

Compiling with Call-by-push-value

Max S. New

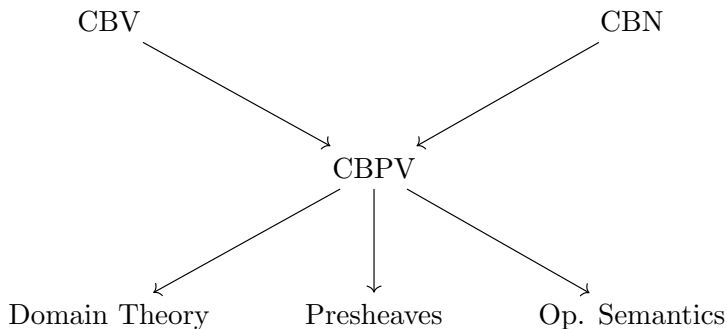
University of Michigan

MFPS 2023

Special Session on Semantics and Compilers

Overview of CBPV

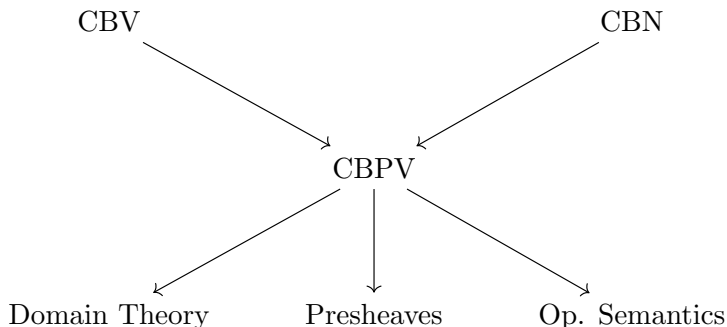
Paul Blain Levy introduced **Call-by-push-value** as a *subsuming paradigm* for effectful computation



- Preserves equational theories

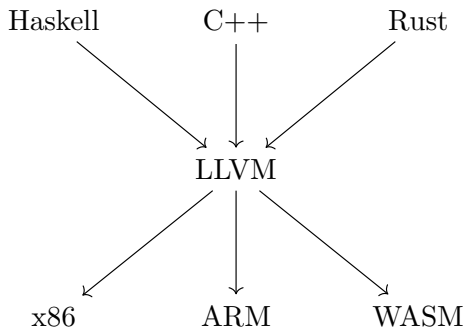
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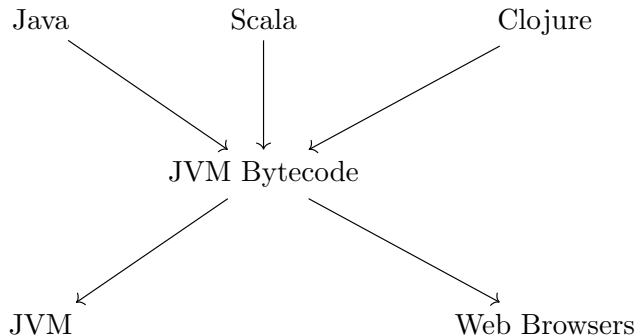


- Preserves equational theories
- Observation: *Denotational models* of CBV/CBN *naturally* decompose into CBPV structure. Semantics of CBPV is *easier* even though it's *more general*

Intermediate Representations



Language Platforms



Compare: Racket, .NET,

CBPV as an IR or Language Platform?

- 1 As an IR: CBPV structure arises in compilation

CBPV as an IR or Language Platform?

- 1 As an IR: CBPV structure arises in compilation
- 2 CBV, CBN embeddings in CBPV preserve and reflect equational theories:
Foundation for a language platform for *verified* language implementations that preserve *reasoning* (equality, logics) not just *whole-program behavior*?

Outline

- 1 Call-by-push-value Overview
- 2 CBPV subsumes Functional IRs
 - CBPV subsumes ANF, MNF
 - Stack-Passing Style subsumes CPS
- 3 Equality-Preserving Compiler Passes in CBPV/SPS
 - Polymorphic Closure Conversion
 - Polymorphic CPS Conversion
- 4 Computation/Stack Types in Compilation
 - Calling Conventions as Types
 - Relative Monads
- 5 Future Work

Call-by-push-value Overview

Basics of CBPV

Refine Moggi's analysis of effects using monads in terms of *adjunctions*
Effectful computation naturally involves two *kinds* of types:

- 1 Value types: the types of pure data, first-class values
- 2 Computation types: the types of effectful computations

Basics of CBPV

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Three notions of term

- 1 Pure functions $\Gamma \vdash V : A$
- 2 Effectful functions $\Gamma \vdash M : B$
- 3 Linear functions aka "Stacks" $\Gamma \mid z : B \vdash L : B'$

Basics of CBPV

Value Types, Values

$$\begin{aligned} A, A' &::= UB \mid \text{Bool} \\ V, V' &::= x \mid \text{thunk } M \\ &\quad \text{true} \mid \text{false} \\ &\quad (V, V') \mid V.\pi_i \end{aligned}$$

A value *is*

- A UB is a “thUnked” B
- A Bool is either true or false

Computation Types, Computations

$$\begin{aligned} B, B' &::= FA \mid A \rightarrow B \\ M, M' &::= z \mid \text{force } V \\ &\quad \text{if } V \ M \ M' \\ &\quad \text{ret } V \\ &\quad \text{let } x \leftarrow M; M' \\ &\quad \lambda x. M \mid M \ V \\ &\quad \text{prints}; M \\ &\quad \text{read } x. M \end{aligned}$$

A computation *does*

- An FA “Feturns” A values
- An $A \rightarrow B$ pops an A , continues as B

Equations in CBPV

Every type has associated $\beta\eta$ equality rules

$$\text{force thunk } M = M \qquad (V : UB) = \text{thunk force } V$$

$$(\lambda x.M)V = M[V/x] \qquad (M : A \rightarrow B) = \lambda x.Mx$$

$$\text{let } x \leftarrow \text{ret } V; N = M[V/x] \qquad N[M : FA/z] = \text{let } z \leftarrow M; N$$

And linear terms are *homomorphisms* of effect operations:

$$M[\text{print } s; N/z] = \text{print } s; M[N/z]$$

$$M[\text{read } x.N/z] = \text{read } x.M[N/z]$$

CBPV Reconstructs CBV and CBN

CBV term $\Gamma \vdash M : A$ becomes

$$\Gamma^{cbv} \vdash M^{cbv} : FA^{cbv}$$

“CBV terms are always returning”

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CBN terms $x_1 : B_1, \dots \vdash M : B$
become

$$x_1 : UB_1^{cbn}, \dots \vdash M^{cbn} : B^{cbn}$$

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CBPV subsumes Functional IRs

A-Normal Form, Monadic Normal Form

A-Normal Form:

Values $::= x \mid \lambda x.M \mid \text{true} \mid \text{false}$
Operations $O ::= \text{ret } V \mid \text{if } V M M' \mid V V' \mid \text{print } s \mid \text{read}$
Terms $M ::= O \mid \text{let } x \leftarrow O; M'$

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Monadic Normal Form:

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With equational theories as well. Every MNF term is equal in the theory to an ANF term.

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Monadic Normal Form:

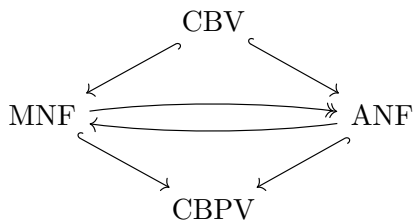
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With equational theories as well. Every MNF term is equal in the theory to an ANF term.

Observe: this is isomorphic a “full” subset of CBPV where the only computation type is FA and $A \rightarrow A'$ is given $\beta\eta$ rules corresponding to $U(A \rightarrow FA')$.

“Fine-grained CBV”, see Levy, Power and Thielecke, Information and Computation 2003.

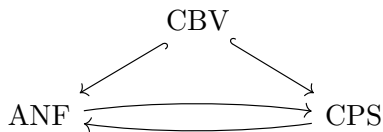
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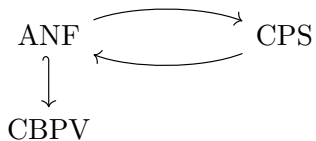
ANF is Equivalent to Continuation Passing Style

A-normal form was introduced in Sabry and Felleisen *Reasoning about Programs in Continuation-Passing Style* Lisp & F.P. 1992.

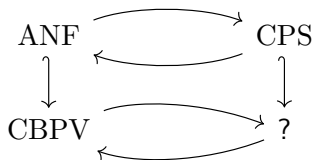
Conversion to A-normal form is equivalent to CPS conversion followed by “unCPS” .



ANF : CPS as CBPV : ?



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Stack-Passing Style: The Opposite of CBPV

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- 2 *Stack* types: the type of the stack a computation runs against

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Three notions of term

- 1 Values $\Gamma \vdash V : A$
- 2 Stacks, i.e., linear values $\Gamma \mid z : B \vdash S : B'$
- 3 Computations, $\Gamma \mid z : B \vdash M$

With “obvious” substitution principles.

Stack-Passing Style: The Opposite of CBPV

Value Types, Values

$$\begin{aligned} A, A' &::= \overset{p}{\neg} B \mid \text{Bool} \\ V, V' &::= x \mid \lambda z. M \mid \text{true} \mid \text{false} \end{aligned}$$

A value *is*

- A $\overset{p}{\neg} B$ is a first class procedure that requires a B stack to run.
- A Bool is true or false.

Stack Types, Stacks

$$\begin{aligned} B, B' &::= \overset{k}{\neg} A \mid A \otimes B \\ S, S' &::= z \mid \lambda x. S \mid (V, S) \end{aligned}$$

A stack *is, linearly,*

- A $\overset{k}{\neg} A$ is a linear *kontinuation* for A values
- An $A \otimes B$ is an A pushed onto a B stack.

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Computations

$$\begin{aligned} M, M' &::= V(S) \mid S(V) \mid \text{if } V \ M \ M' \\ &\quad \text{let}(x, z) = S \ \text{in } M \\ &\quad \text{prints}; M \mid \text{read } x. M \end{aligned}$$

A computation *isn't* (no output)

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A stack *is, linearly,*

- A $\overset{k}{\neg} A$ is a linear *continuation* for A values
- An $A \otimes B$ is an A pushed onto a B stack.

CBPV to SPS and Back

$$\text{Bool}^{sps} = \text{Bool}$$

$$(UB)^{sps} = \overset{p}{\neg} B^{sps}$$

$$(A \rightarrow B)^{sps} = A^{sps} \otimes B^{sps}$$

$$(FA)^{sps} = \overset{k}{\neg} A^{sps}$$

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Linear duality!

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CBPV and SPS as Flavors of Linear Logic

Different “flavors” of linear logic based on the allowed sequents

$$\Gamma \mid \Delta \vdash M : \Delta'$$

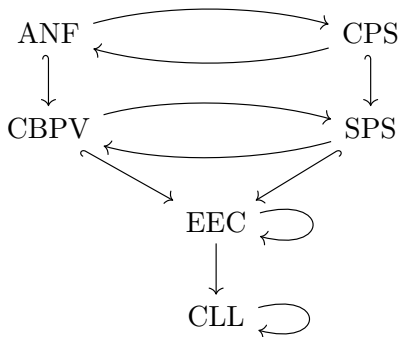
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Calculus	Allowed $ \Delta $	Allowed $ \Delta' $
Enriched-Effect Calculus	$= 1$	$= 1$
Call-by-push-value	≤ 1	$= 1$
Stack-passing Style	$= 1$	≤ 1
Intuitionistic	$< \omega$	$= 1$
Co-Intuitionistic	$= 1$	$< \omega$
Classical	$< \omega$	$< \omega$

ANF-CPS Correspondence as Linear Duality



Equality-Preserving Compiler Passes in CBPV/SPS

Two “Polymorphic” Compiler Passes

- *Typed Closure conversion*, Minamide, Morrisett and Harper, POPL '96

$$(A \rightarrow A')^{cc} = \exists X. X \times (X, A \rightarrow_{code} A')$$

- *Polymorphic Continuation Passing style*

$$(A \rightarrow A')^{cps} = \forall X. A, (A' \rightarrow X) \rightarrow X$$

From control effects to typed continuation passing, Thielecke, POPL '03

Both passes are type preserving, equivalence preserving*.

Polymorphic Closure Conversion

Target architectures don't have built in support for closures, need to implement them as a pair of an environment and a *code pointer*.

$$(A \multimap A')^{cc} = \exists X. X \times (X, A \multimap_{code} A')$$

(For equality preservation: need quotient/parametricity)

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In SPS, the closures are the *procedures*:

$$(\overset{p}{\multimap} B)^{cc} = \exists X : \text{ValTy}. X \times \overset{code}{\multimap} (X \otimes B^{cc})$$

Polymorphic CPS Conversion

Target architectures only support jumps, not calls with return, need to pass continuations as arguments.

To support arbitrary calls, functions must pass return continuations as arguments.

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In SPS, FA becomes $\overset{k}{\neg}A$ the *linear continuations*:

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In the dual “polymorphic CPS” is “polymorphic closure conversion” of continuations!

Does Polymorphic CPS Conversion Preserve Equivalence?

Ahmed and Blume, ICFP '11: polymorphic CPS does not preserve equivalence in CBV evaluation order:

$$\Lambda X. \lambda x : 1, k : (\text{Bool} \rightarrow X). y \leftarrow k(\text{true}); k(\text{false})$$

Polymorphic but still “abuses” the continuation.

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$$\begin{aligned} ((A \rightarrow A')^{cps})^{cbv} &= (\forall X. A^{cps}, (A'^{cps} \rightarrow X) \rightarrow X)^{cbv} \\ &= \forall X : \text{ValTy}. A^{cps, cbv} \rightarrow U(A'^{cps, cbpv} \rightarrow FX) \rightarrow FX \\ &\not\cong \forall R : \text{CompTy}. A^{cps, cbv} \rightarrow U(A'^{cps, cbpv} \rightarrow R) \rightarrow R \end{aligned}$$

Computation/Stack Types in Compilation

(Stack-based) Calling Conventions as Computation Types

$$A_1, \dots, A_n \rightarrow A'$$

(Stack-based) Calling Conventions as Computation Types

- 1 Left-to-right

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$$A_0 \rightarrow A_1 \rightarrow \dots \rightarrow \forall R. \text{CODE}(A' \rightarrow R) \rightarrow R$$

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- ④ Caller-cleanup (cdecl)

$$\forall R. \text{CODE}(A' \rightarrow A_0 \rightarrow A_1 \rightarrow \dots \rightarrow R) \rightarrow A_0 \rightarrow A_1 \rightarrow \dots \rightarrow R$$

(Stack-based) Calling Conventions as Stack Types

Can dualize the same translations to SPS:

$$A_1, \dots, A_n \multimap A'$$

e.g.,

$$A_0 \otimes A_1 \otimes \dots \exists S. \overset{\text{code}}{\multimap} (A' \otimes S) \otimes S$$

Compare: Stack-based calling conventions in *Stack-Based Typed Assembly Language* Morrisett, Kravy, Glew and Walker JFP 2002

Monads

A monad T in λ calculus is an operation on types T with

$$\eta : B \rightarrow TB' \qquad -^* : (B \rightarrow TB') \rightarrow (TB \rightarrow TB')$$

satisfying 3 equations.

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Example: “error monad”

$$TA = E + A$$

Good for equational reasoning, but not a good model of how exceptions are *implemented*. Monads for effects fundamentally *conflate* two aspects: TA is a *first class value* representing a *computation that can run*.

Relative Monads

A *relative monad*¹² in CBPV consists of a type constructor

$$\text{Eff} : \text{ValTy} \rightarrow \text{CompTy}$$

with operations

$$\eta : A \rightarrow \text{Eff } A \qquad \frac{x : A \vdash N : \text{Eff } A'}{z : \text{Eff } A \vdash x \leftarrow^{\text{Eff}} z; N : \text{Eff } A'}$$

satisfying 3 equations.

¹Altenkirch, Chapman and Uustalu, LMCS 2015

²Relative to F , or to the profunctor of computations

Relative Exception Monads

Naïve implementation:

$$F(A + E)$$

Double barreled continuations:

$$\forall R. U(A \rightarrow R) \rightarrow U(E \rightarrow R) \rightarrow R$$

Double barreled code pointers:

$$\forall R. \text{CODE}(A \rightarrow R) \rightarrow \text{CODE}(E \rightarrow R) \rightarrow R$$

Relative Exception Monads

Stack-walking exception³:

¹Caveat: Need to restrict to well-behaved elements to get a monad

²Caveat: need to restrict to a well-behaved subset to get a monad

Relative Exception Monads

Stack-walking exception³:

$$\text{Exn } E \text{ } A \cong F(A + E)$$

$$\&(\forall X : \text{ValTy}. U(A \rightarrow \text{Exn } E \text{ } X) \rightarrow \text{Exn } E \text{ } X)$$

$$\&\forall X : \text{ValTy}. U(E \rightarrow \text{Exn } X \text{ } A) \rightarrow \text{Exn } X \text{ } A$$

¹Caveat: Need to restrict to well-behaved elements to get a monad

²Caveat: need to restrict to a well-behaved subset to get a monad

Relative Exception Monads

Stack-walking exception³:

$$\begin{aligned}\text{Exn } E A &\cong F(A + E) \\ &\&(\forall X : \text{ValTy}. U(A \rightarrow \text{Exn } E X) \rightarrow \text{Exn } E X) \\ &\&\forall X : \text{ValTy}. U(E \rightarrow \text{Exn } X A) \rightarrow \text{Exn } X A\end{aligned}$$

Easier to see as the dual in SPS:

$$\begin{aligned}\text{Exn } E A &\cong \overset{k}{\neg}(A + E) \\ &\oplus (\exists X : \text{ValTy}. U(A \otimes \text{Exn } E X) \otimes \text{Exn } E X) \\ &\oplus (\exists X : \text{ValTy}. U(E \otimes \text{Exn } X A) \otimes \text{Exn } X A\end{aligned}$$

(Caveat: need to quotient to get a monad)

¹Caveat: Need to restrict to well-behaved elements to get a monad

²Caveat: need to restrict to a well-behaved subset to get a monad

Future Work

Future: Beyond The Stack, Beyond Sequentiality

- ① Only have *stack*-based calling conventions in CBPV proper. Can registers be incorporated in a similarly well-behaved type theory?

Future: Beyond The Stack, Beyond Sequentiality

- 1 Only have *stack*-based calling conventions in CBPV proper. Can registers be incorporated in a similarly well-behaved type theory?
- 2 CBPV gives a foundation for sequential composition, can we combine CBPV with Intuitionistic/Classical LL to similarly analyze IRs for concurrent/parallel code?

WIP: Implementation

- 1 Zydeco, a CBPV Surface Language + Polymorphism
<https://github.com/zydeco-lang/zydeco>
- 2 Surface language where we can experiment with writing code using new abstractions like relative monads.
- 3 Ongoing work on a backend using a CBPV IR
- 4 Extend to Dependent CBPV, compile Dependent CBPV...

CBPV as an IR

- CBPV structure arises naturally in compilation
- Foundation for verified equality preserving compilation
- Computation/Stack types useful for typing low-level programming idioms
- An implementation called Zydeco in progress:
<https://github.com/zydeco-lang/zydeco>

BONUS: Relative Monads in SPS

A *relative monad* in SPS consists of a type constructor

$$\text{Not} : \text{ValTy} \rightarrow \text{StkTy}$$

with operations

$$x : A \mid z : \text{Not } A \vdash \text{call}(z, x) \qquad \frac{x : A \mid z : \text{Not } A' \vdash M}{z : \text{Not } A' \vdash \lambda^{\text{Not}}_x.M : \text{Not } A}$$

satisfying 3 equations.