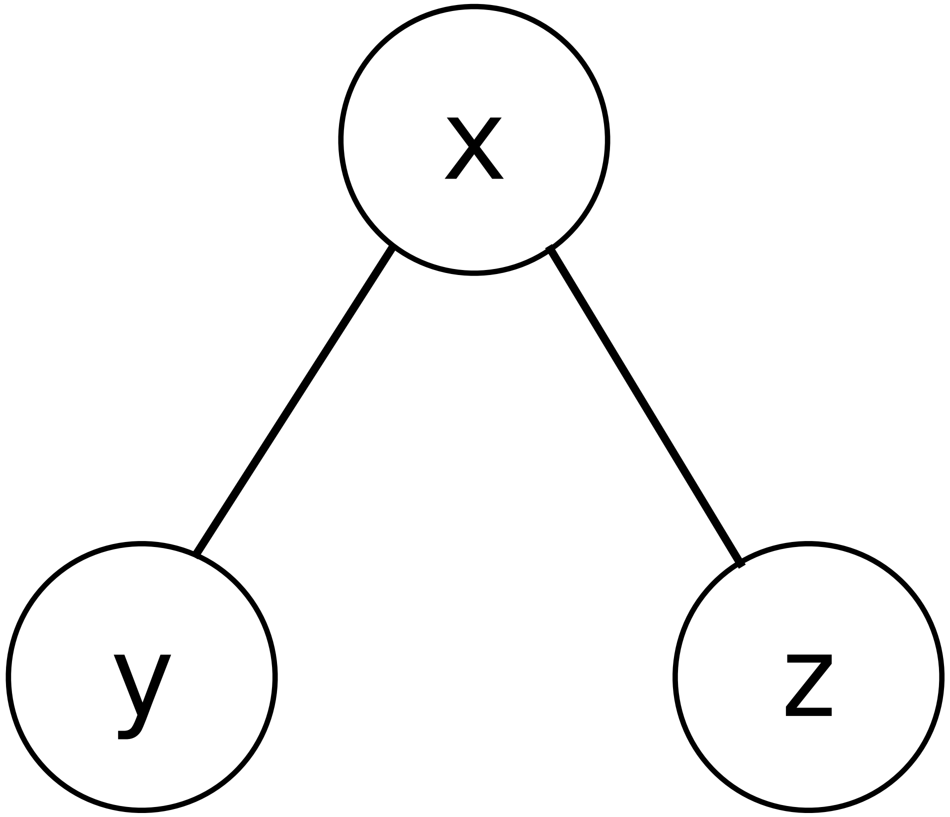

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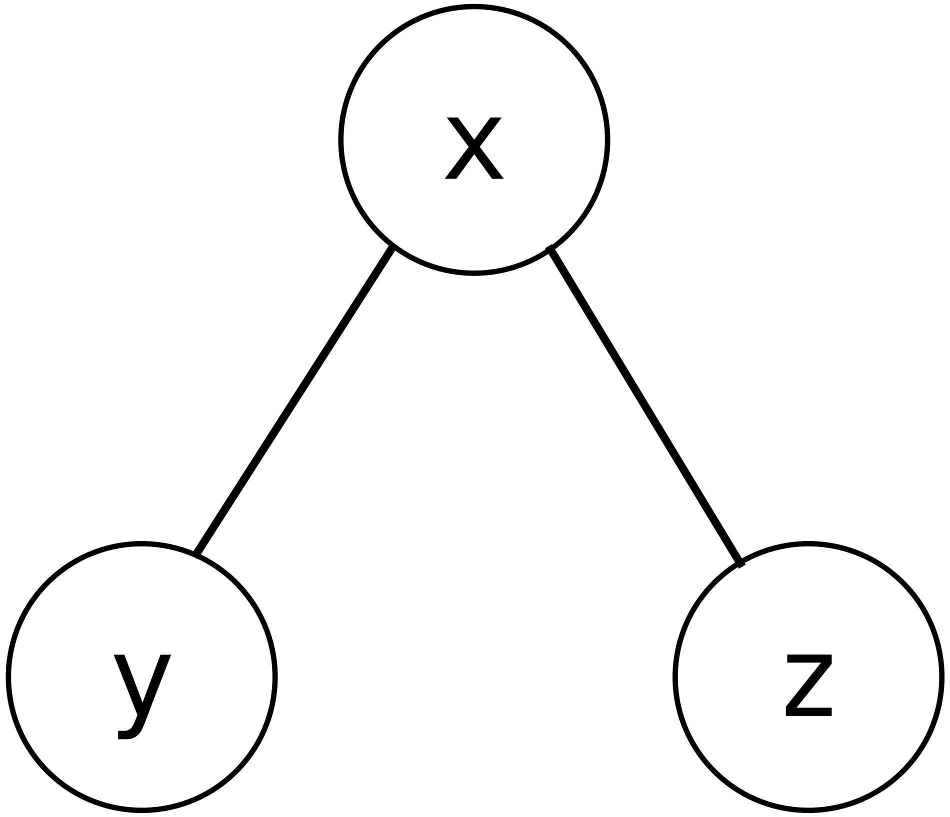

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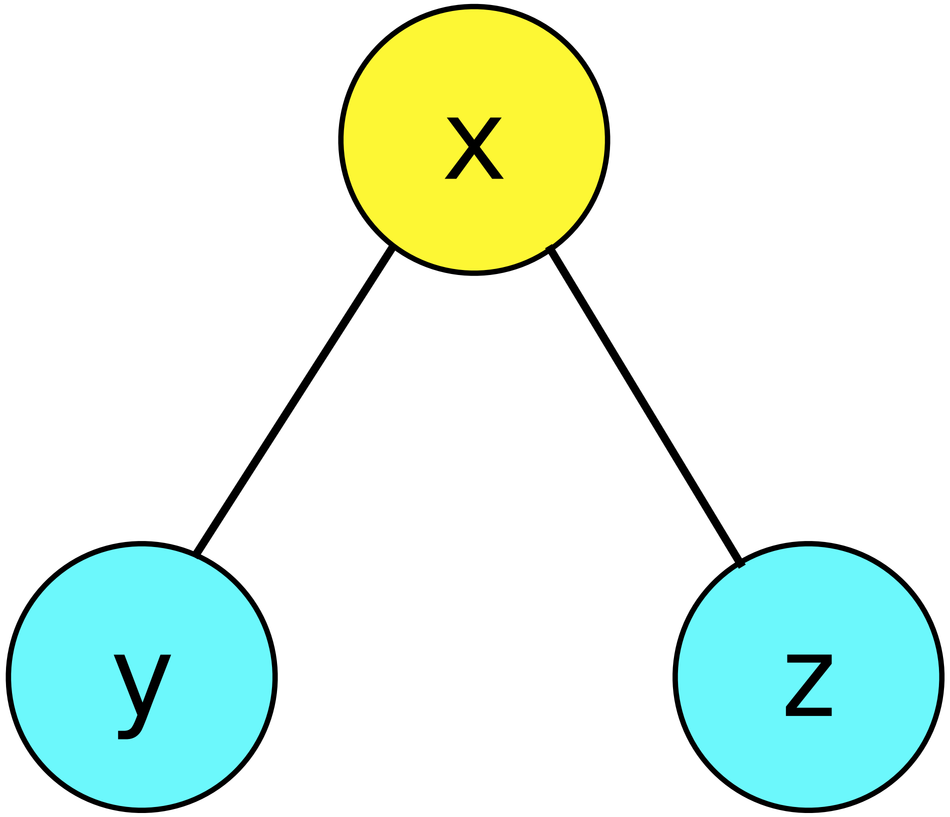

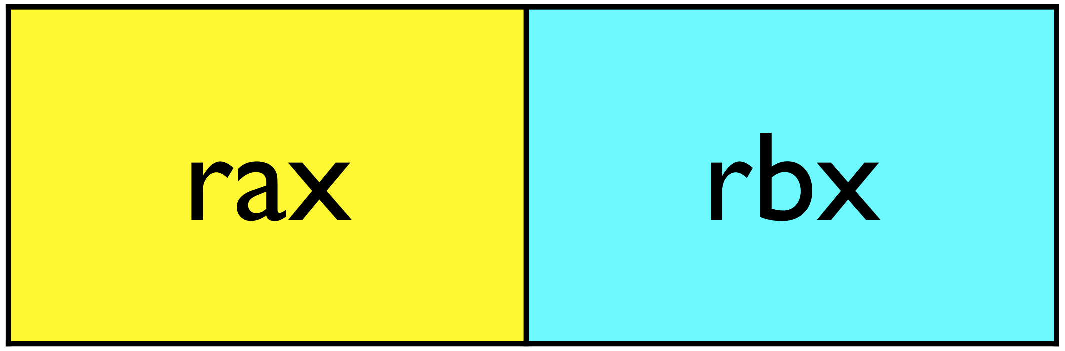

Graph Coloring Register Allocation

Given our register conflict graph, want to assign a register to each variable so that no adjacent variables are assigned the same register.

Equivalent to graph coloring: think of each register as a "color" and we want to paint each node so that no adjacent nodes are the same color.

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Even with heuristics, the algorithm is still n^2, and the slowest part of the compiler.

Chaitin's Algorithm

- Intuition:
  - Suppose we are trying to *k*-color a graph and find a node with fewer than *k* edges.
  - If we delete this node from the graph and color what remains, we can find a color for this node if we add it back in.
  - Reason: With fewer than *k* neighbors, some color must be left over.
- Algorithm:
  - Find a node with fewer than *k* outgoing edges.
  - Remove it from the graph.
  - Recursively color the rest of the graph.
  - Add the node back in.
  - Assign it a valid color.

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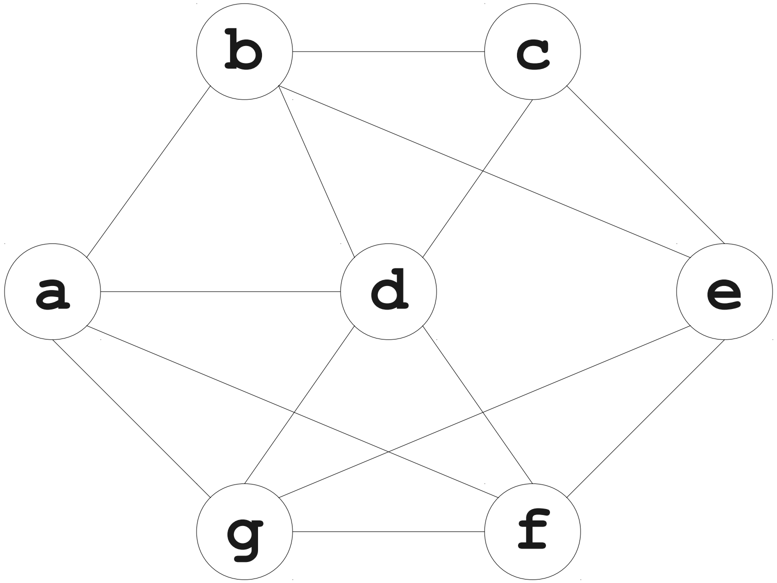

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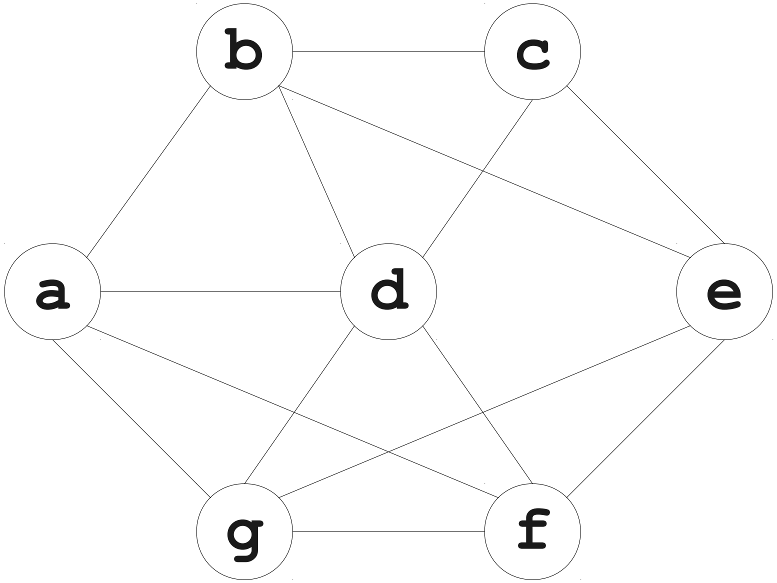

Registers

R_0	R_1	R_2	R_3

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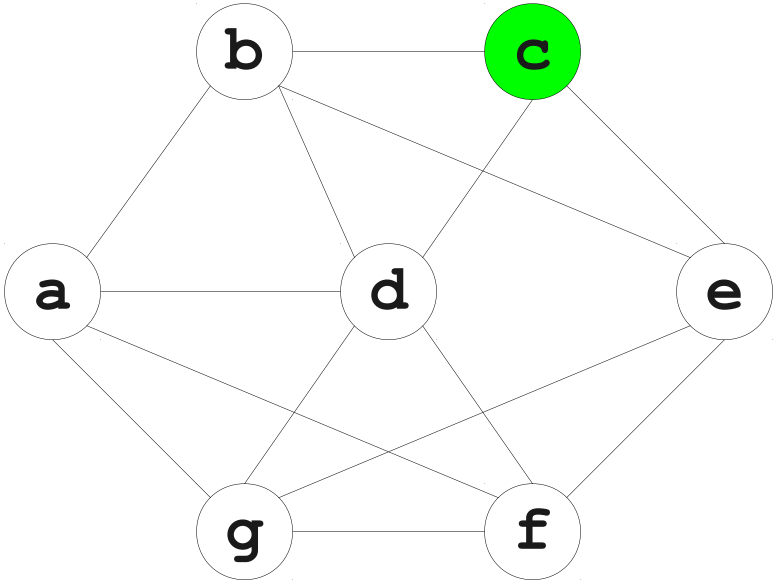

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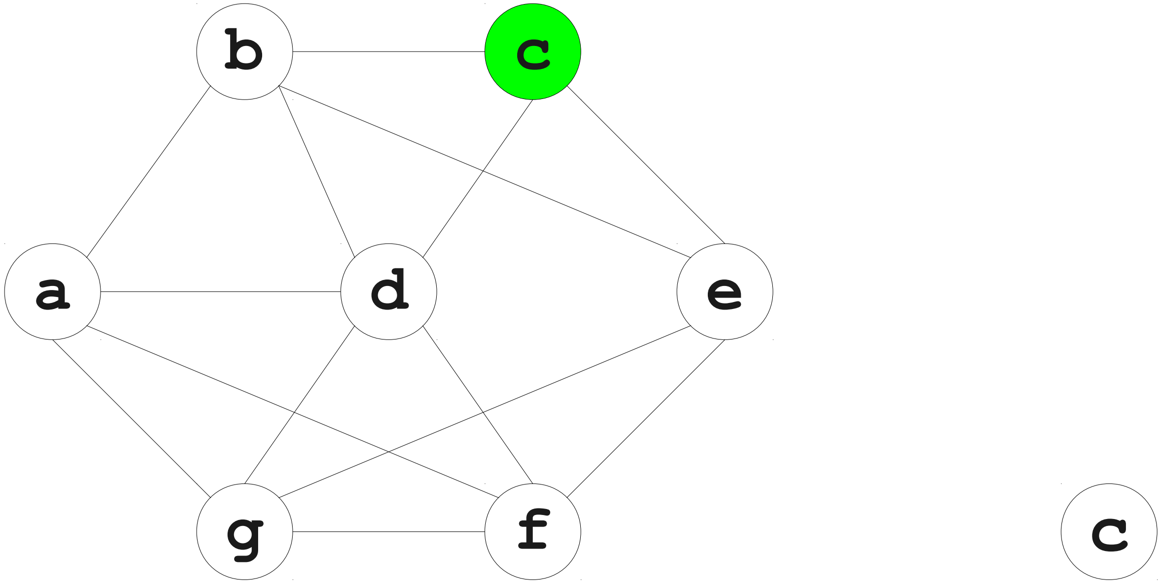

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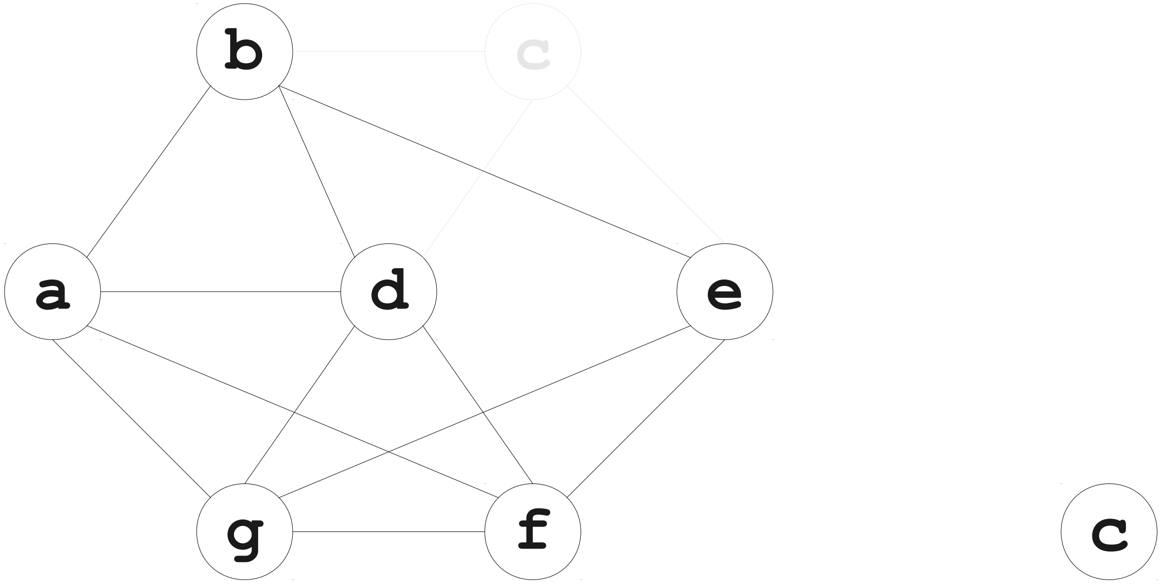

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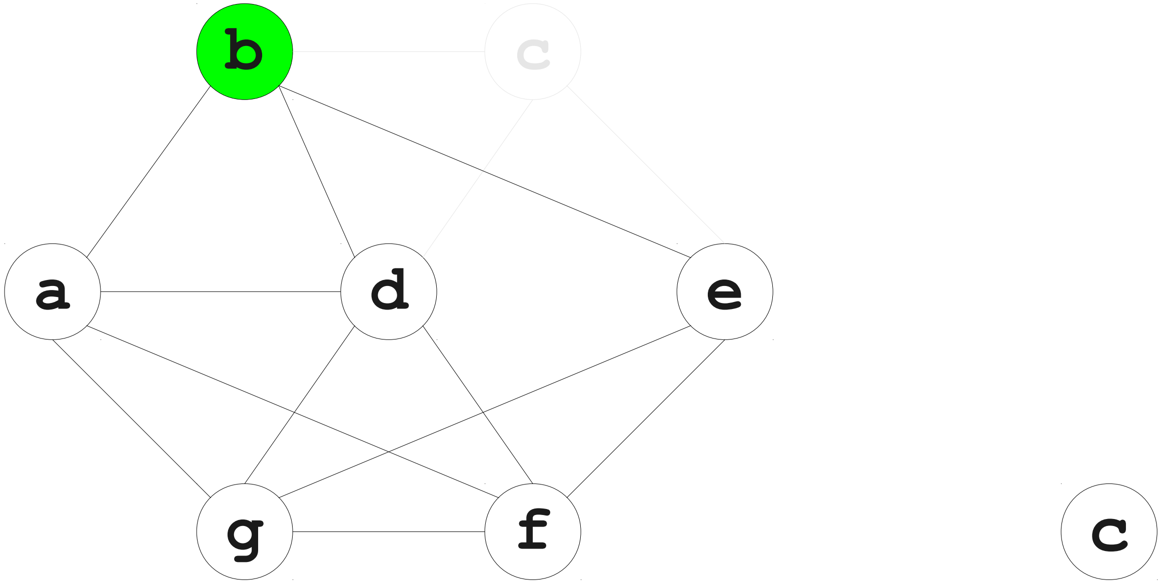

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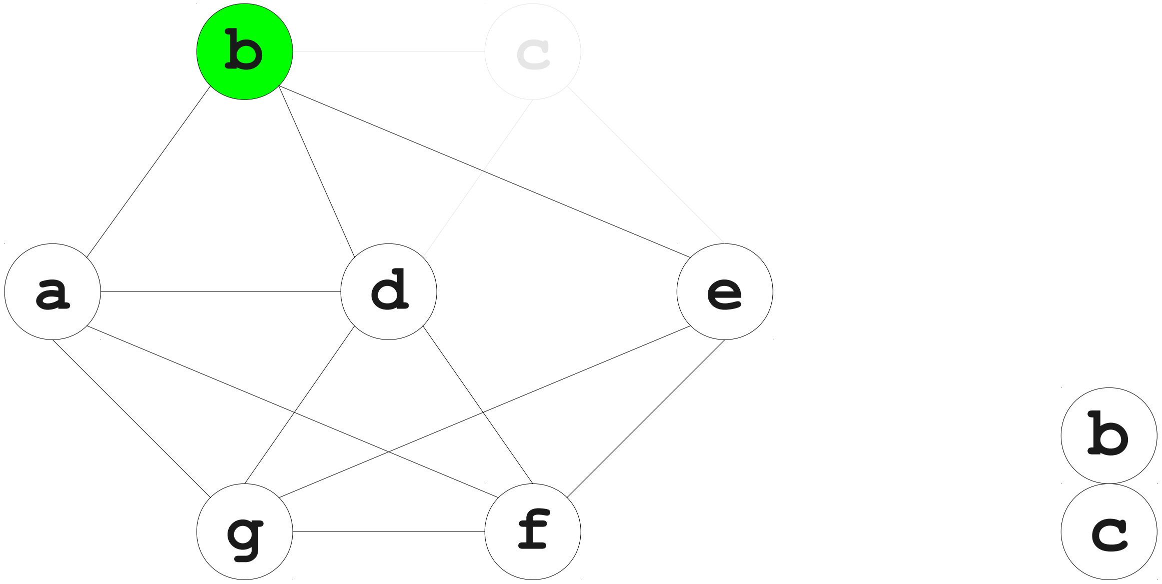

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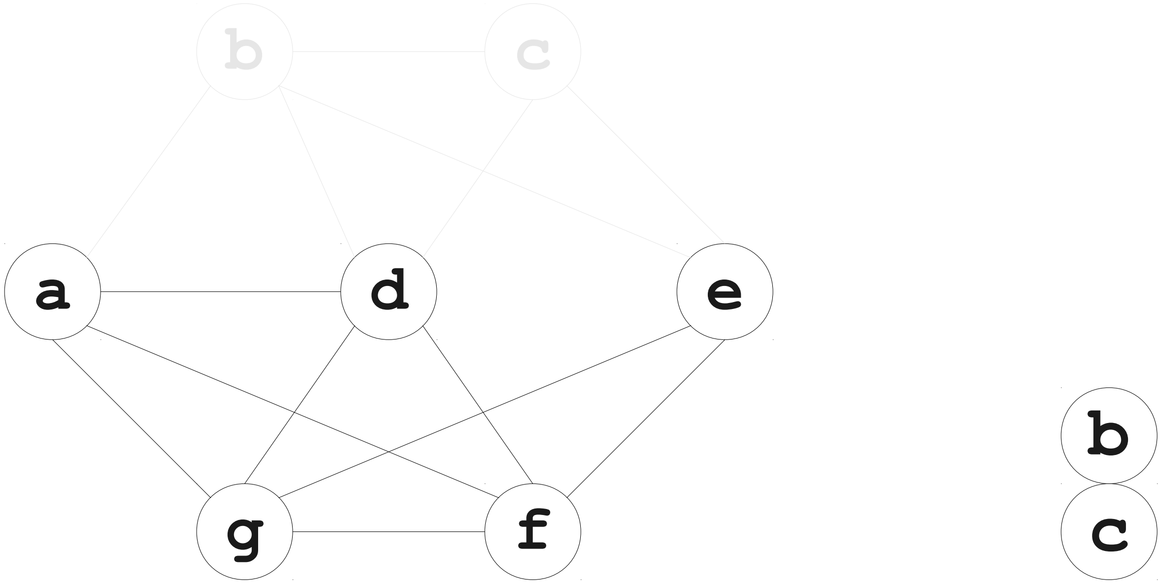

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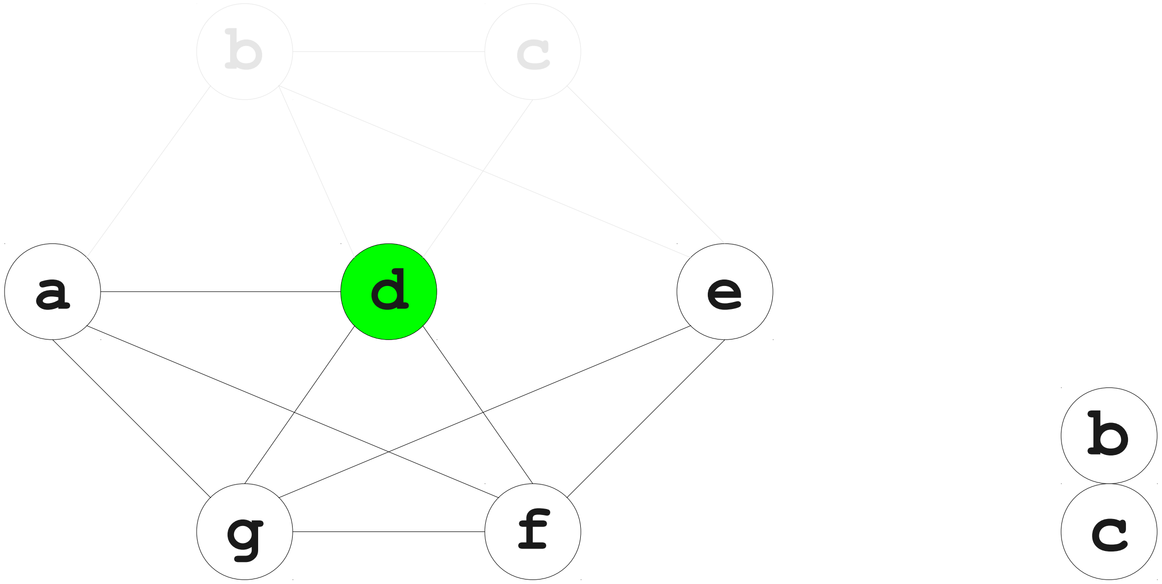

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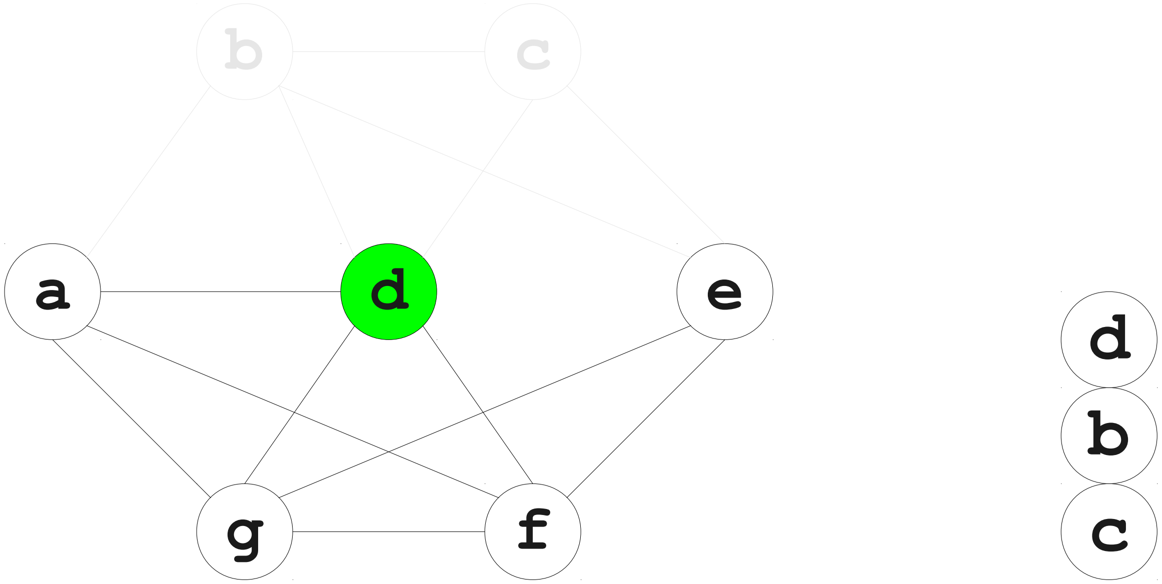

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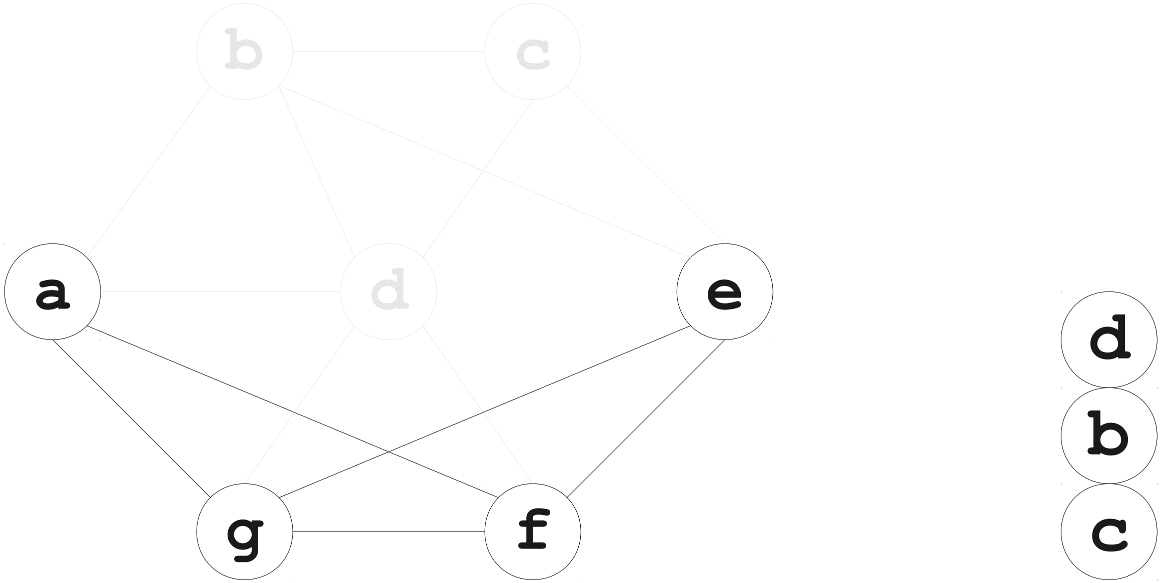

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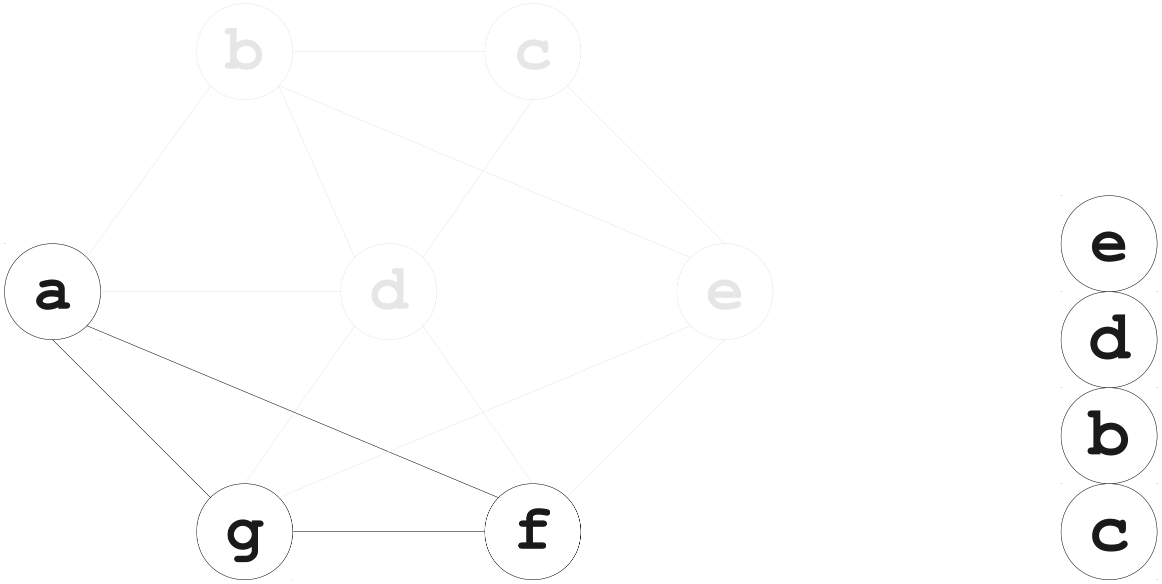

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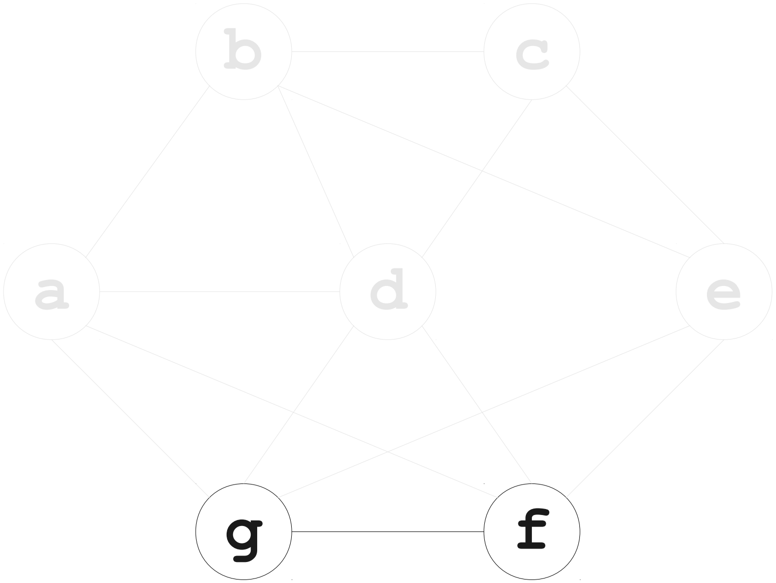

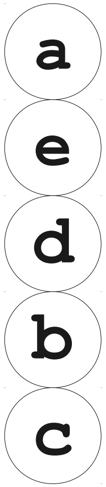

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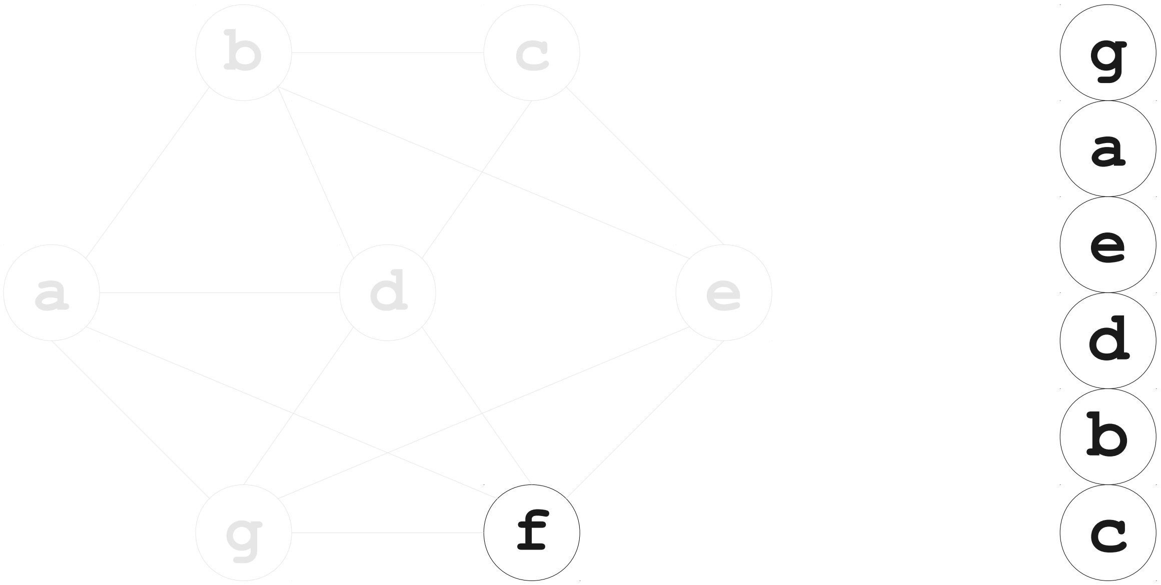

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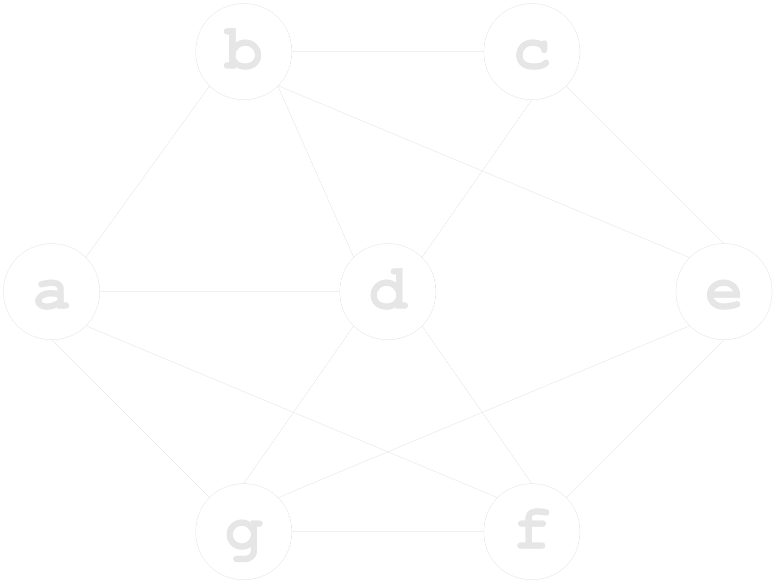

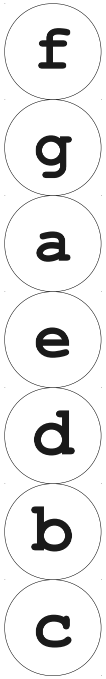

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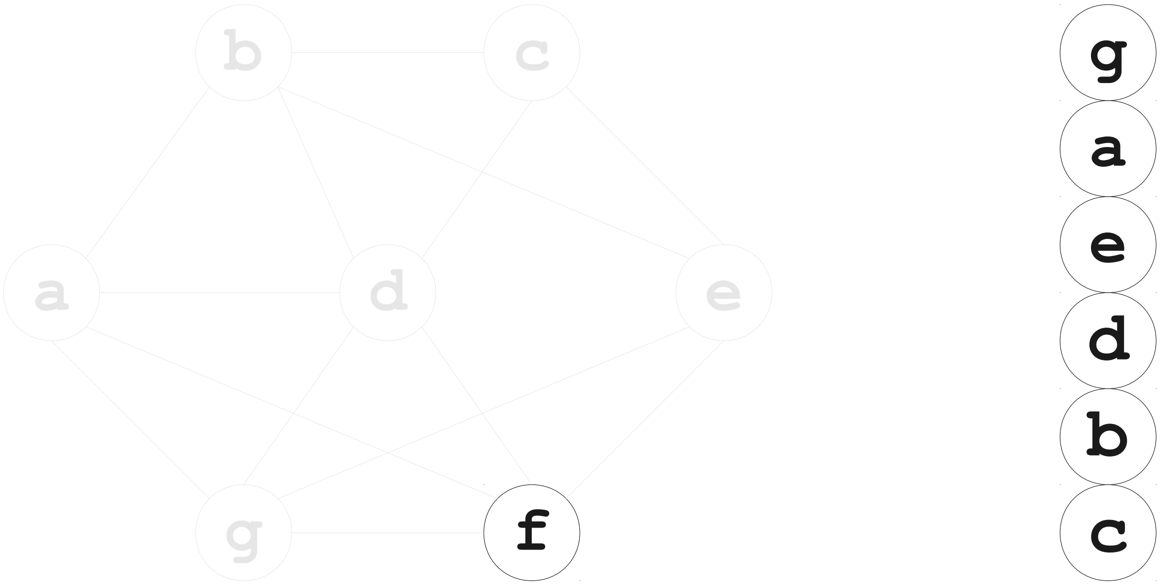

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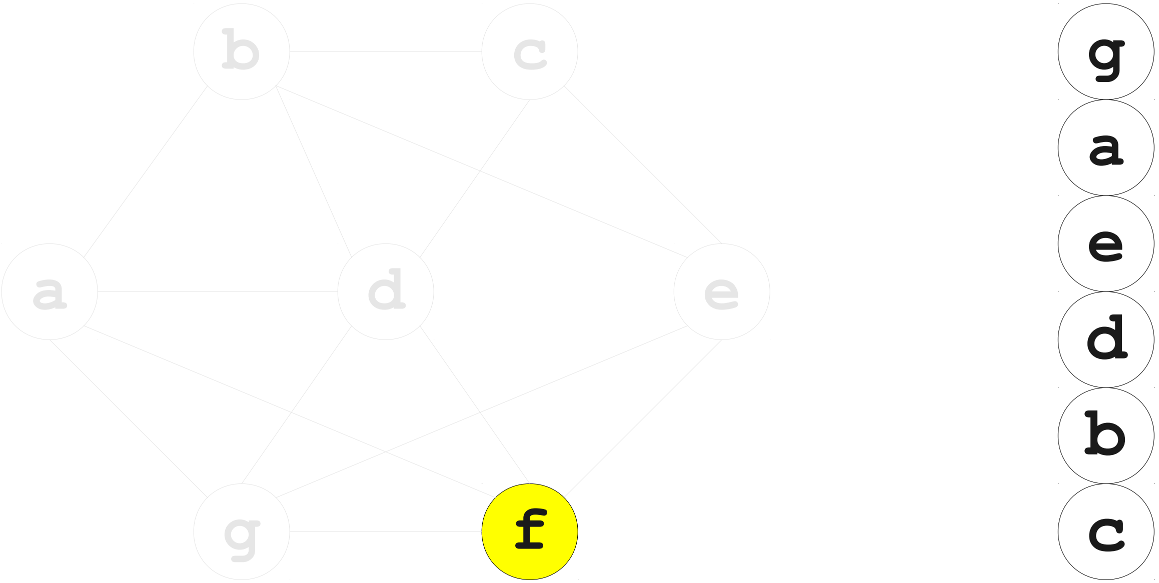

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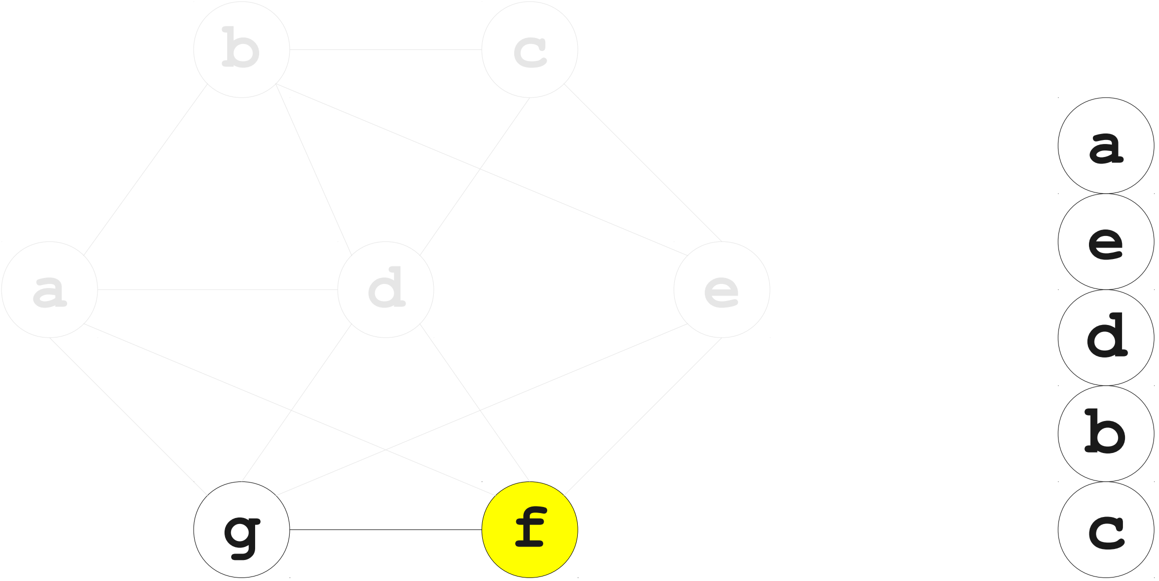

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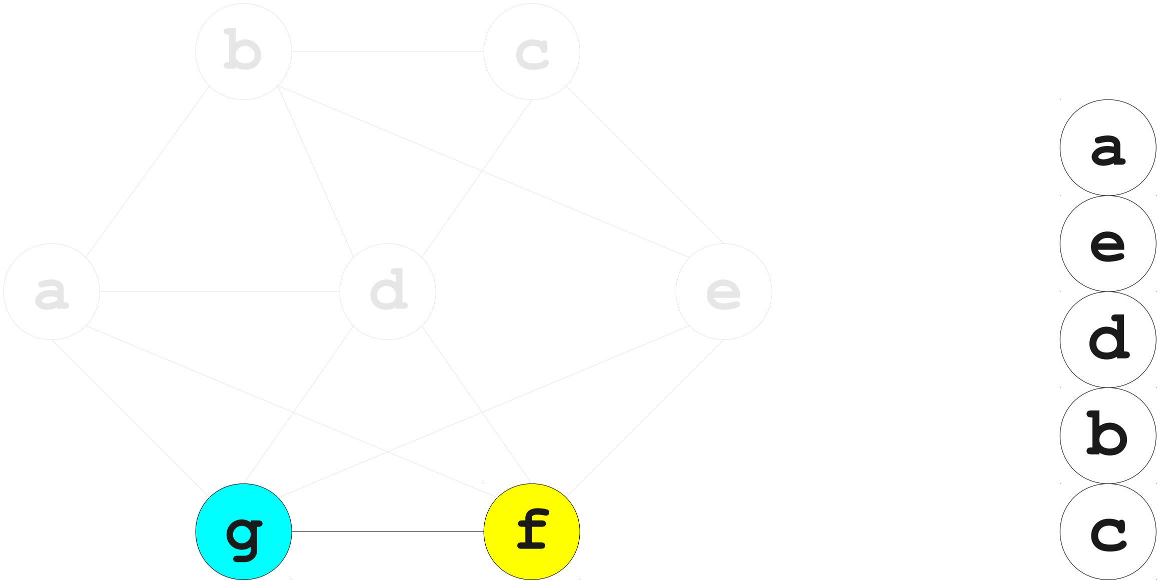

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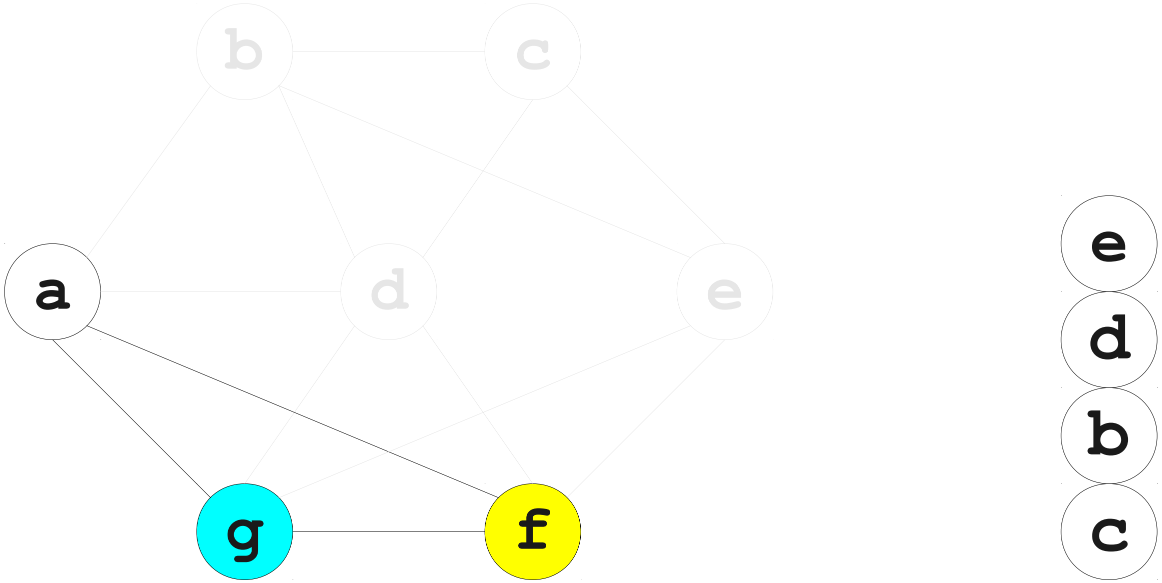

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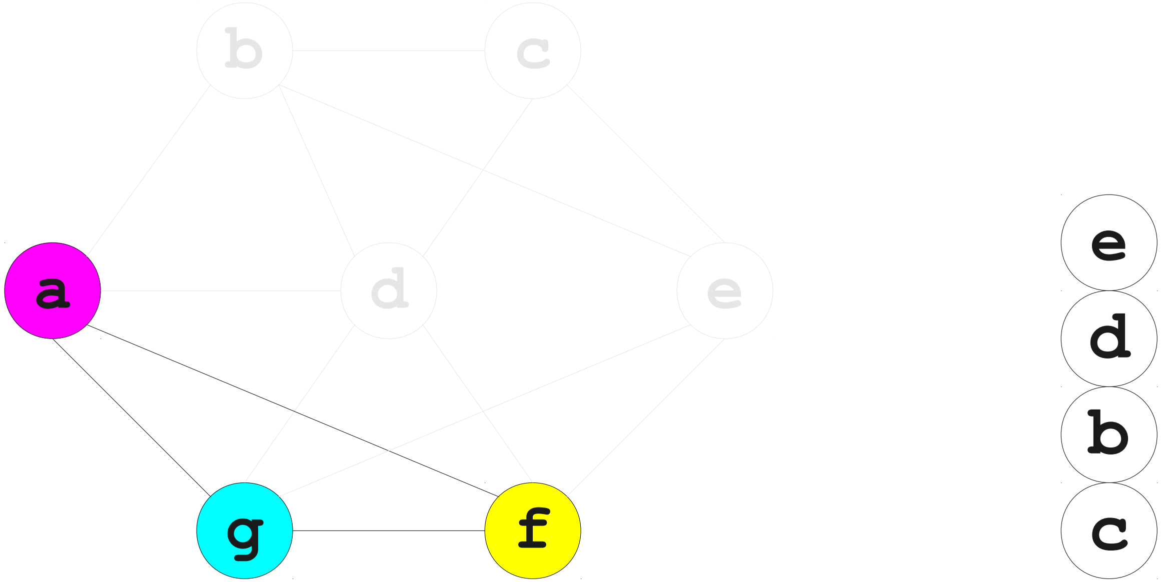

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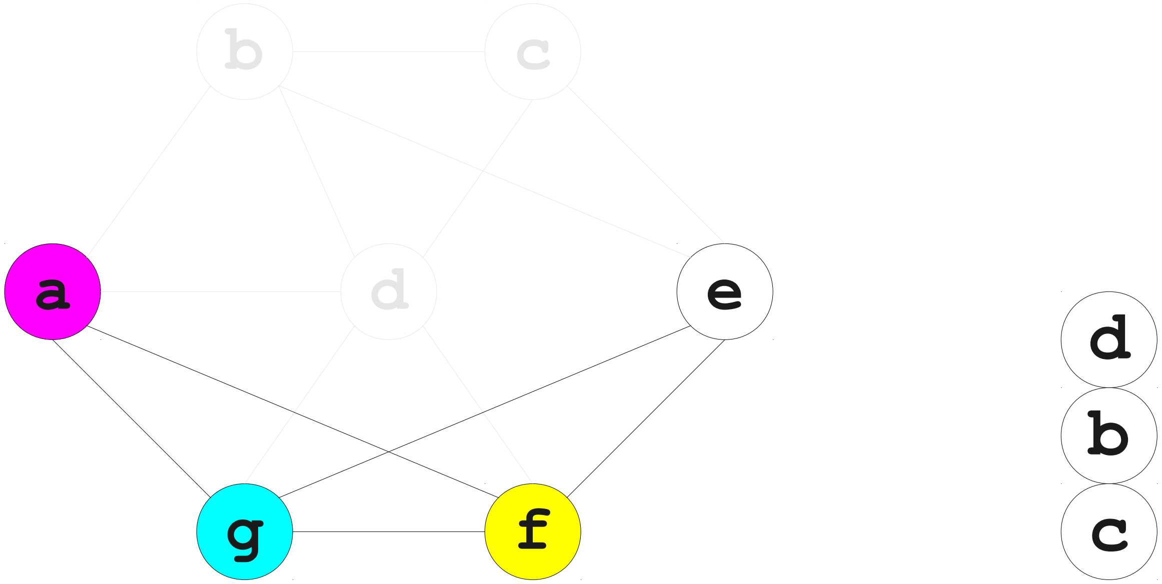

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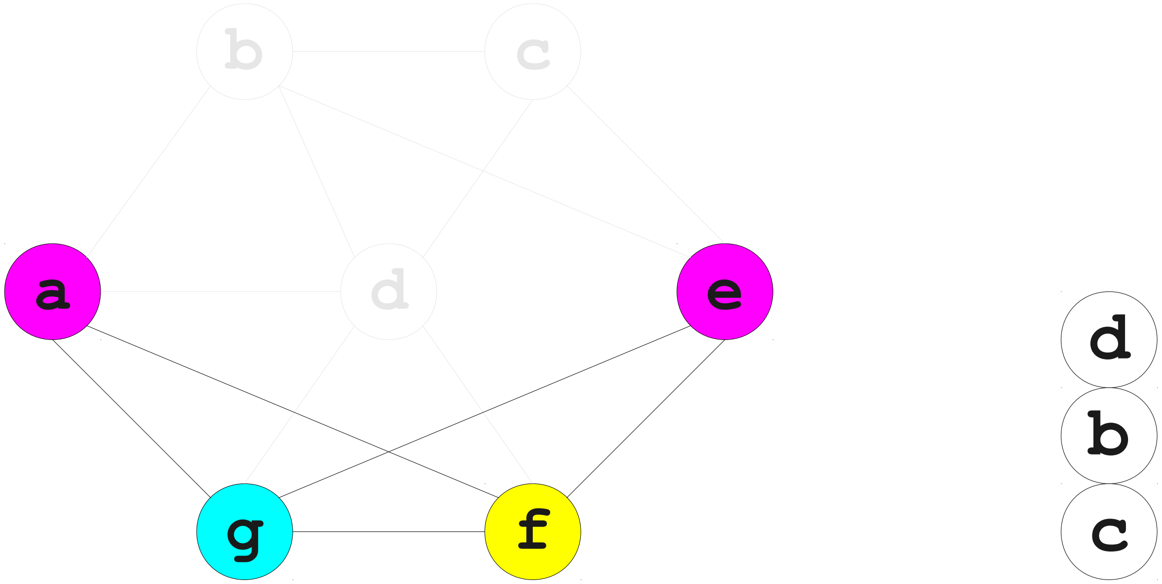

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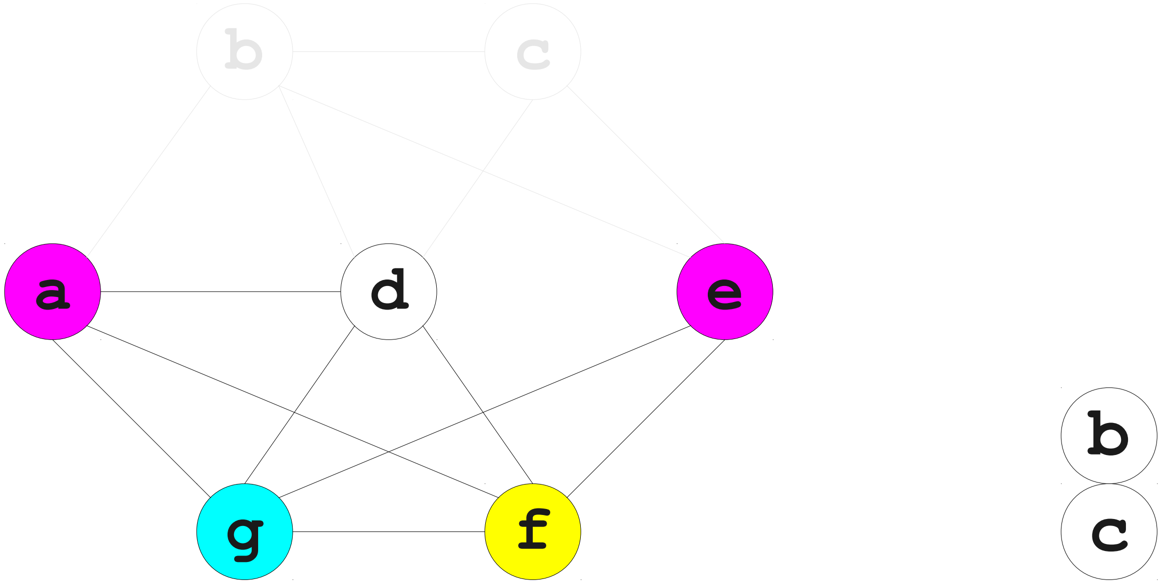

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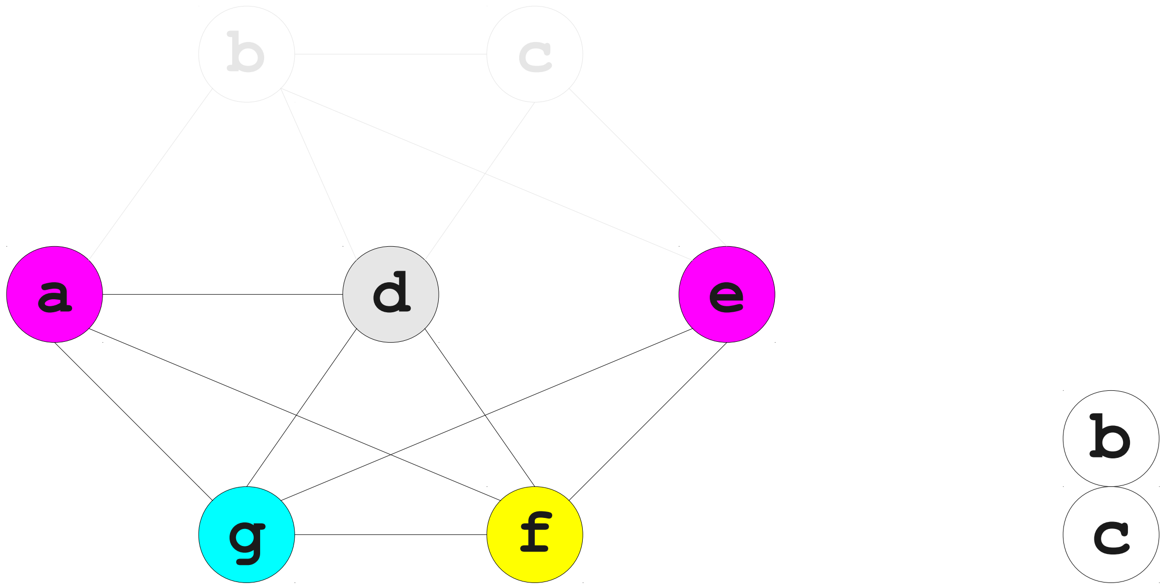

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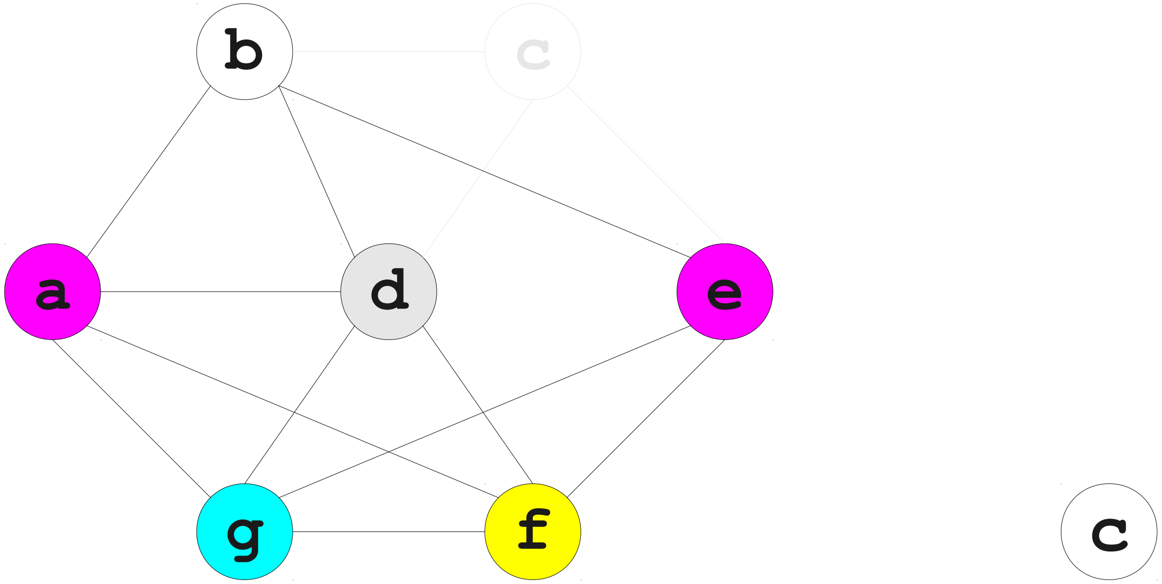

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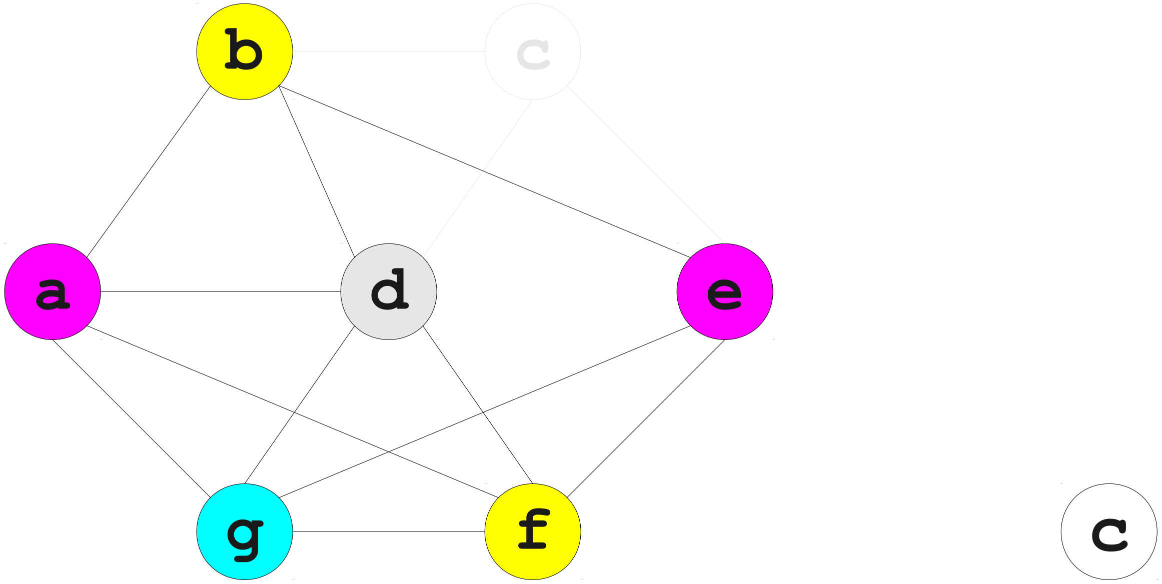

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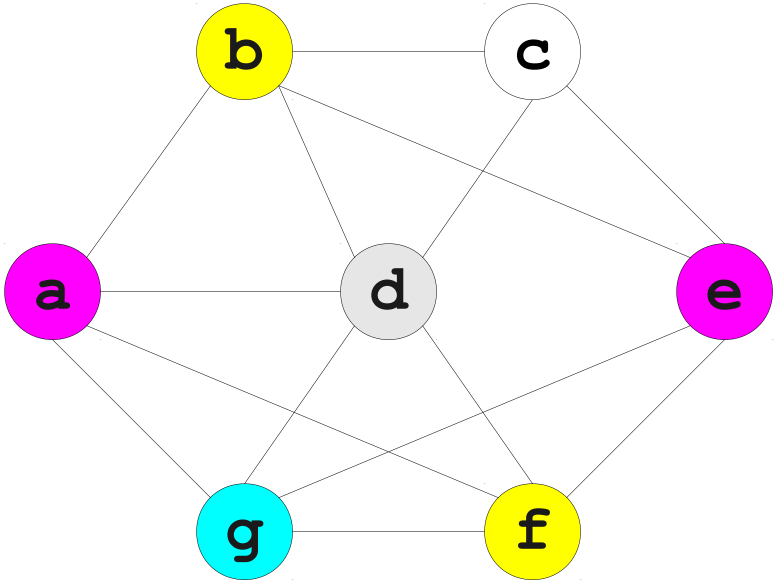

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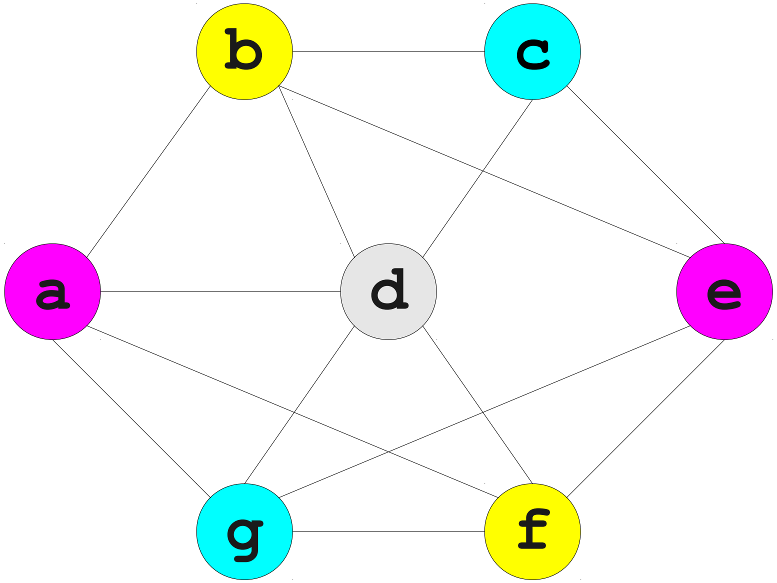

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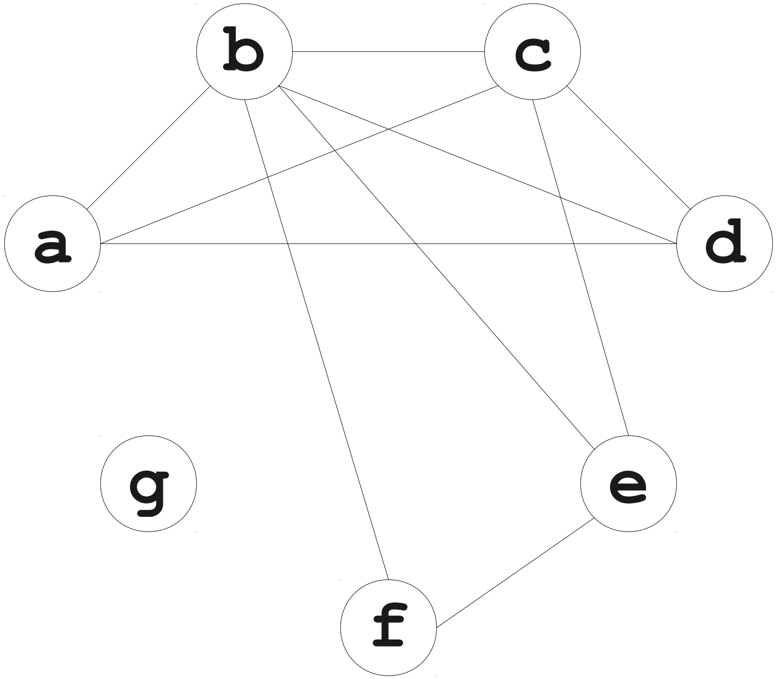
One Problem

- What if we can't find a node with fewer than *k* neighbors?
- Choose and remove an arbitrary node, marking it “troublesome.”
  - Use heuristics to choose which one.
- When adding node back in, it may be possible to find a valid color.
- Otherwise, we have to spill that node.

Chaitin's Algorithm Reloaded



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R_0 R_1 R_2

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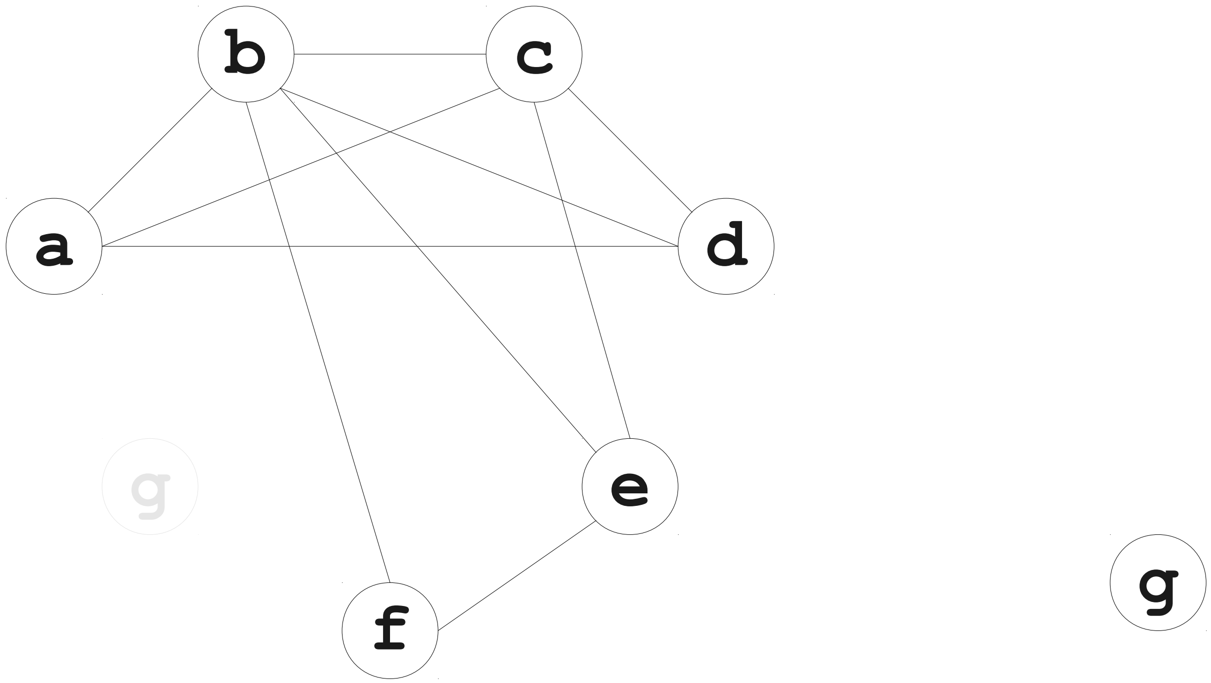

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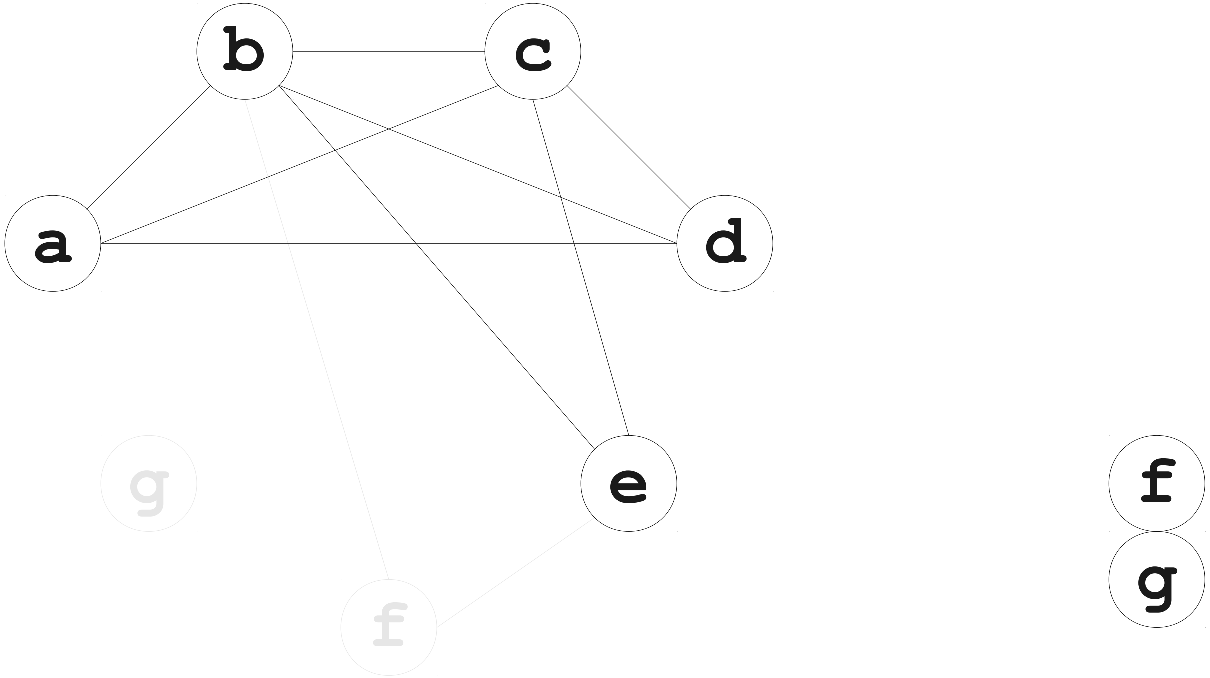

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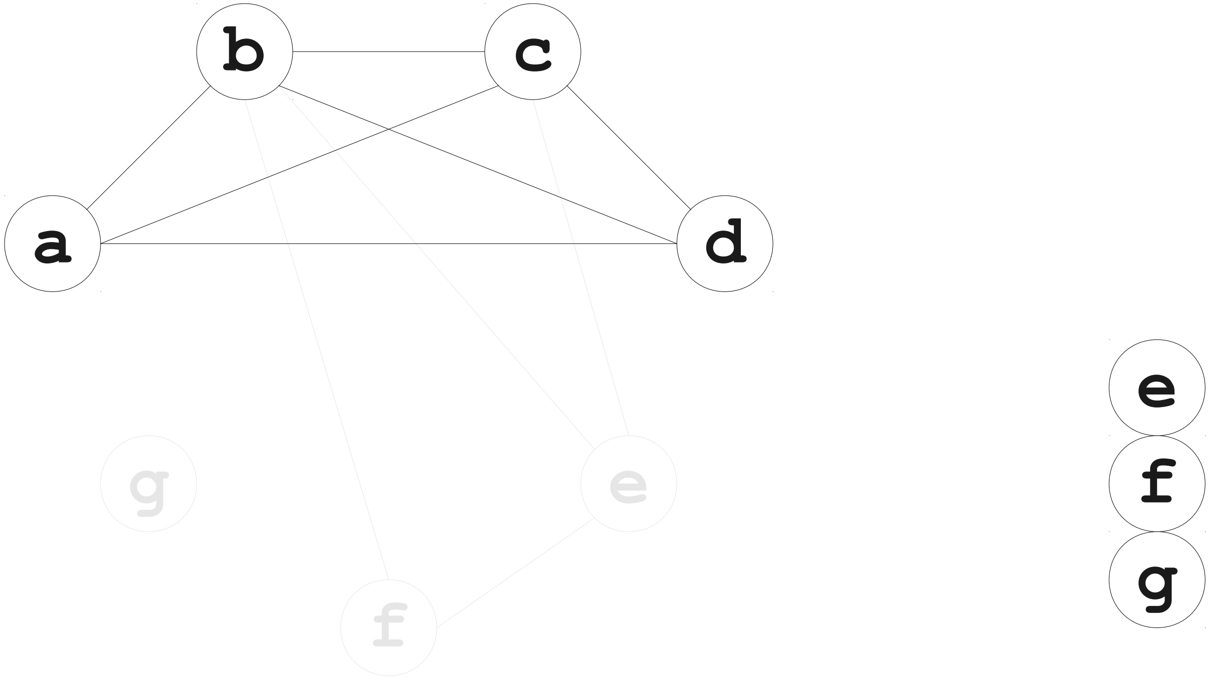

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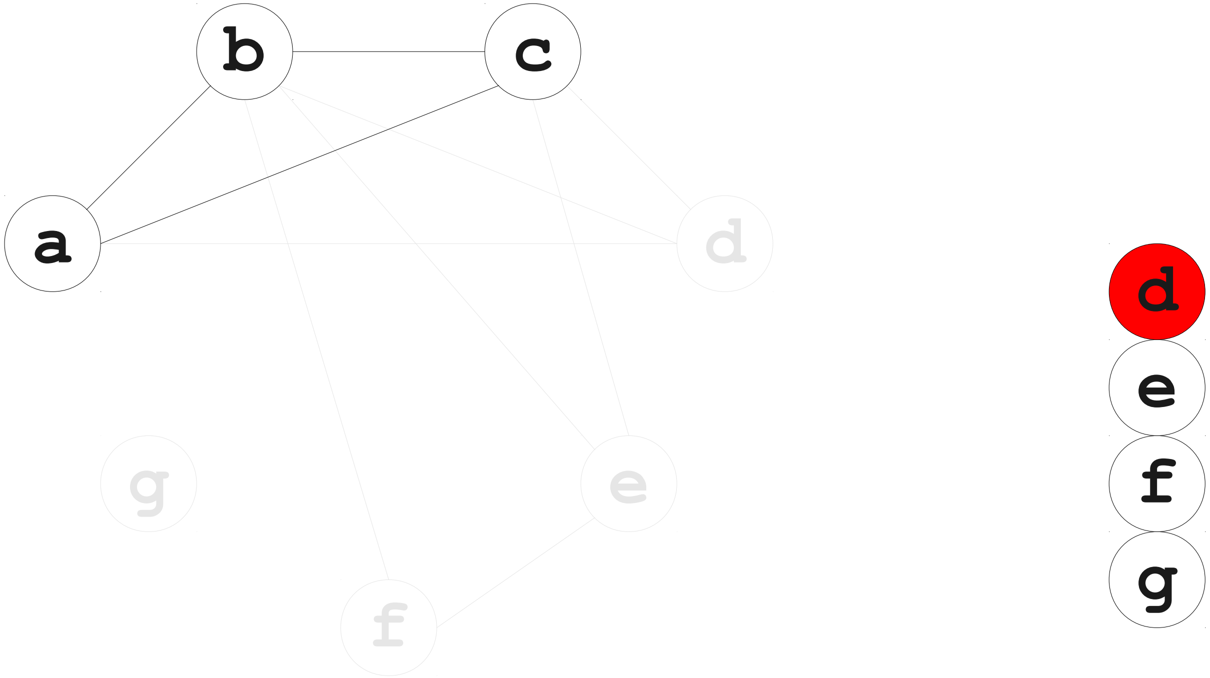

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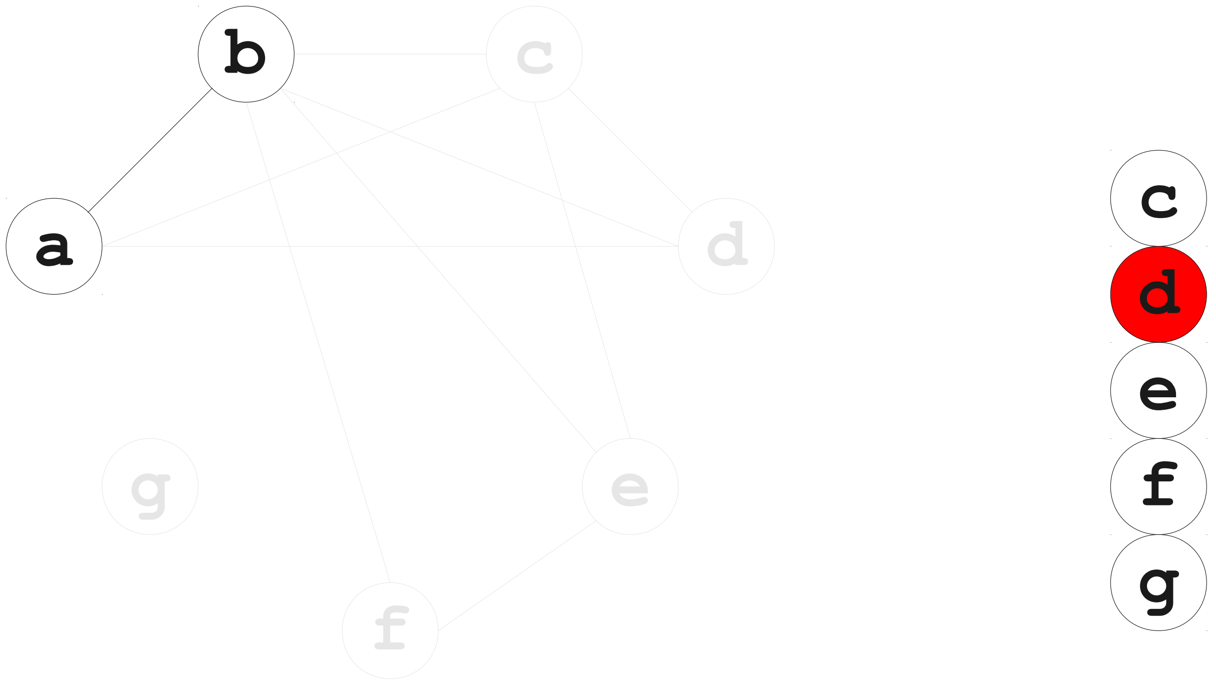

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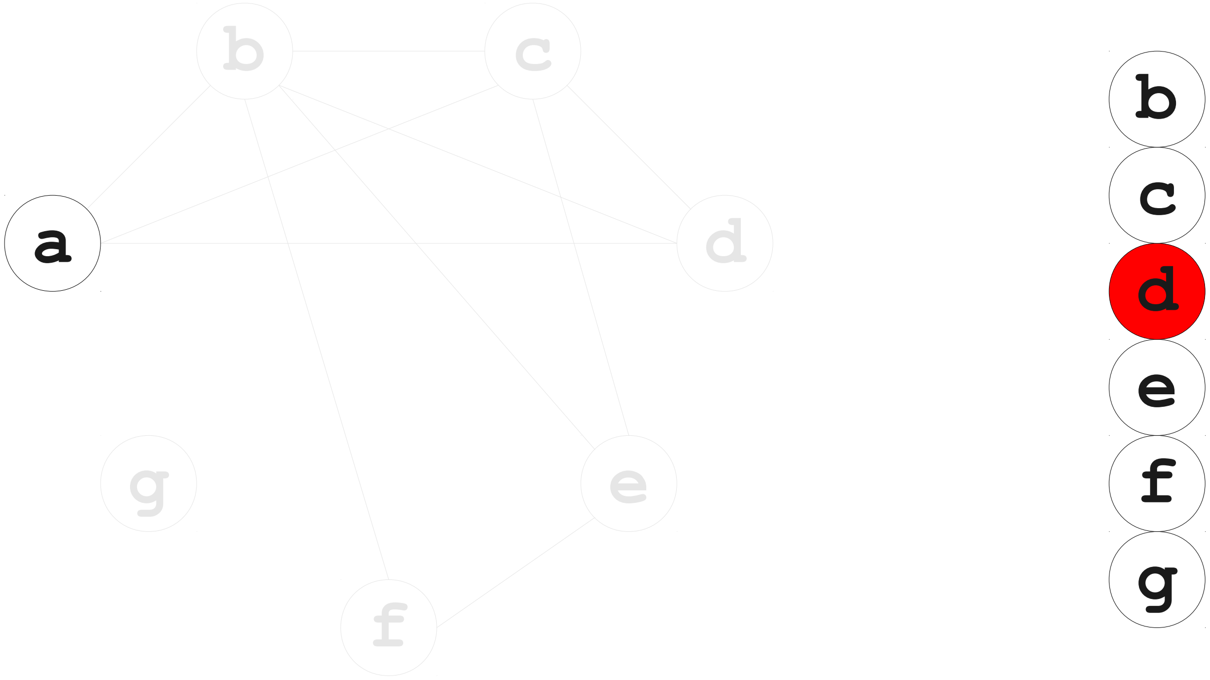

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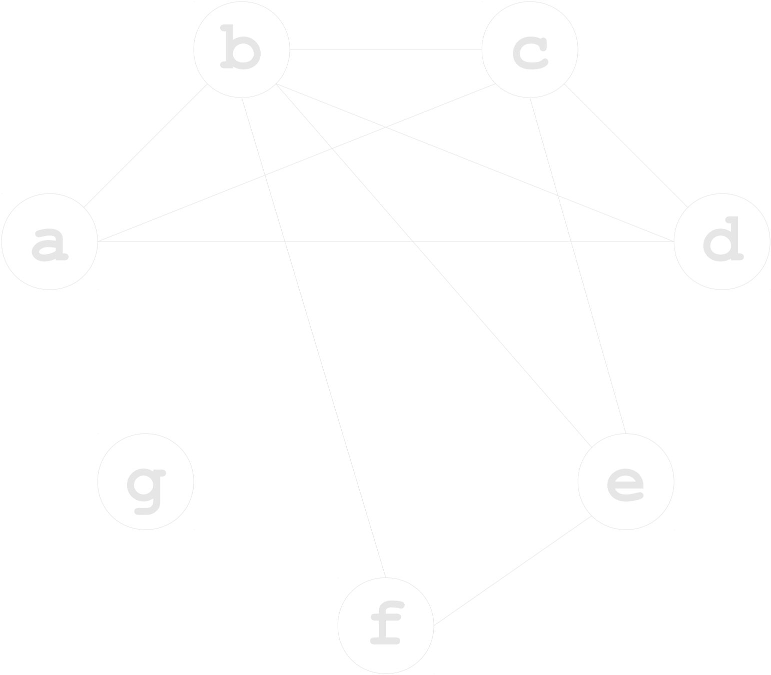

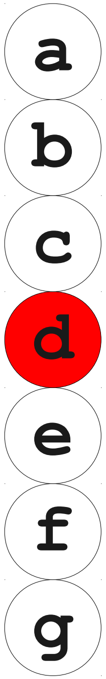

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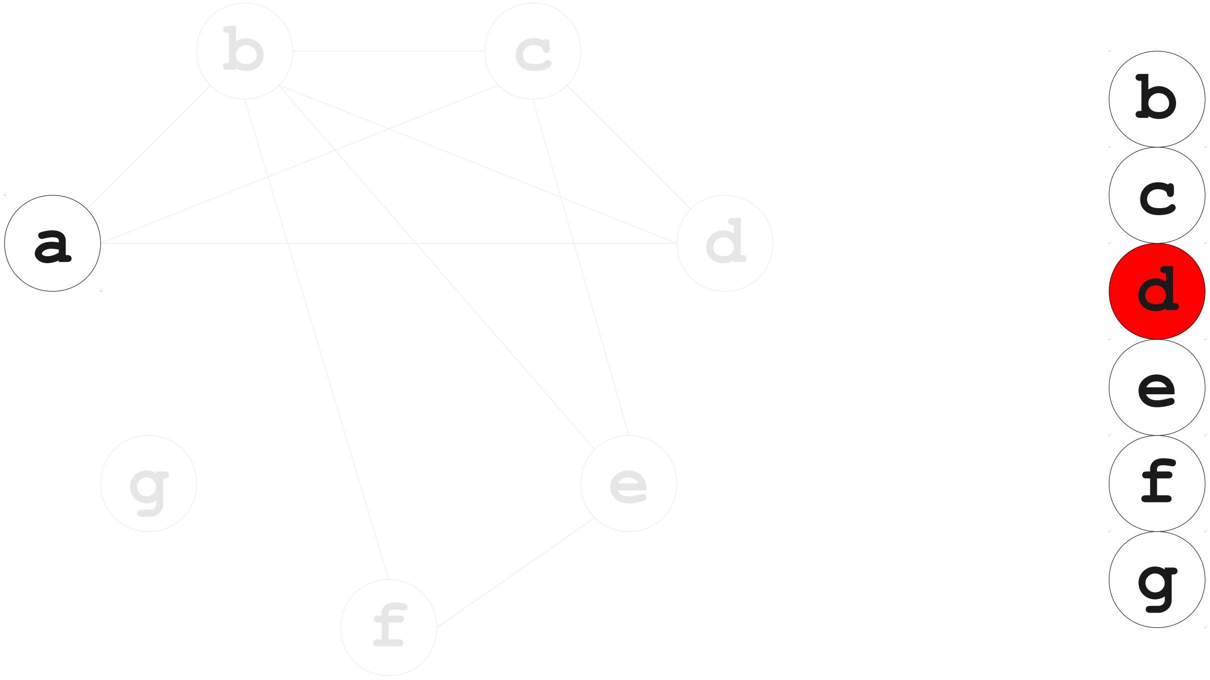

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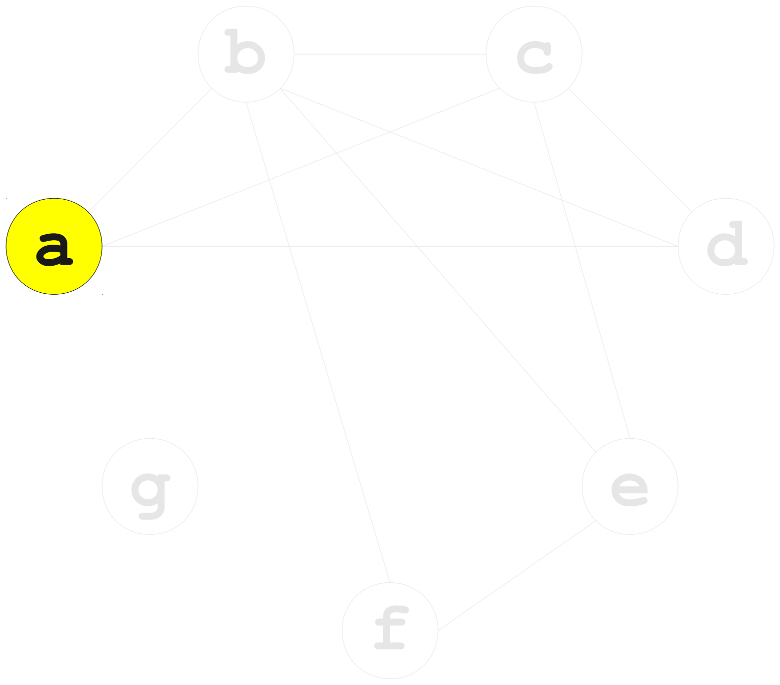

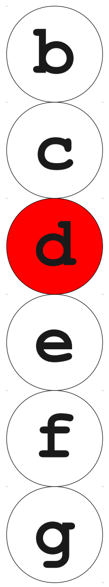

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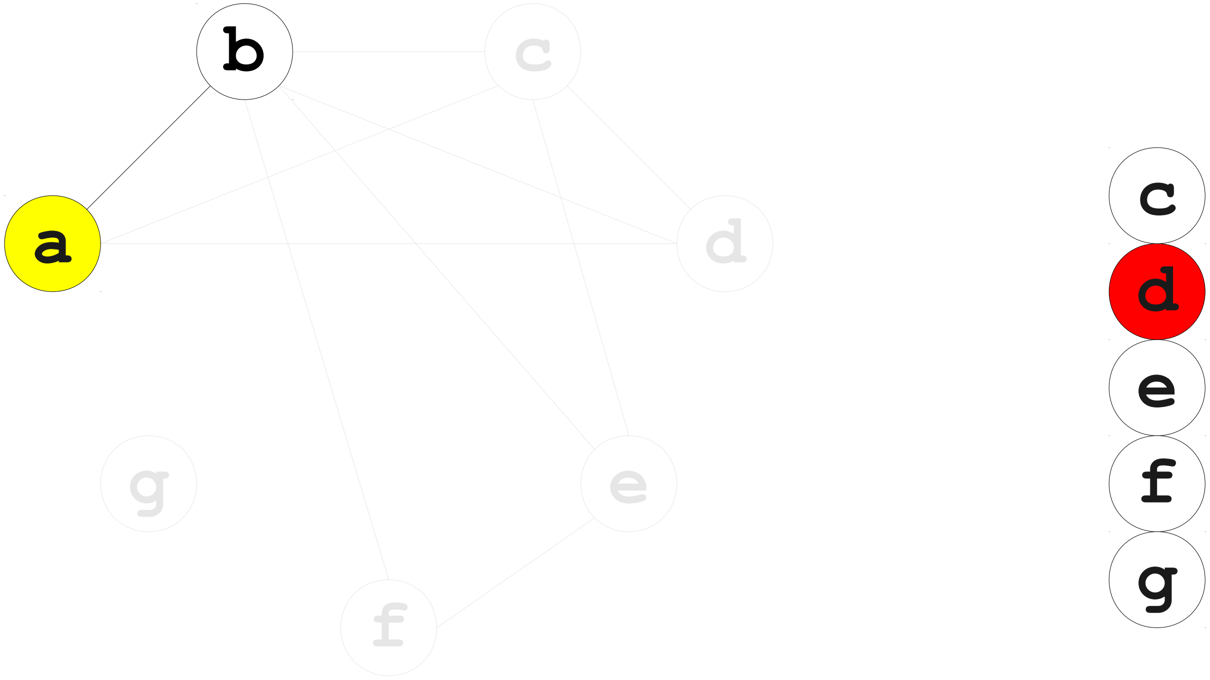

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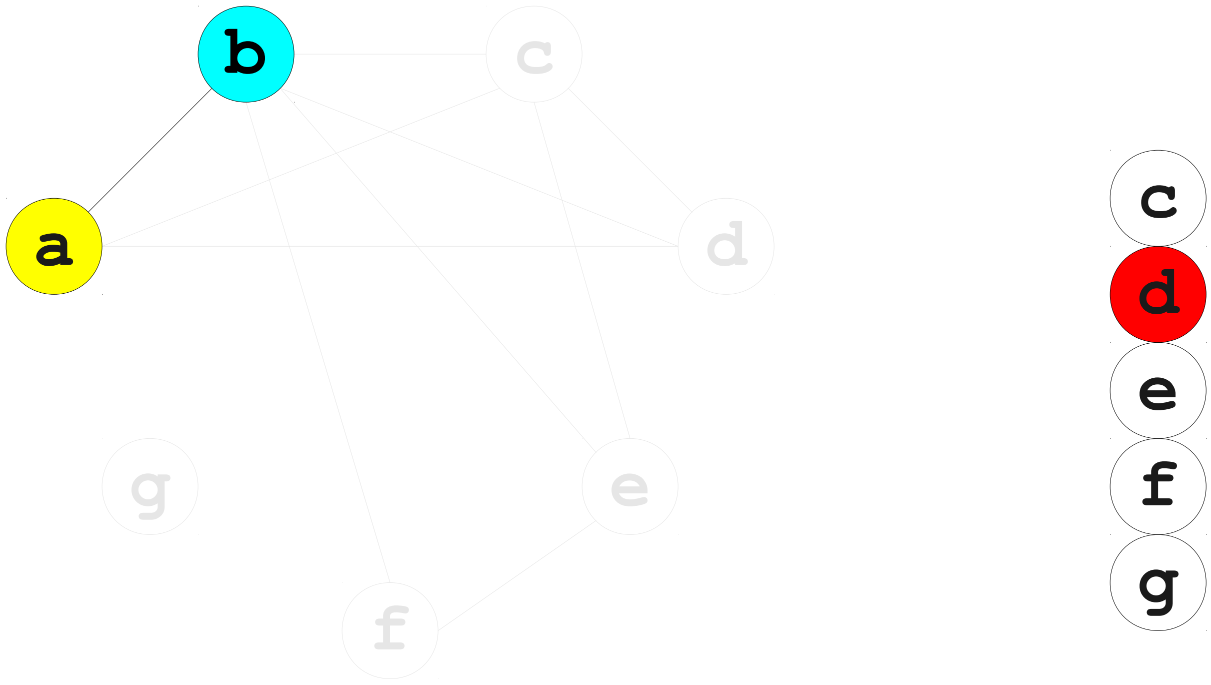

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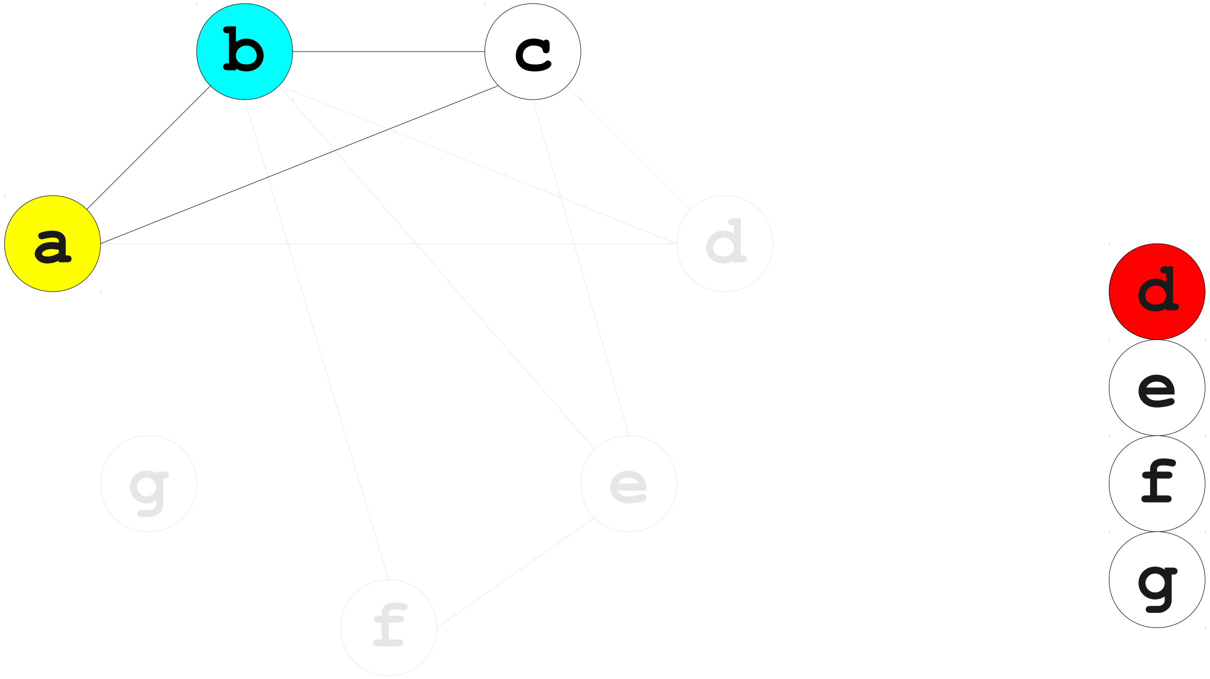

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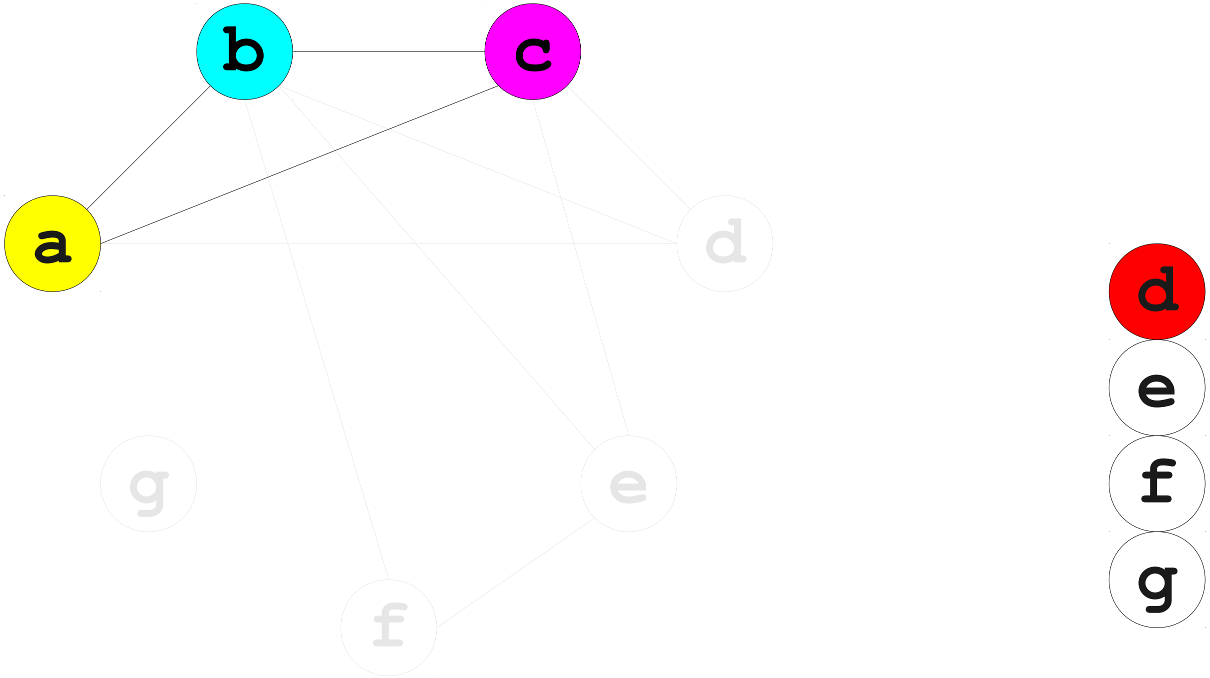

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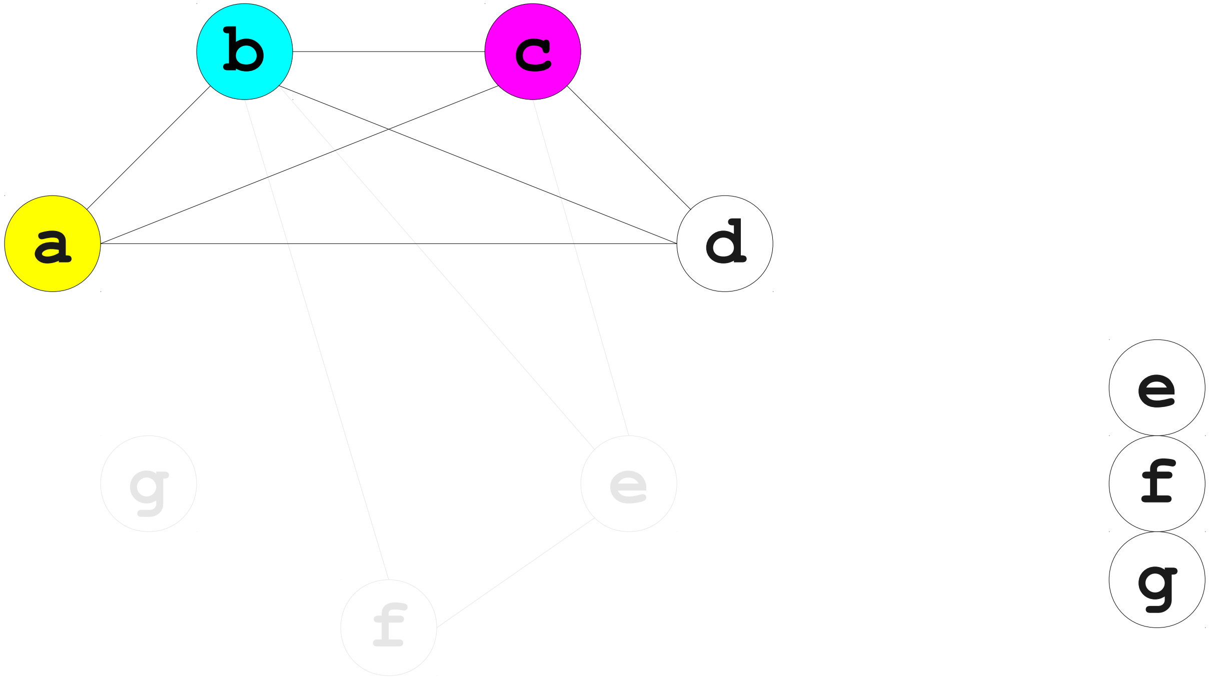

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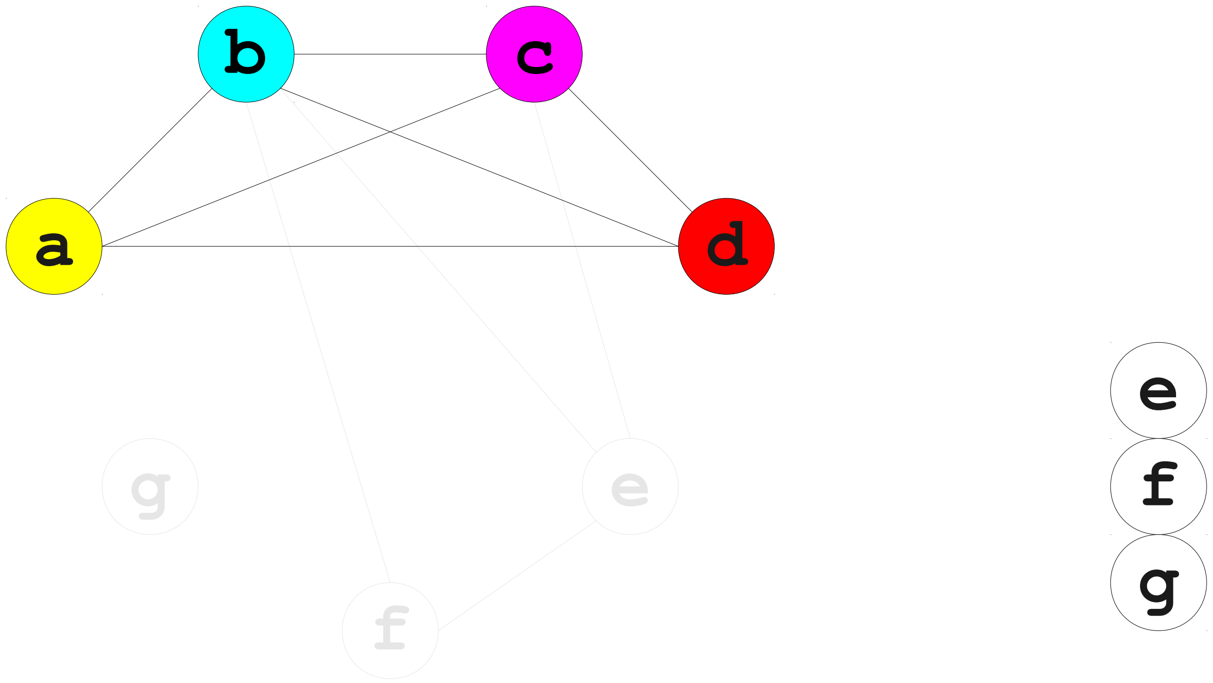

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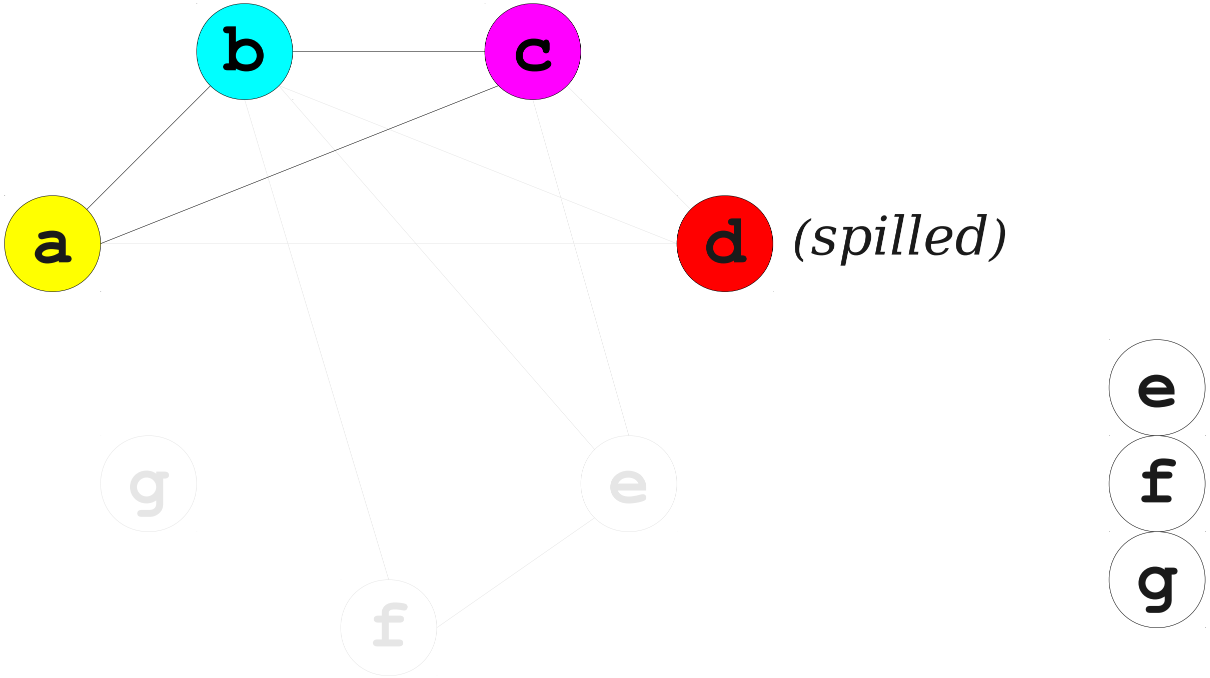

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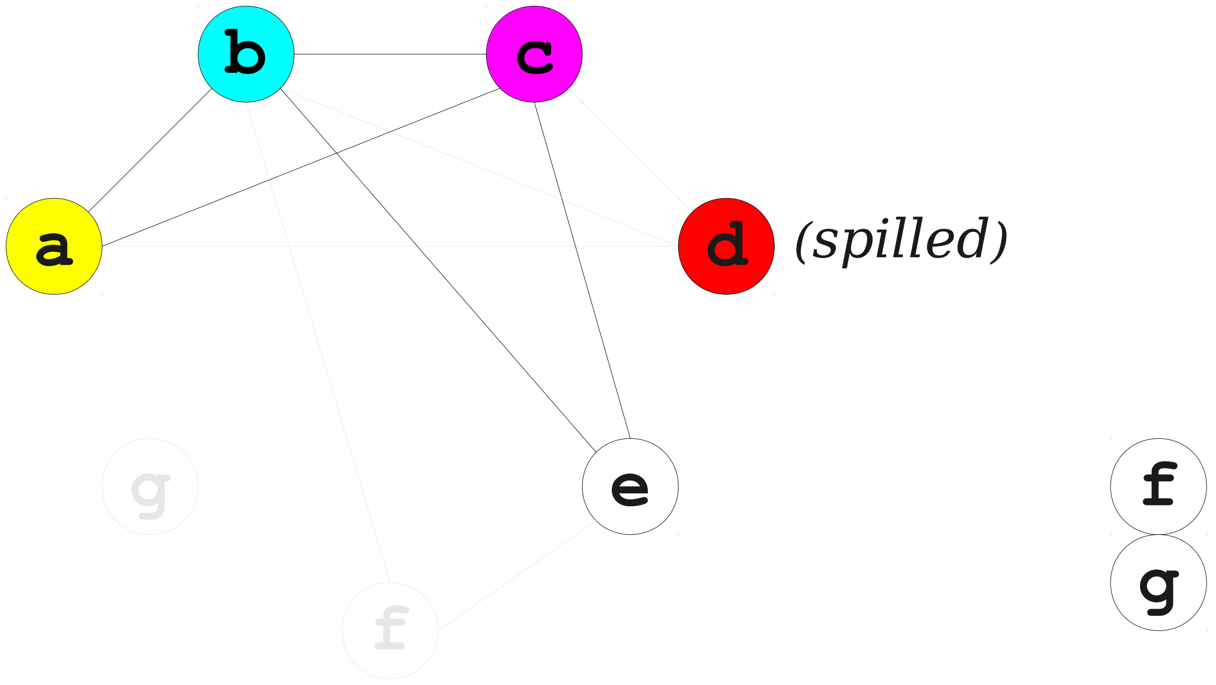
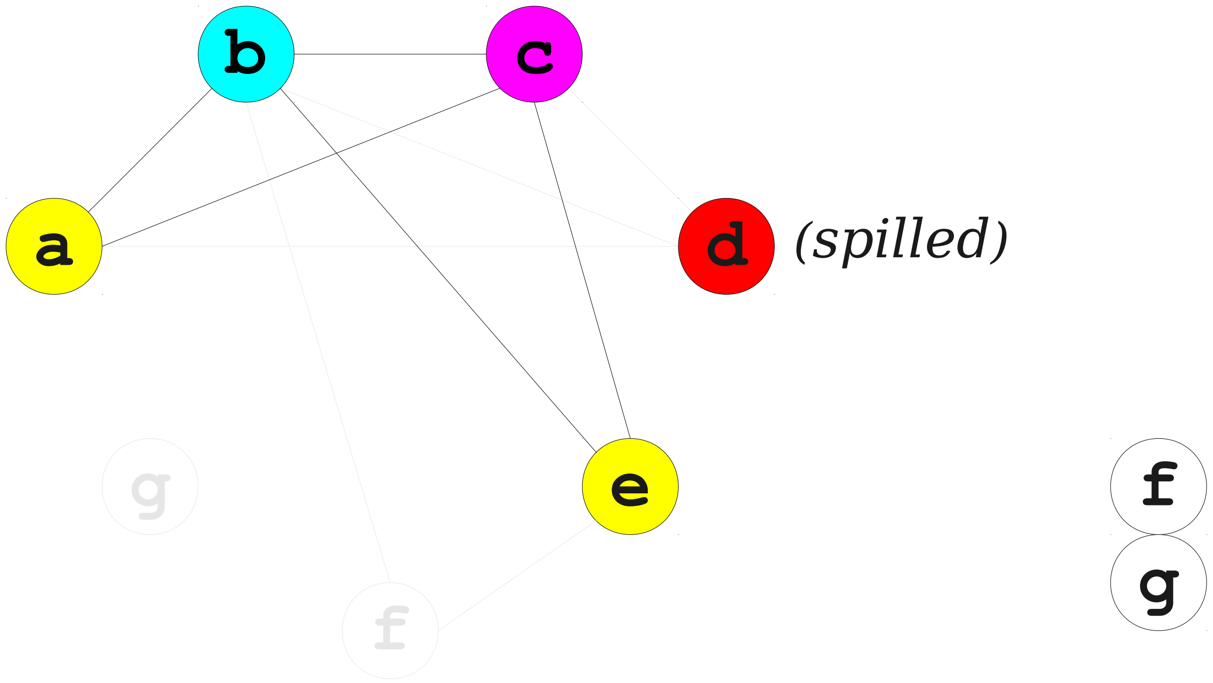
Chaitin's Algorithm Reloaded



Registers

R_0	R_1	R_2

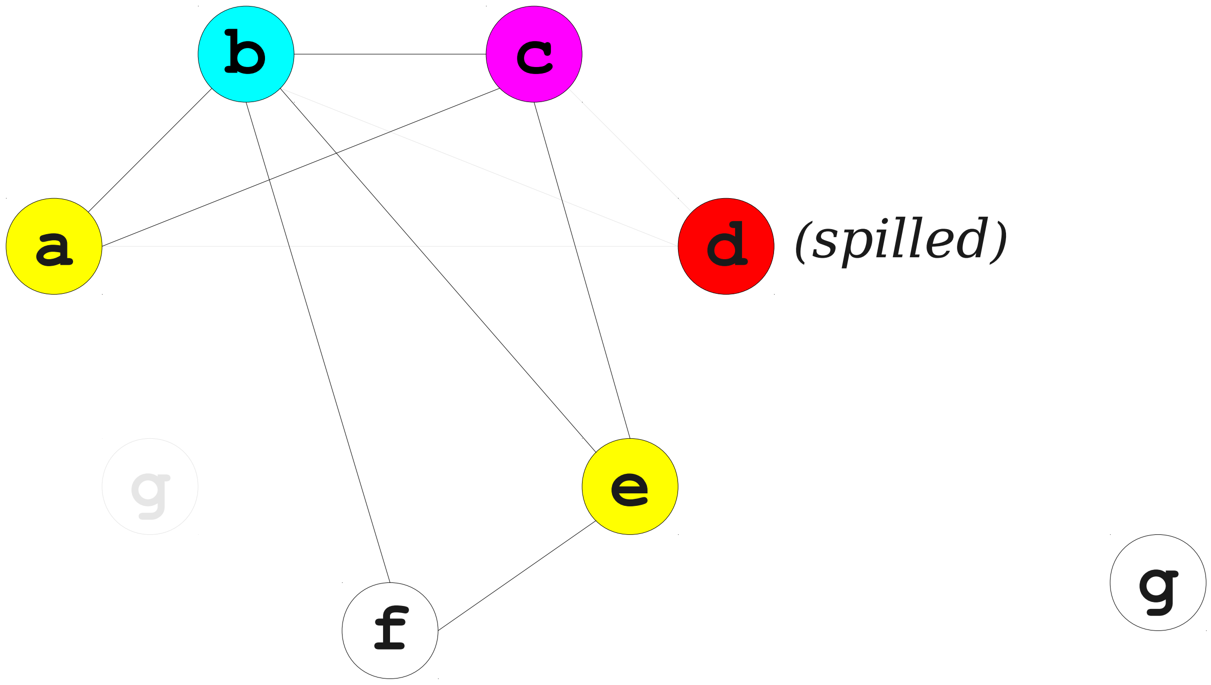
Chaitin's Algorithm Reloaded



Registers

R_0	R_1	R_2

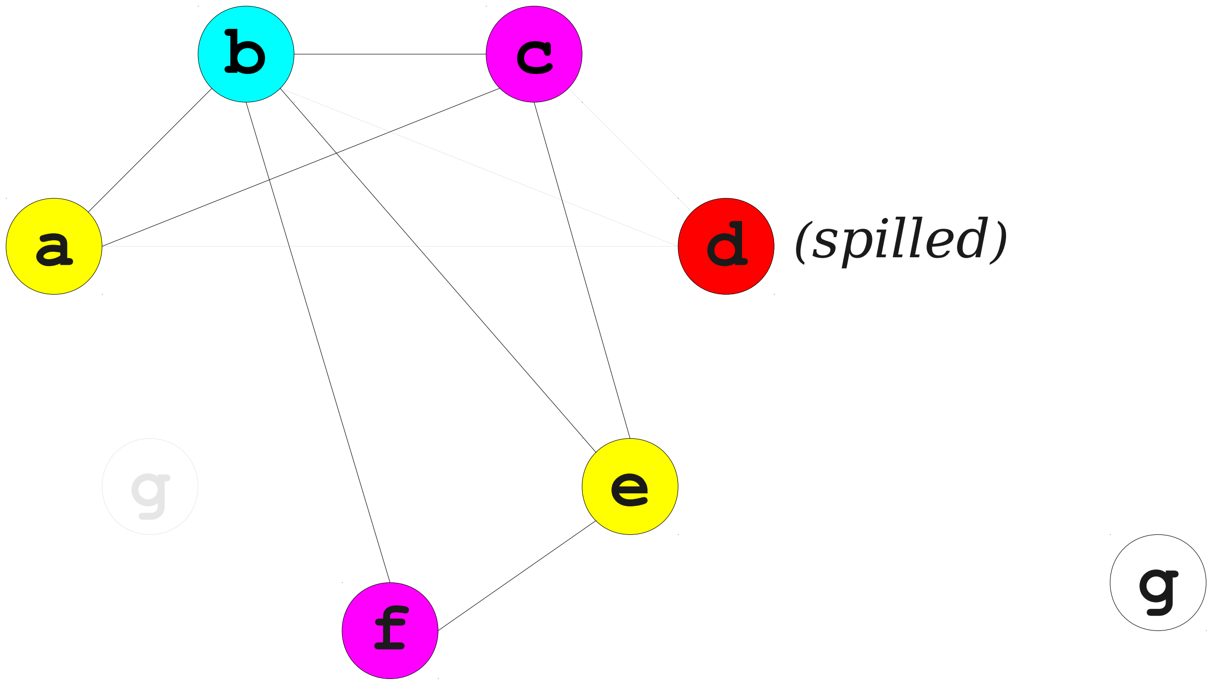
Chaitin's Algorithm Reloaded



Registers

R_0	**R_1**	**R_2**

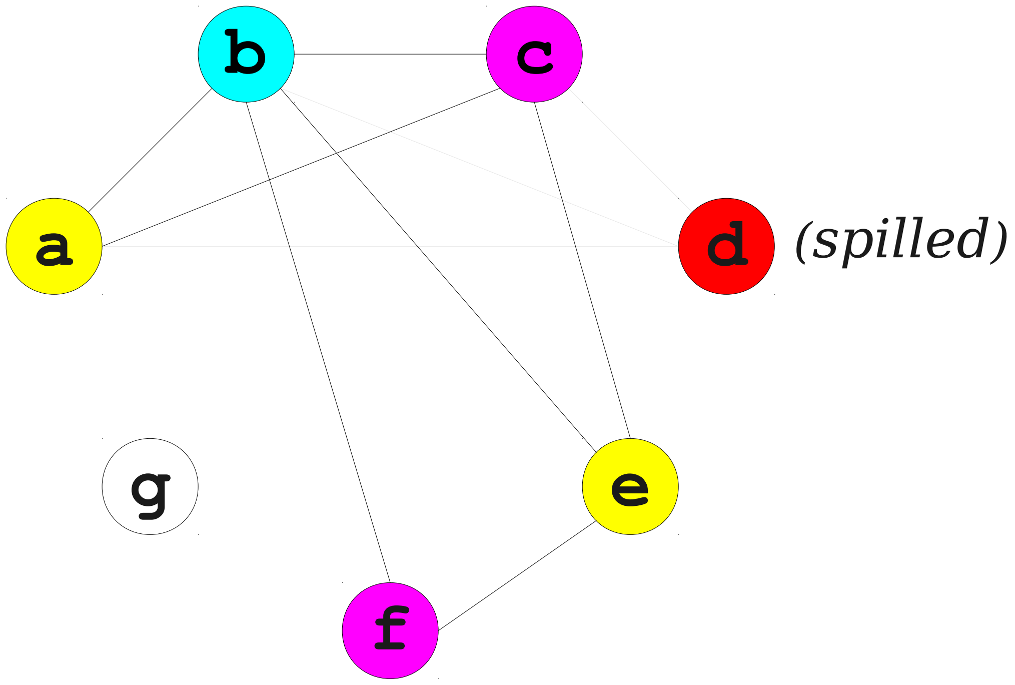
Chaitin's Algorithm Reloaded



Registers

R_0	R_1	R_2

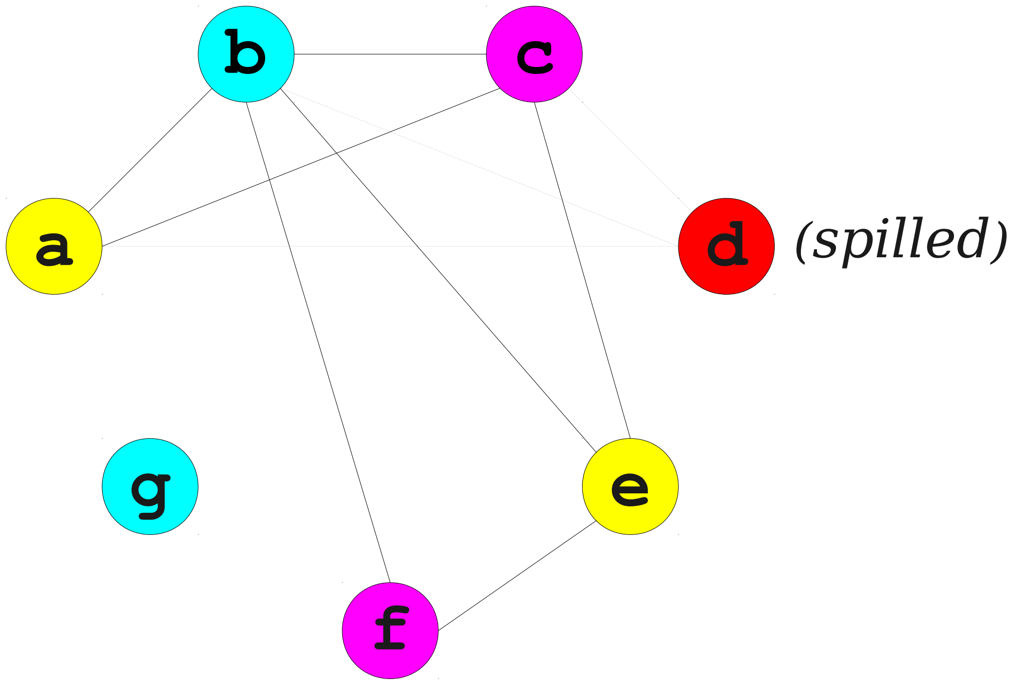
Chaitin's Algorithm Reloaded



Registers

R_0	R_1	R_2

Chaitin's Algorithm Reloaded



Registers

R_0	R_1	R_2

Another Example

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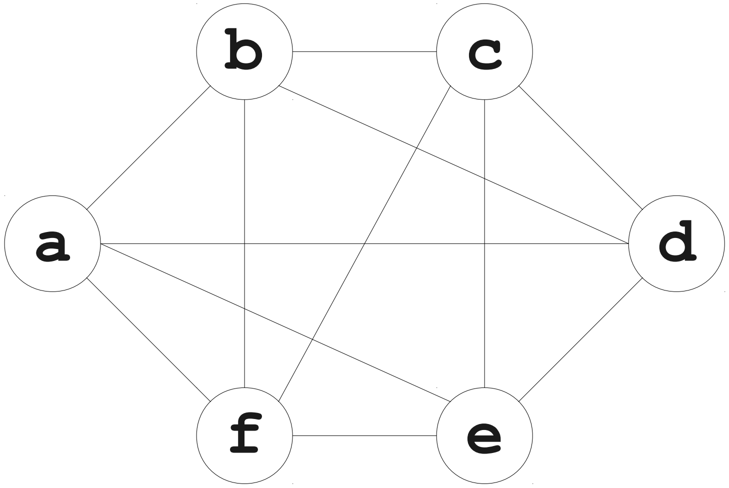
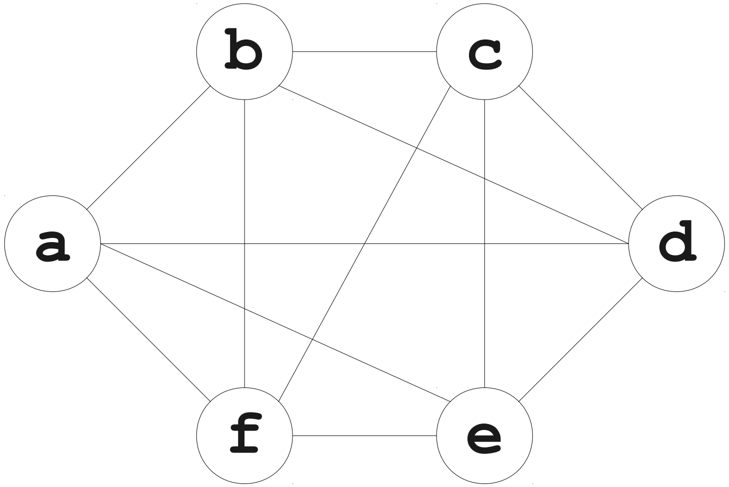


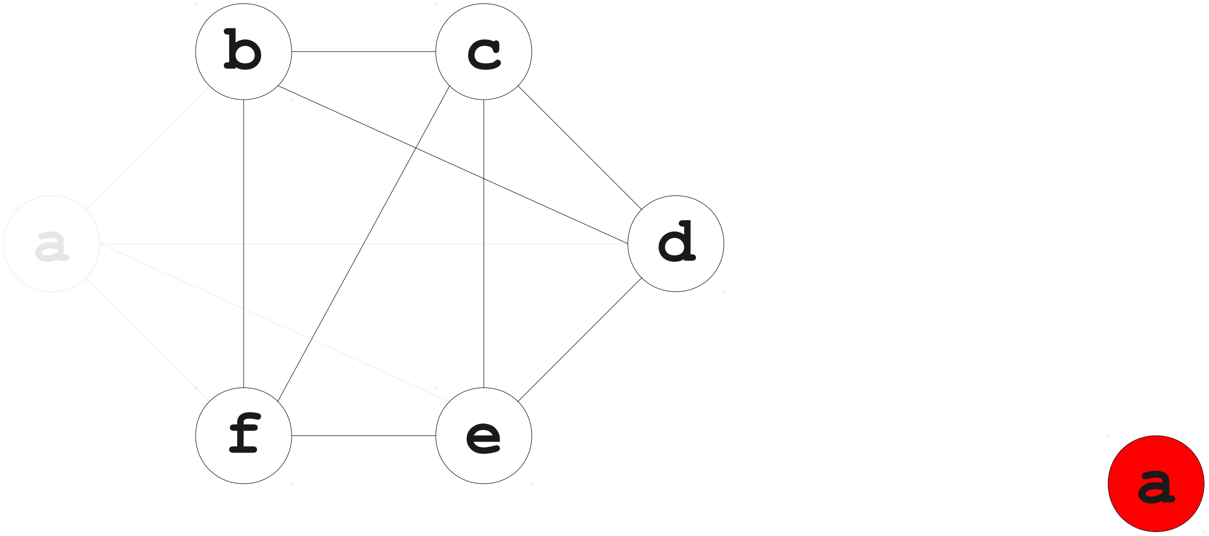
Another Example



Registers

R_0	R_1	R_2

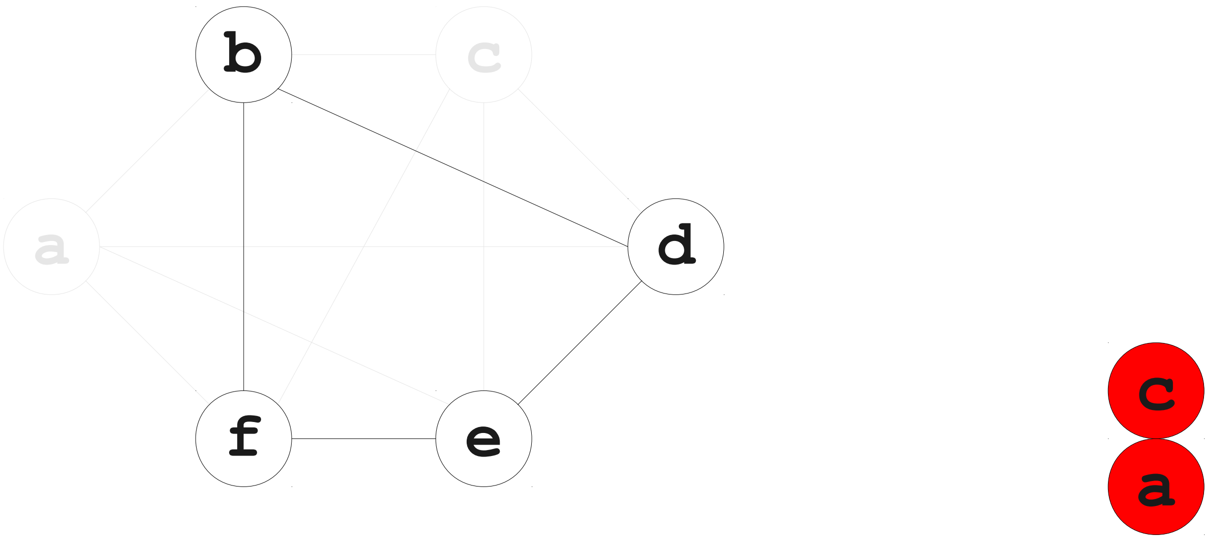
Another Example



Registers

R_0	R_1	R_2

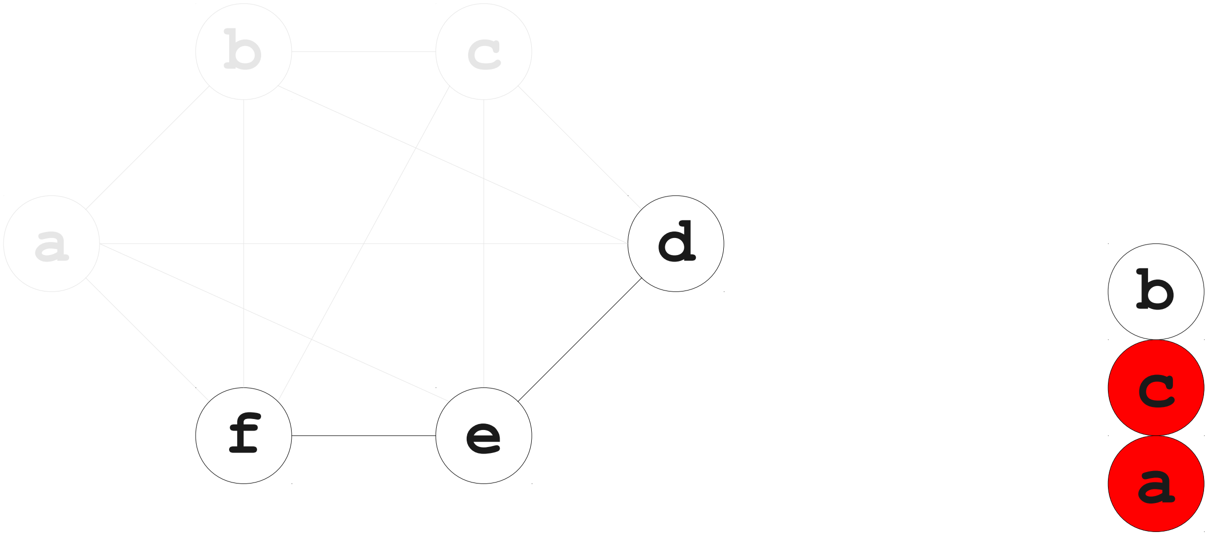
Another Example



Registers

R_0	R_1	R_2

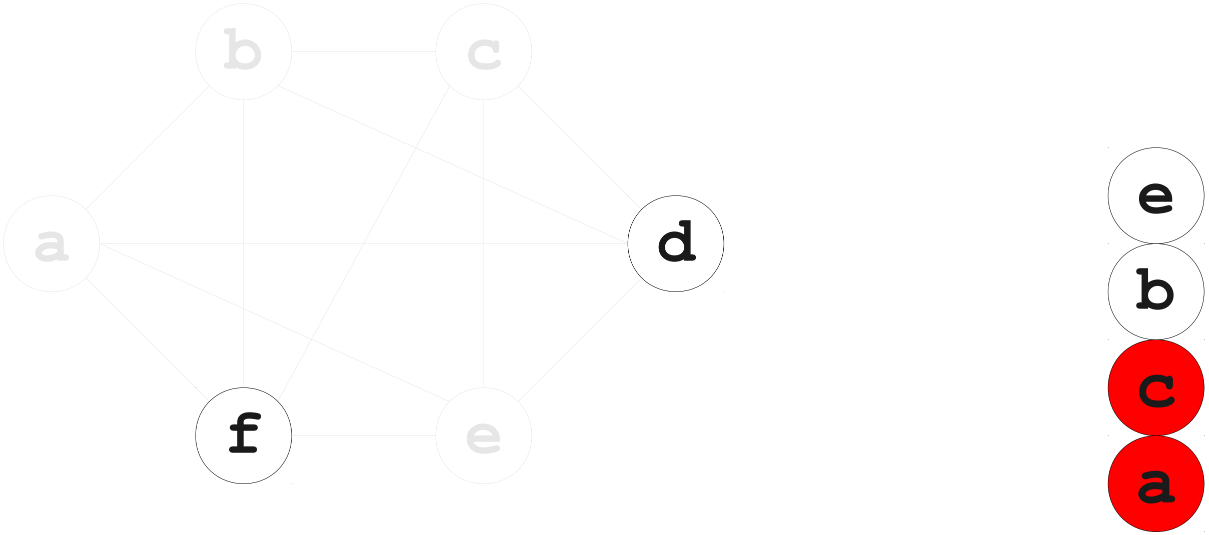
Another Example



Registers

R_0	R_1	R_2

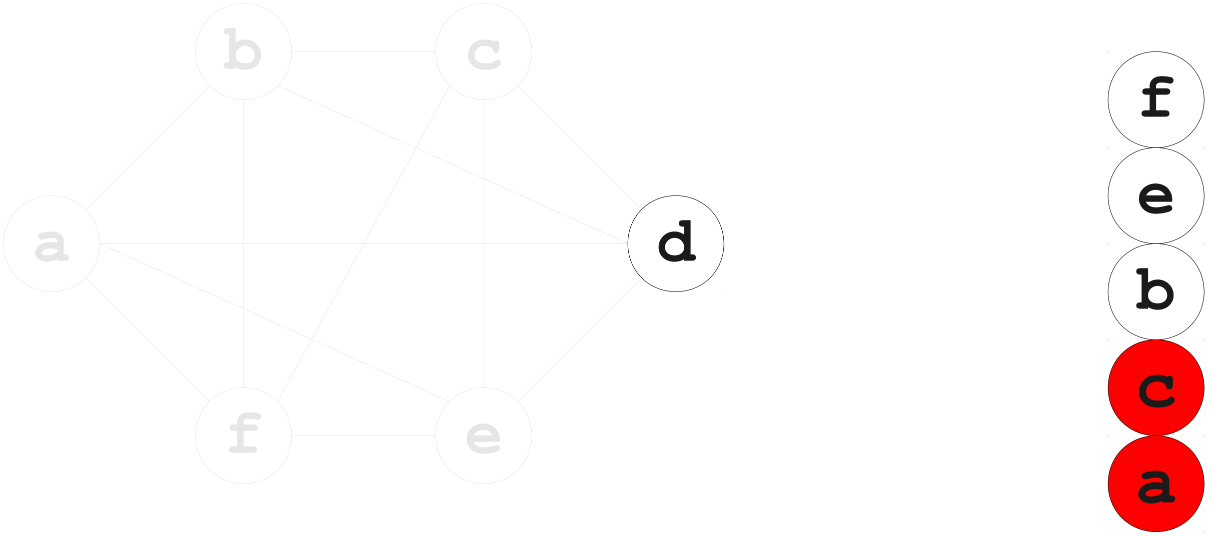
Another Example



Registers

R_0	R_1	R_2

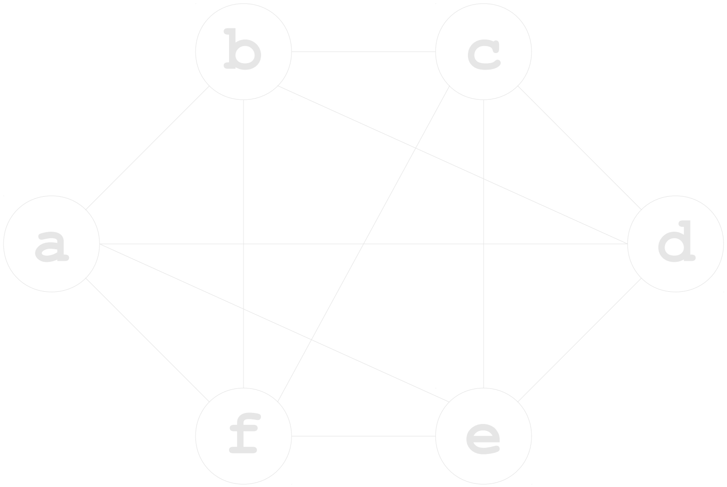
Another Example

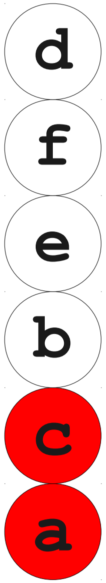


Registers

R_0	R_1	R_2

Another Example

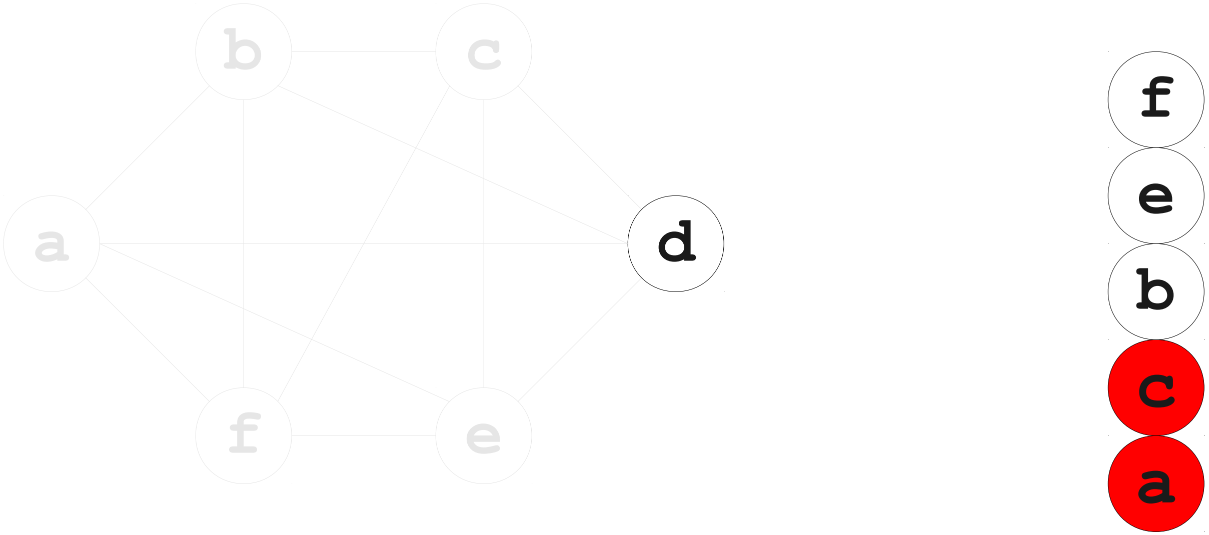




Registers

R_0	R_1	R_2

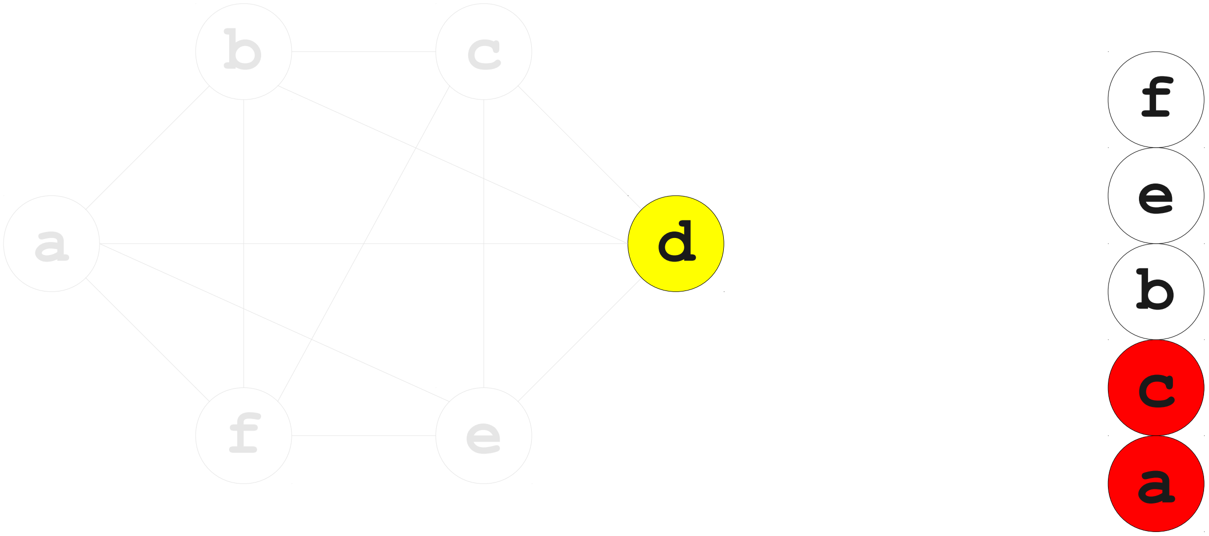
Another Example



Registers

R_0	R_1	R_2

Another Example



Registers

R_0	R_1	R_2

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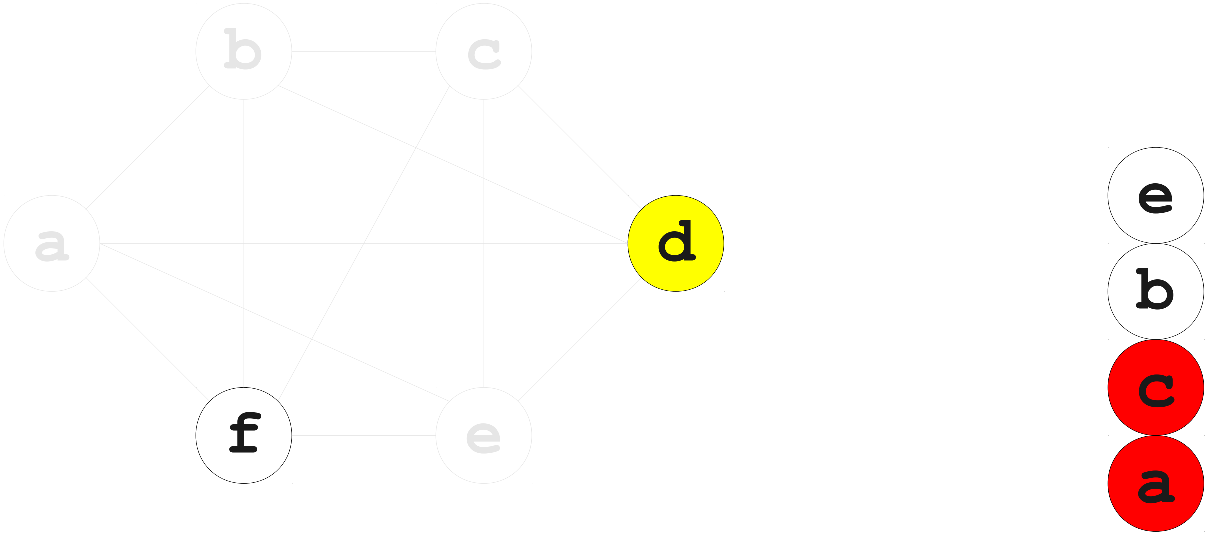
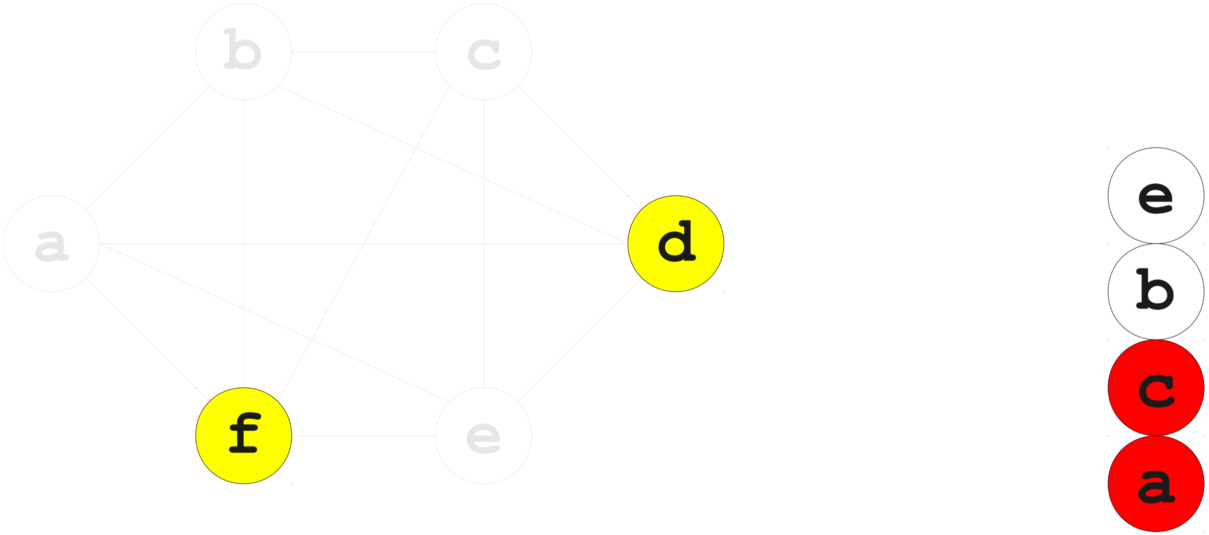


Registers

R_0	R_1	R_2

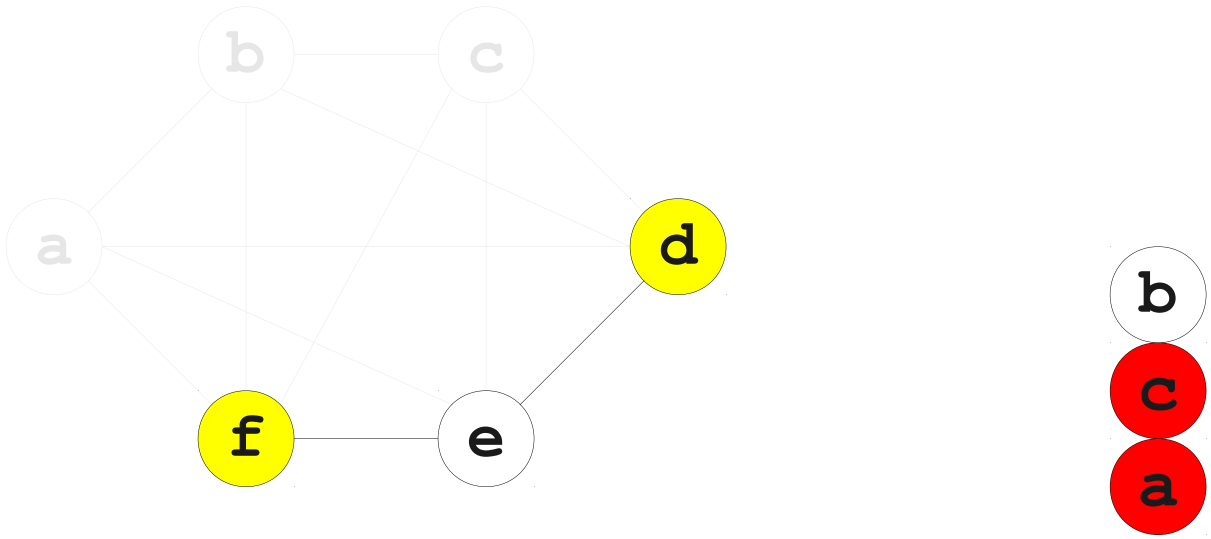
Another Example



Registers

R$_0$	R$_1$	R$_2$

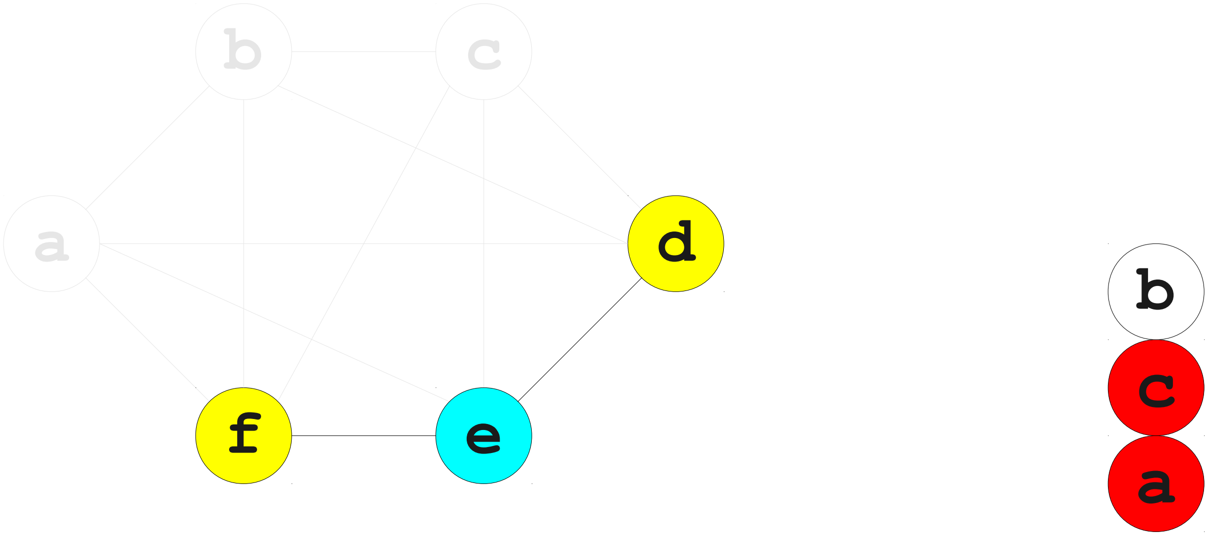
Another Example



Registers

R_0	R_1	R_2

Another Example



Registers

R_0	R_1	R_2

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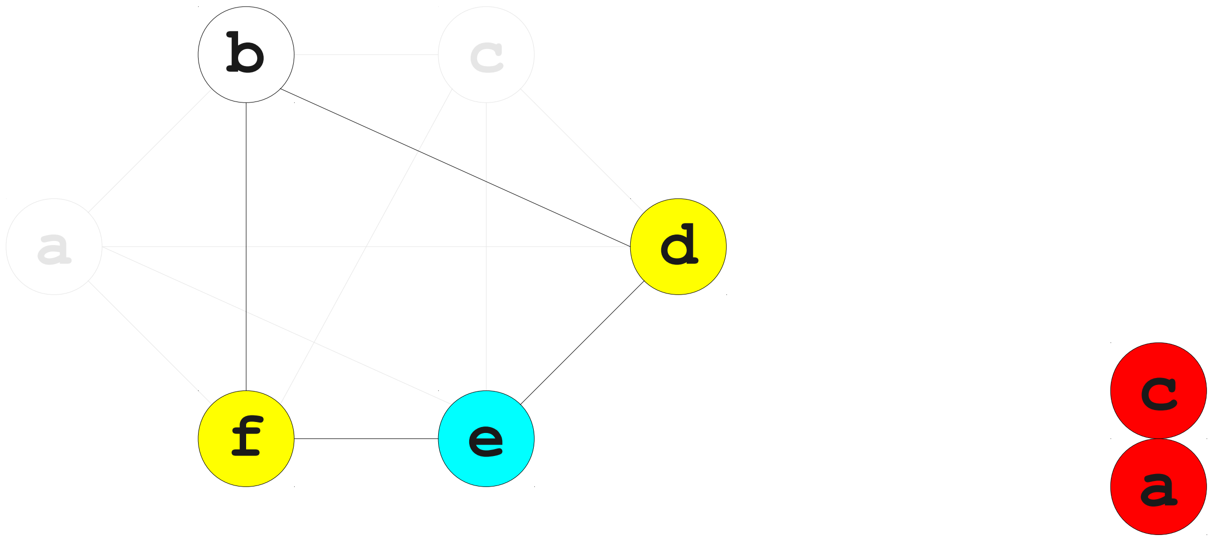
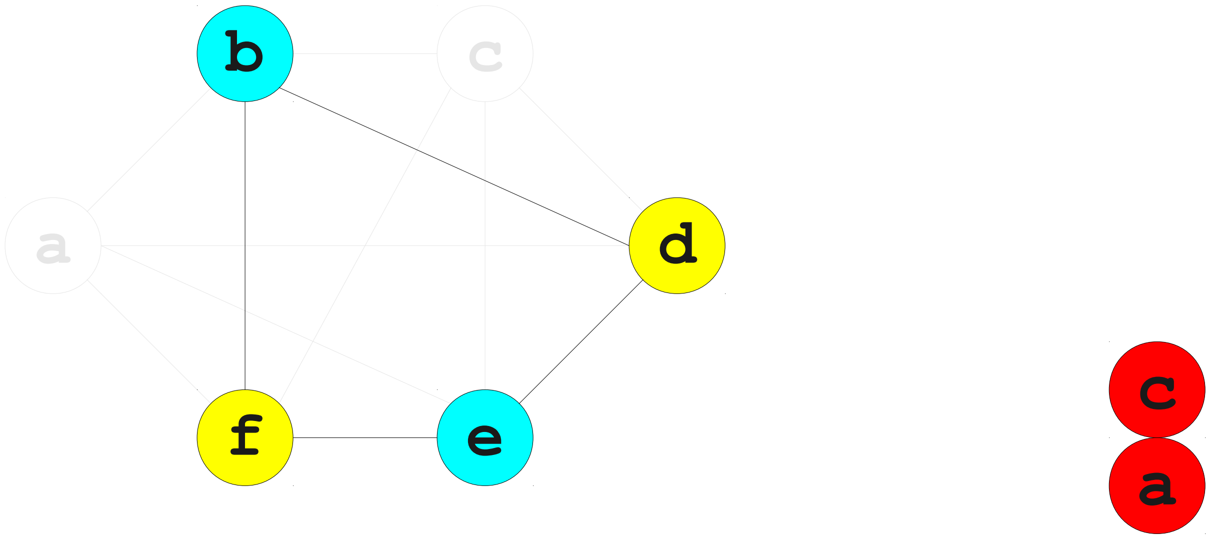


Registers

R_0	R_1	R_2

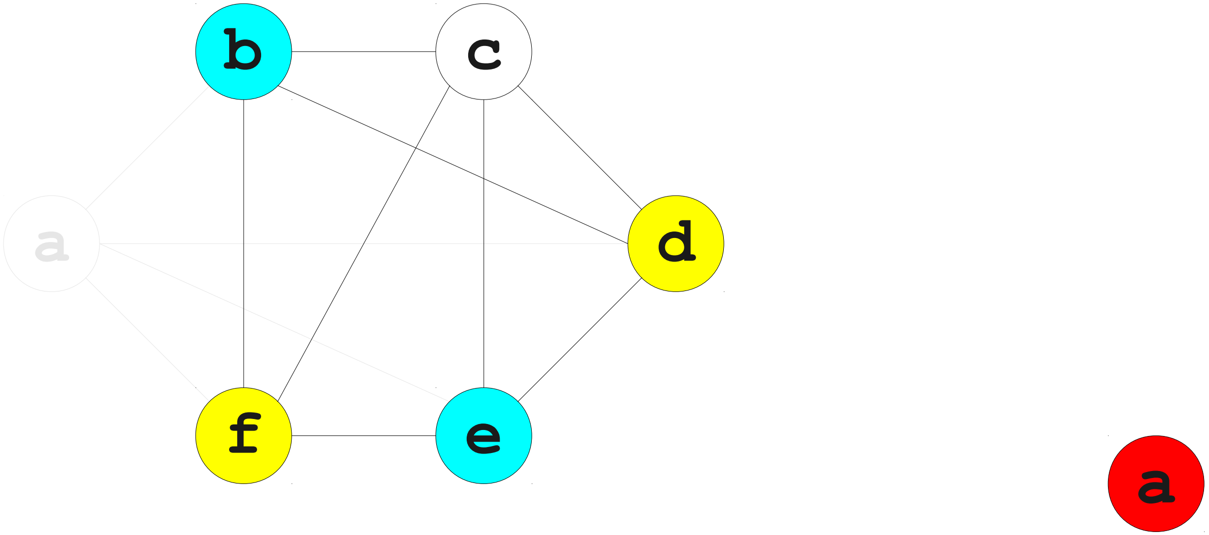
Another Example



Registers

R_0	R_1	R_2

Another Example



Registers

R_0	R_1	R_2

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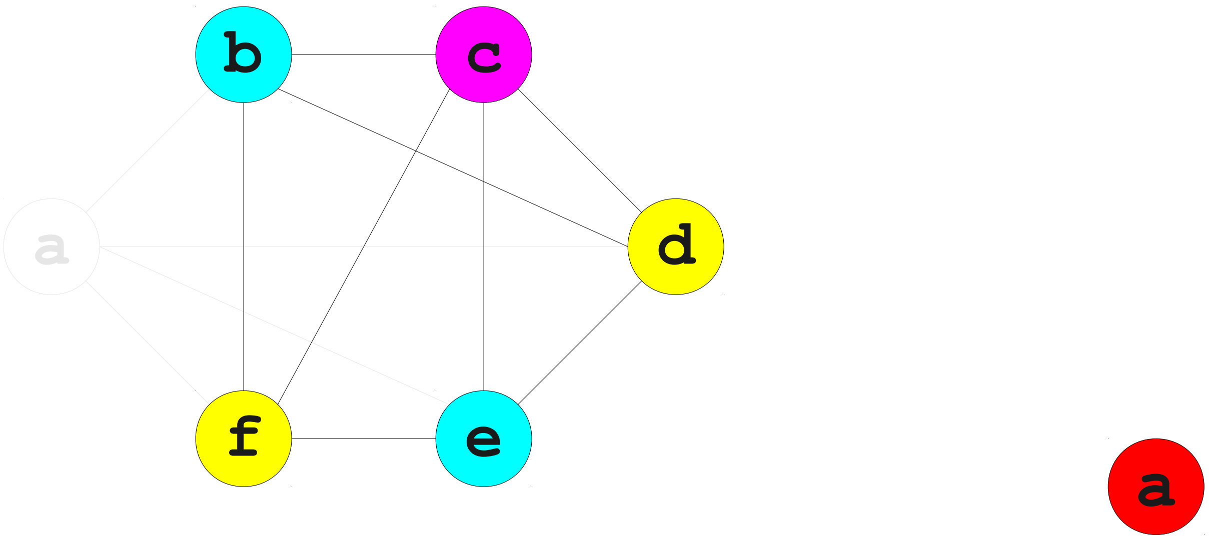

Registers

R_0	R_1	R_2

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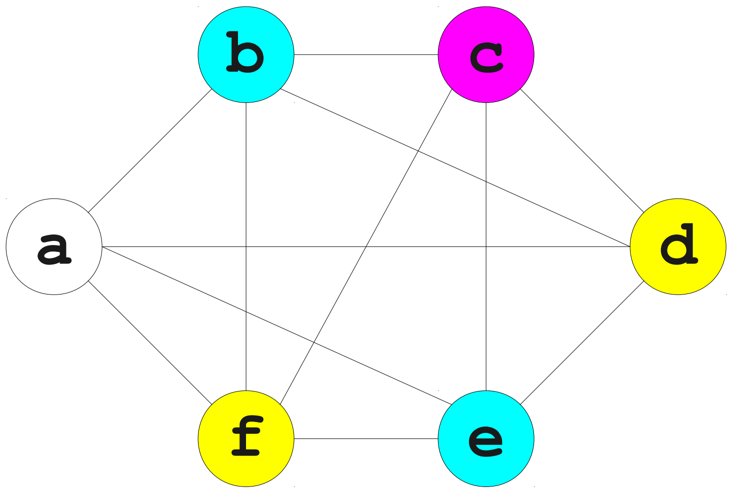

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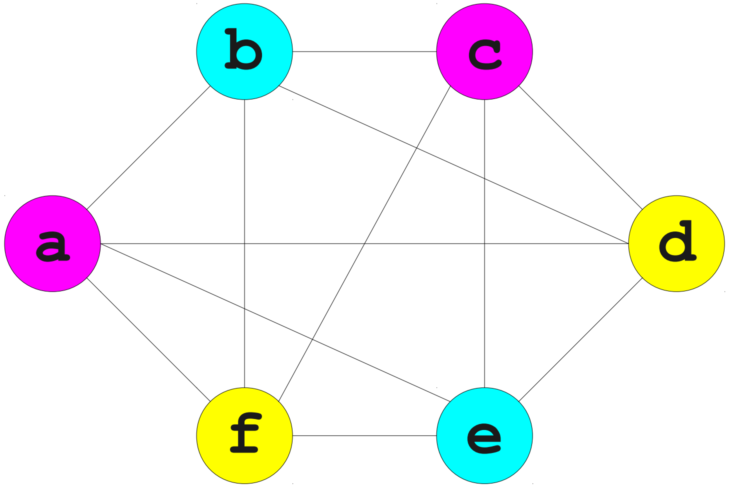
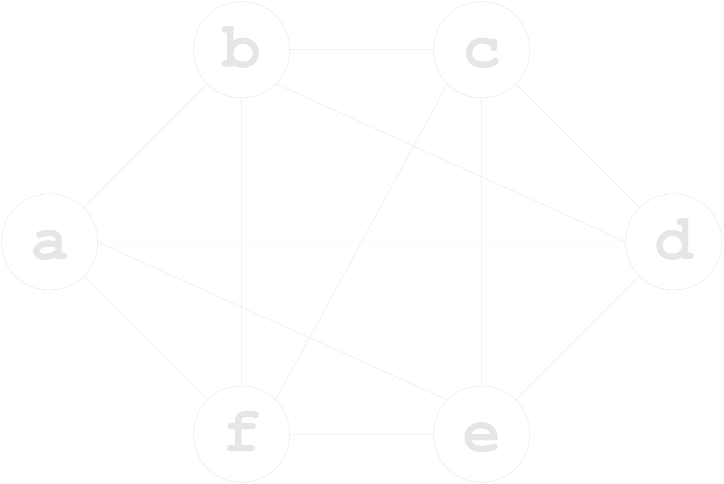
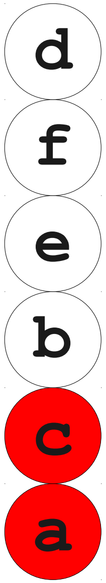


Registers

R_0	R_1	R_2

Another Example





Registers

R_0	R_1	R_2

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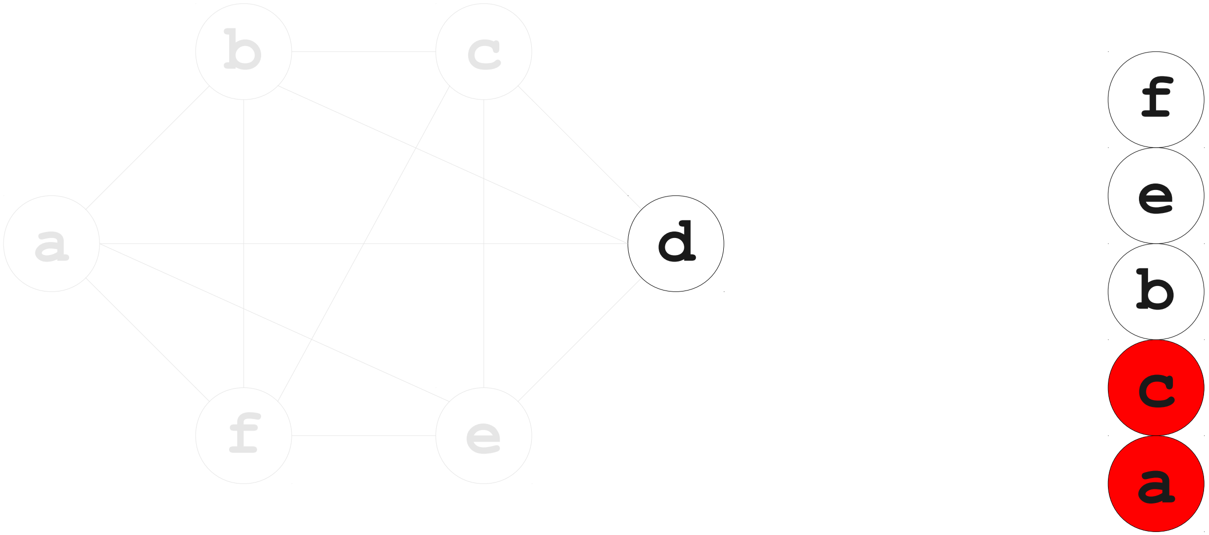

Registers

R_0	R_1	R_2

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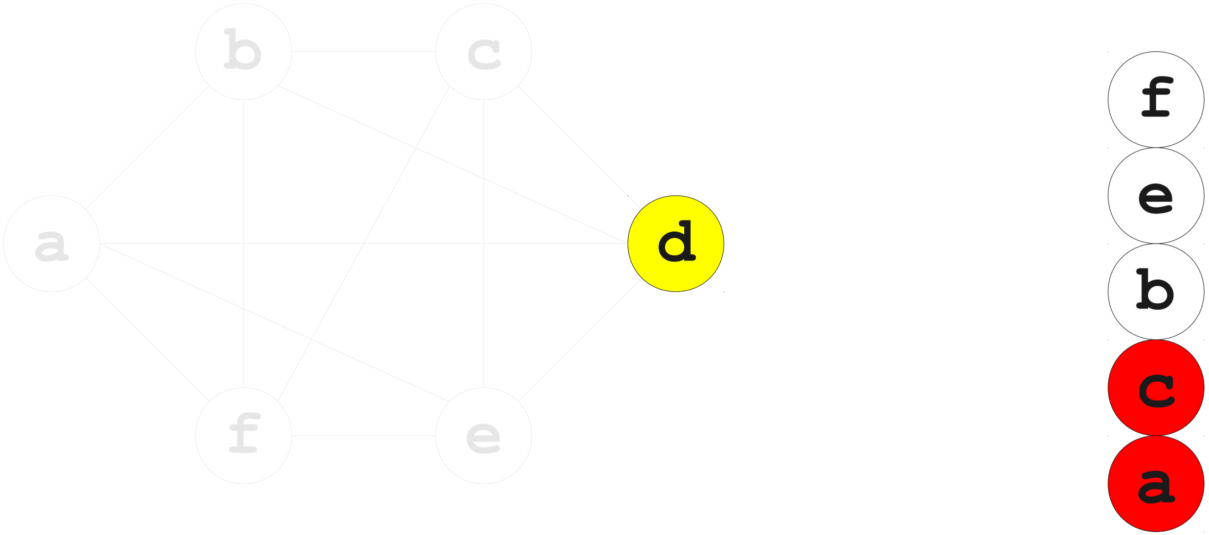

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R_0	R_1	R_2

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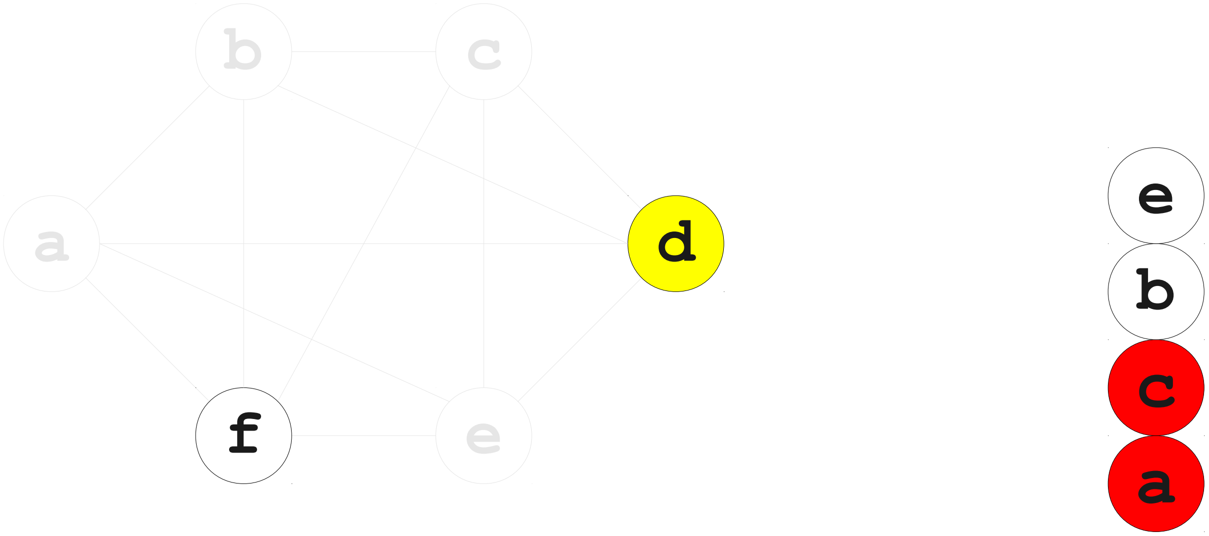
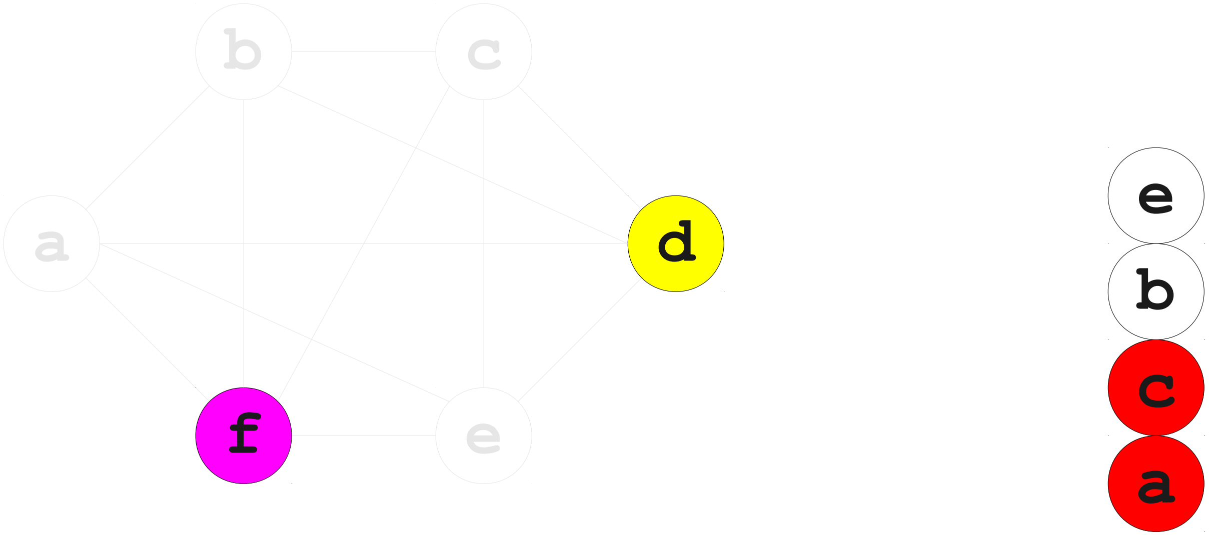


Registers

R_0	R_1	R_2

Another Example



Registers

R_0 R_1 R_2

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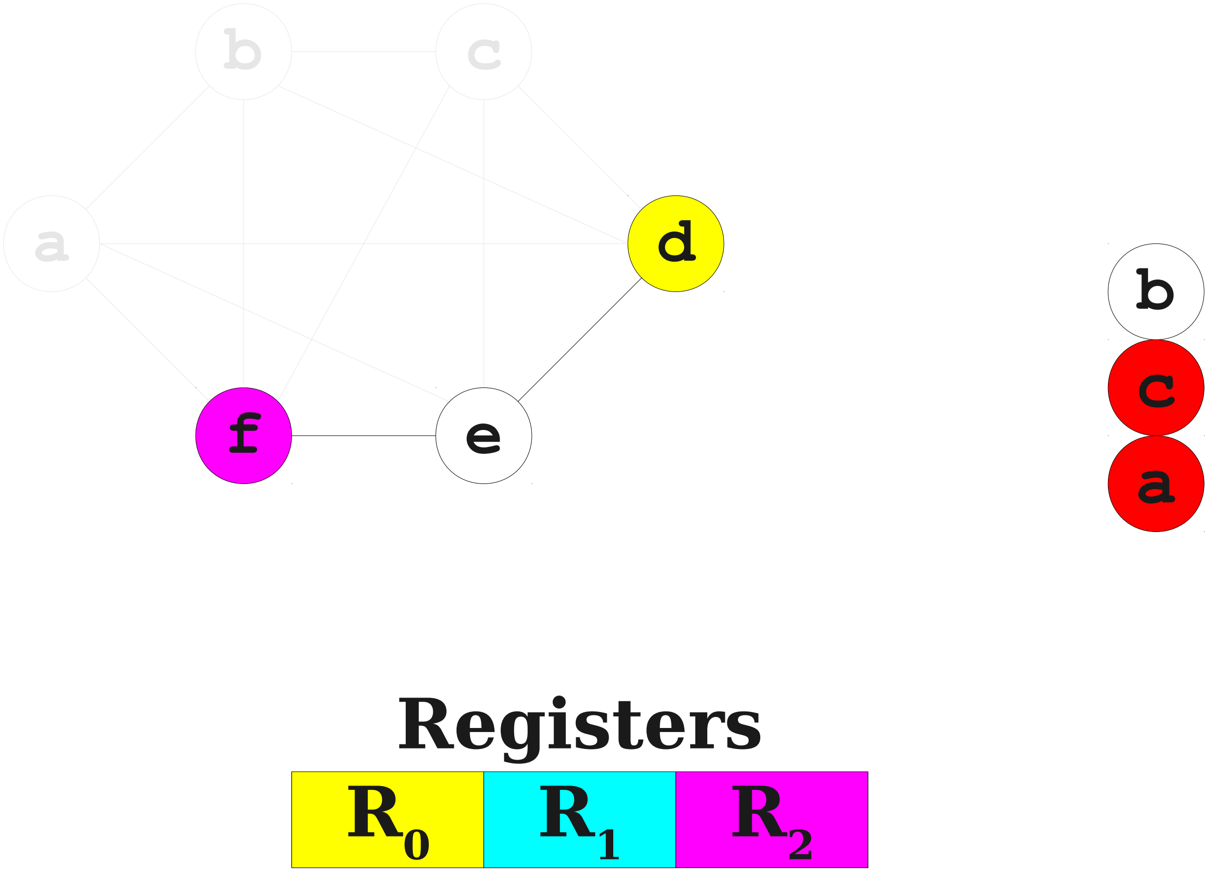

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R_0 R_1 R_2

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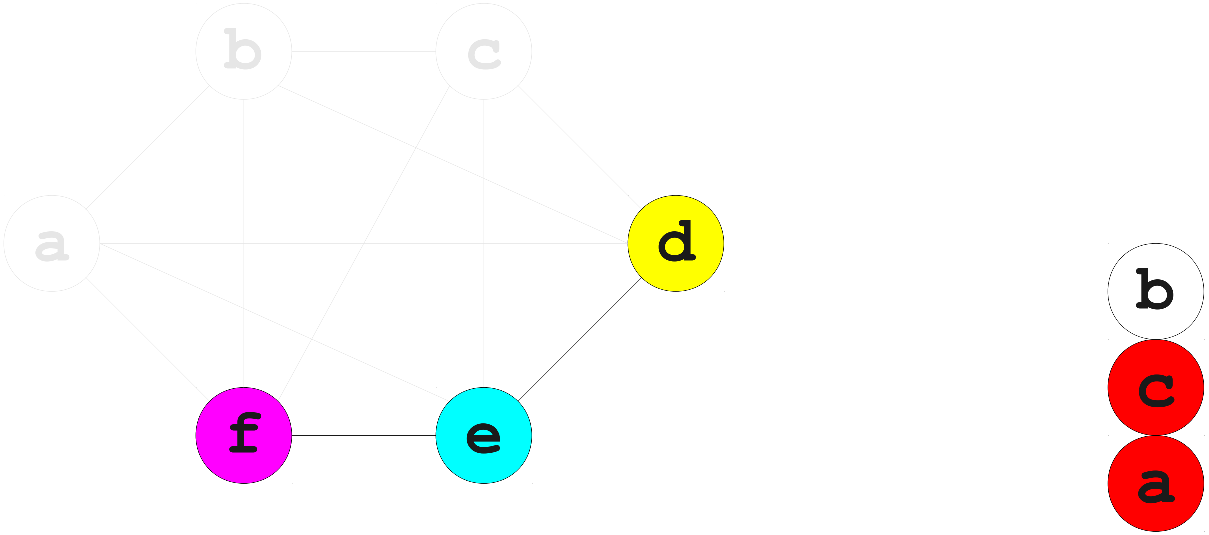

Registers

R_0	R_1	R_2

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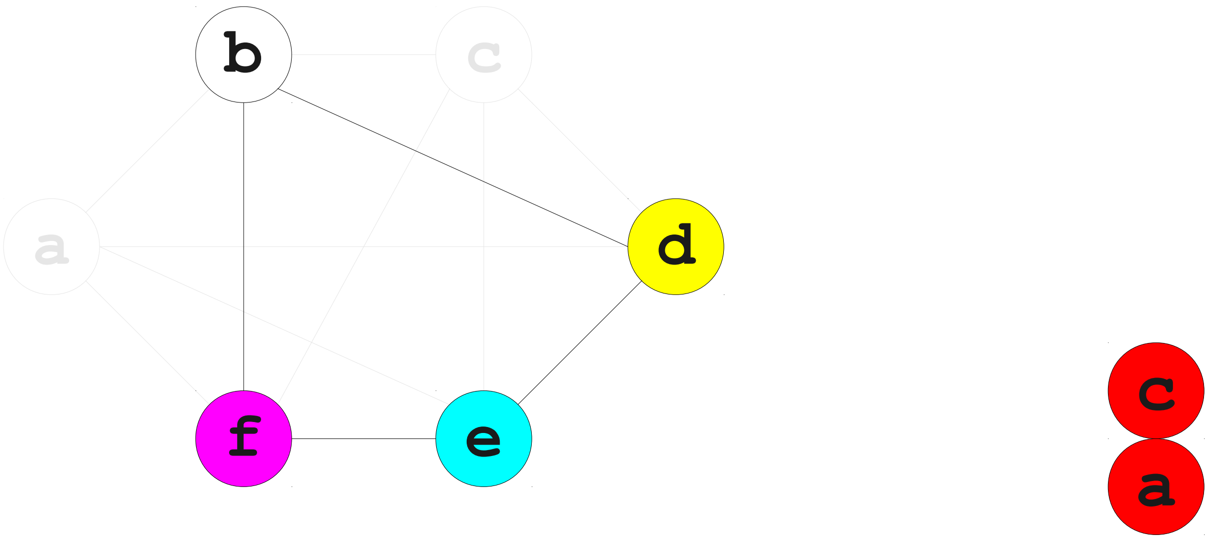

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R_0	R_1	R_2

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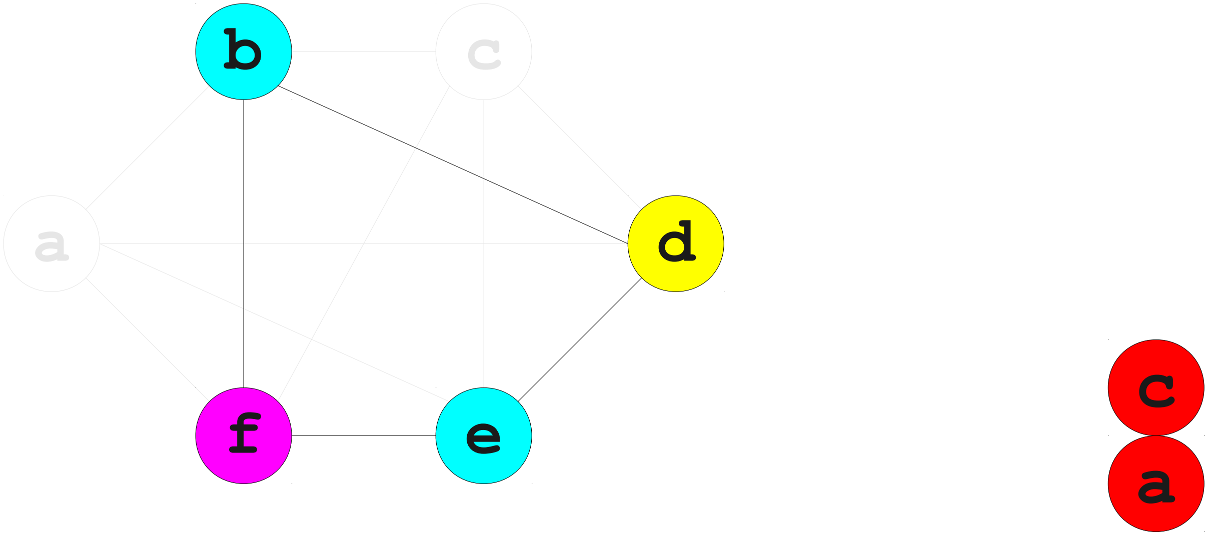

Registers

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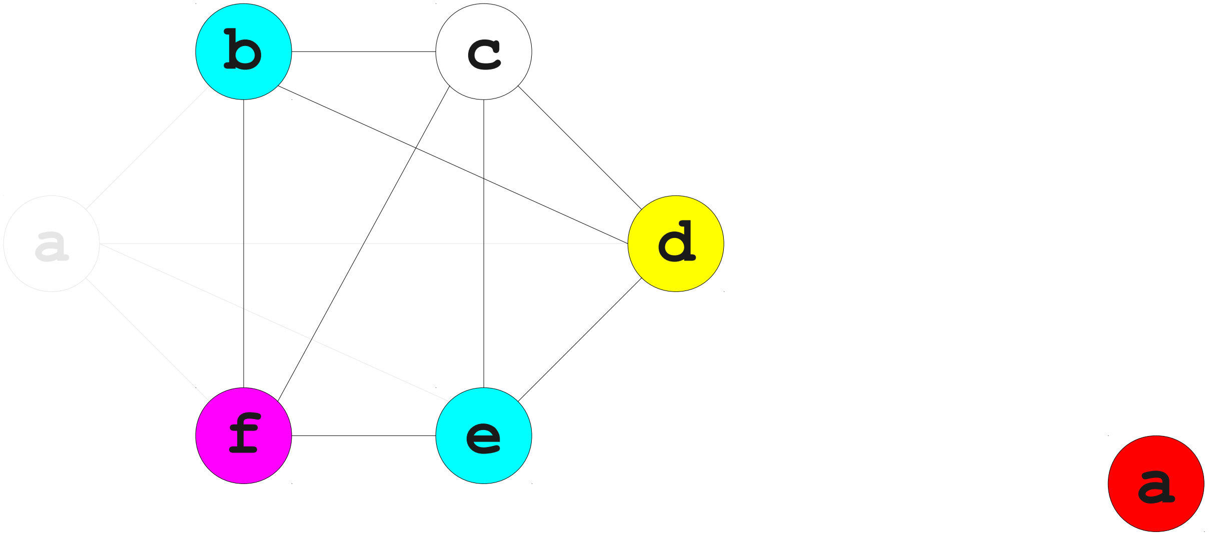

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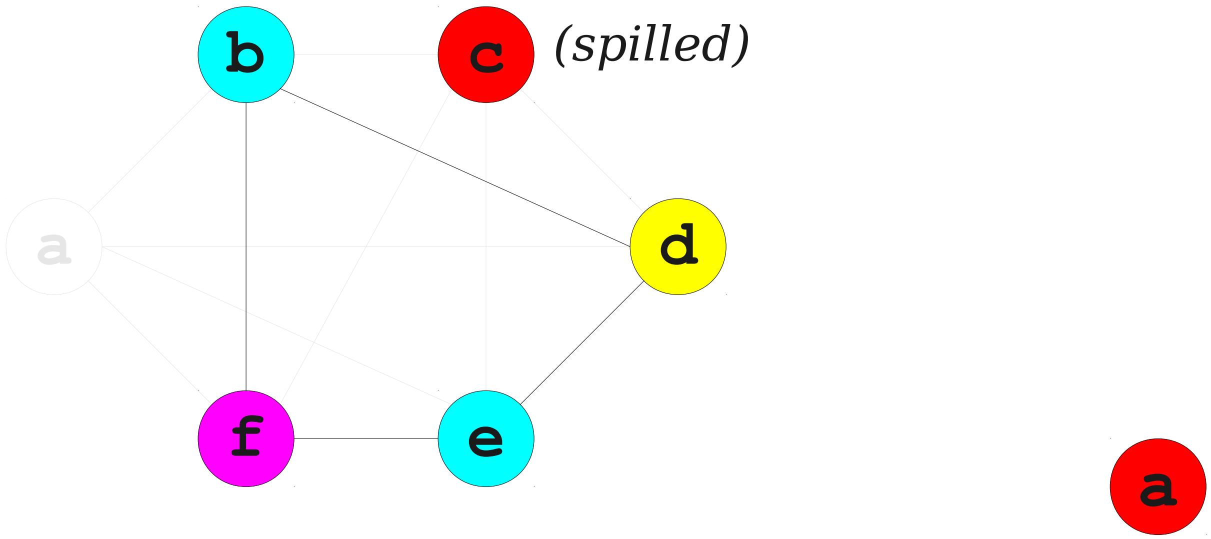

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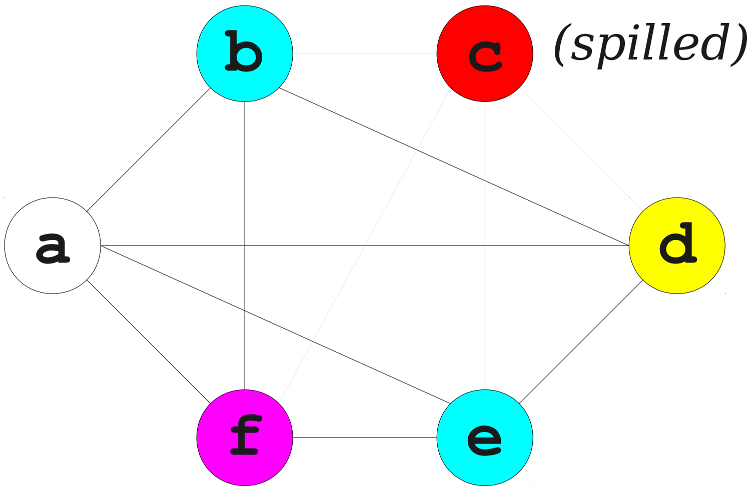

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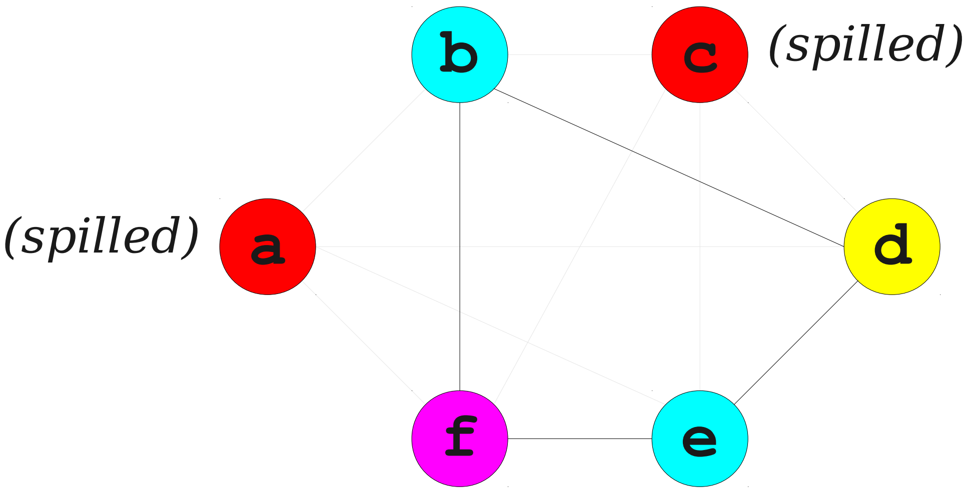

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Improvements on Spilling

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1. Use heuristics to decide which variables to spill

- least frequently used variable

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- least frequently used variable

2. Instead of always keeping the spilled variable on the stack, break the live range of the variable up by introducing new temporaries and reconstruct the conflict graph.

- more nodes but fewer edges

Implementing Reg Allocation

1. For each function definition, we’ll run liveness, conflict analysis and register allocation, producing a mapping from variable names to registers/stack offsets.

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- Function prelude/epilogue
- Function calls